Me 'n' Mine™

X
CBSE

Answer Book

Pullout Worksheets
Mathematics

NEW SARASWATI HOUSE

Second Floor, MGM Tower, 19 Ansari Road, Daryaganj, New Delhi-110002 (India)
Phone : +91-11-43556600
Fax : +91-11-43556688
E-mail : delhi@saraswatihouse.com
Website : www.saraswatihouse.com
CIN : U22110DL2013PTC262320
Import-Export Licence No. 0513086293

Branches:

- Ahmedabad (079) 22160722 • Bengaluru (080) 26619880, 26676396
- Bhopal +91-7554003654 • Chennai (044) 28416531 • Dehradun 09837452852
- Guwahati (0361) 2457198 • Hyderabad (040) 42615566 • Jaipur (0141) 4006022
- Jalandhar (0181) 4642600, 4643600 • Kochi (0484) 4033369 • Kolkata (033) 40042314
- Lucknow (0522) 4062517 • Mumbai (022) 28737050, 28737090
- Nagpur +91-7066149006 • Patna (0612) 2275403 • Ranchi (0651) 2244654

Revised Edition 2018

TM Code: 2017007000073

Published by: New Saraswati House (India) Pvt. Ltd.
19 Ansari Road, Daryaganj, New Delhi-110002 (India)

Printed at: Vikas Publishing House Pvt. Ltd., Sahibabad (Uttar Pradesh)

Product Code: NSS3PCS106MATAA17CBN

CONTENTS

Chapter 1

REAL NUMBERS

WORKSHEET – 1

1. We know that the factors of a prime are 1 and the prime itself only.

Therefore, the common factor of p and q will be 1 only. Hence, HCF $(p, q) = 1$.

2. As prime factors of 1005 are:

$1005 = 5 \times 3 \times 67$.

$\therefore$ 7 is not a prime factor of 1005.

3. $\frac{125}{2^4 \cdot 5^3} = \frac{5^3}{16 \times 5^3} = \frac{1}{16} = 0.0625$

Clearly, the decimal form of $\frac{125}{2^4 \cdot 5^3}$ terminates after four places.

4. Terminating

Hint: $\frac{24}{125} = \frac{192}{1000} = 0.192$.

5. $\text{LCM} = \frac{\text{First number} \times \text{Second number}}{\text{HCF}}$

$= \frac{96 \times 404}{4} = 24 \times 404 = 9696.$

6. (*i*) 660 (*ii*) 330

Hint: Going in opposite direction to the factor tree, we obtain

$2 \times 165 = 330$ ***(ii)*** and $2 \times 330 = 660$ ***(i)***.

7. HCF = 3; LCM = 420

Hint: $12 = 2^2 \times 3$; $15 = 3 \times 5$; $21 = 3 \times 7$.

8. (*i*) Terminating

Hint: $\frac{543}{250} = \frac{543}{2^1 \times 5^3}$.

(*ii*) Non-terminating repeating

Hint: $\frac{9}{108} = \frac{1}{12} = \frac{1}{2^2 \times 3^1}$.

9. Hint: Let $5 - 2\sqrt{3} = \frac{a}{b}$; $b \neq 0$

$\Rightarrow \quad \sqrt{3} = \frac{5b - a}{2b}$

As RHS of this equation is rational, but LHS is irrational so a contradiction.

10. Let a be any odd positive integer and $b = 4$. By Euclid's lemma there exist integers q and r such that

$a = 4q + r, \; 0 \leq r < 4$

$\therefore \quad a = 4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$.

Therefore, for a to be odd, we have to take

$a = 4q + 1$ or $4q + 3$.

11. The maximum capacity (in kg) of a bag will be the HCF of 490, 588 and 882. Let us find out the required HCF by prime factorisation method.

$490 = 2 \times 5 \times 7^2$

$588 = 2^2 \times 3 \times 7^2$

$882 = 2 \times 3^2 \times 7^2$

$\therefore \quad \text{HCF} = 2 \times 7^2 = 98$

Thus, the maximum capacity of a bag is 98 kg.

WORKSHEET – 2

1. Smallest composite number = 4

Smallest prime number = 2

HCF of 4 and 2 = 2.

2. Going to opposite direction to the factor tree, we obtain

$3 \times 7 = 21$ (*ii*) and $2 \times 21 = 42$ (*i*).

3. $\sqrt{2} = 1.414...$ and $\sqrt{3} = 1.732...$

Therefore, we can take $1.5 = \frac{3}{2}$

as $\sqrt{2} < \frac{3}{2} < \sqrt{3}$.

4. Required number $= \frac{23 \times 1449}{161}$

$= \frac{1449}{7} = 207.$

5. Hint: As 12576 > 4052

$\therefore \quad 12576 = 4052 \times 3 + 420$

Further $\quad 4052 = 420 \times 9 + 272$

Further $\quad 420 = 272 \times 1 + 148$

Further $272 = 148 \times 1 + 124$
Further $148 = 124 \times 1 + 24$
Further $124 = 24 \times 5 + 4$
Further $24 = 4 \times 6 + 0$.
In the last equation, remainder is zero.
Hence, the required HCF = 4.

6. First given number is composite as
$5 \times 3 \times 11 + 11 = 11\ (15 + 1) = 11 \times 16$
$= 11 \times 2 \times 8$
But second given number is prime as
$5 \times 7 + 7 \times 3 + 3 = 35 + 21 + 3 = 59$.

7. No. Prime factors of 6^n will be of type $2^n \times 3^n$. As it doesn't have 5 as a prime factor, so 6^n can't end with the digit 5.

8. **Hint:** Let a be any positive integer
$\therefore$ $a = 3q$ or $3q + 1$ or $3q + 2$
$\therefore$ $a^2 = 9q^2 = 3m;\ m = 3q^2$
or $a^2 = (3q + 1)^2 = 3m + 1,\ m = q\ (3q + 2)$
or $a^2 = (3q + 2)^2 = 3m + 1,\ m = 3q^2 + 4q + 1$.

9. We represent 6,72 and 120 in their prime factors.
$6 = 2 \times 3$
$72 = 2^3 \times 3^2$
$120 = 2^3 \times 3 \times 5$
Now, HCF $= 2 \times 3 = 6$
And LCM $= 2^3 \times 3^2 \times 5 = 360$.

10. **Hint:** Let $\sqrt{2} - \sqrt{5} = x$, a rational number
$\Rightarrow$ $\sqrt{2} = x + \sqrt{5}$
Squaring both sides, we get
$2 = x^2 + 5 + 2x\sqrt{5}$
$\Rightarrow$ $\sqrt{5} = \dfrac{-x^2 - 3}{2x}$
RHS of this last equation is rational, but LHS is irrational which is a contradiction.

11. Number of members in group 1 (army group) = 308
Number of members in group 2 (army band) = 24
Greatest number which divides both 308, 24 = HCF (308, 24)
$\therefore$ Consider
$308 = 24 \times 12 + 20$
$24 = 20 \times 1 + 4$
$20 = 4 \times 5 + 0$

$\because$ 24) 308 (12
24
68
48
20

$\therefore$ HCF of (308, 24) = 4.
$\therefore$ Maximum number of column in which the two group can march = 4.

WORKSHEET – 3

1. $\dfrac{43}{2^4 \times 5^3} = \dfrac{43 \times 5}{(2 \times 5)^4} = \dfrac{215}{10^4} = 0.0215$
Hence, the number terminates after four places of decimal.

2. LCM $= \dfrac{45 \times 105}{15}$
LCM = 315.

3. $128 = 2^7$; $240 = 2^4 \times 3 \times 5$.
Now, HCF (128, 240) $= 2^4 = 16$.

4. $1.033 = \dfrac{1033}{1000}$
$\therefore$ $1000 = 2^3 \times 5^3$

2	1000
2	500
2	250
5	125
5	25
5	5
	1

5. No.
Hint: Prime factors of 15^n does not contain $2^p \times 5^q$ in factor, p, q being positive integers.

6. Rational number = 0.27
Irrational number = 0.26010010001... .

7. (*i*) $\dfrac{145}{625} = \dfrac{29}{125} \times \dfrac{8}{8} = \dfrac{232}{1000} = 0.232$.

(*ii*) $\dfrac{7}{80} \times \dfrac{125}{125} = \dfrac{875}{10000} = 0.0875$.

8. Let us assume, to the contrary that $\sqrt{2}$ is rational. We can take integers a and $b \neq 0$ such that
$\sqrt{3} = \dfrac{a}{b}$, where a and b are coprime.
$\Rightarrow$ $3b^2 = a^2$
$\Rightarrow$ a^2 is divisible by 3
$\Rightarrow$ a is divisible by 3 ...(*i*)
We can write $a = 3c$ for some integer c
$\Rightarrow$ $a^2 = 9c^2$
$\Rightarrow$ $3b^2 = 9c^2$ $(\because a^2 = 3b^2)$
$\Rightarrow$ $b^2 = 3c^2$
$\Rightarrow$ b^2 is divisible by 3
$\Rightarrow$ b is divisible by 3 ...(*ii*)
From (*i*) and (*ii*), we observe that a and b have atleast 3 as a common factor. But

this contradicts the fact that a and b are co-prime. This means that our assumption is not correct.

Hence, $\sqrt{3}$ is an irrational number.

9. As: $1032 = 408 \times 2 + 216$ ...(*i*)

$408 = 216 \times 1 + 192$ ...(*ii*)

$216 = 192 \times 1 + 24$ ...(*iii*)

$192 = 24 \times 8 + 0$ ...(*iv*)

$\Rightarrow$ HCF = 24

$\therefore$ From (*iii*)

$\Rightarrow$ $24 = 216 - 192$

$= 216 - [408 - 216]$ [Use (*ii*)]

$= 2 \times 216 - 408$

$= 2[1032 - 2 \times 408] - 408$ [Use (*i*)]

$\Rightarrow$ $24 = 1032 \times 2 - 5 \times 408$

$\Rightarrow$ $m = 2$.

10. Hint: Let x be any positive integer.

Then it is of the form $3q$ or $3q + 1$ or $3q + 2$.

If $x = 3q$, then

$x^3 = (3q)^3 = 9m;\ m = 3q^3$

If $x = 3q + 1$, then

$x^3 = (3q + 1)^3$

$= 9m + 1;\ m = q(3q^2 + 3q + 1)$.

If $x = 3q + 2$, then

$x^3 = (3q + 2)^3$

$= 9m + 8;\ m = q(3q^2 + 6q + 4)$.

11. The maximum number of columns must be the highest common factor (HCF) of 616 and 32. Let us find out the HCF by the method of Euclid's division lemma.

Since 616 > 32, we apply division lemma to 616 and 32, to get

$616 = 32 \times 19 + 8$

Since the remainder $8 \neq 0$, we apply the division lemma to 32 and 8, to get

$32 = 8 \times 4 + 0$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 8, the HCF of 616 and 32 is 8

Hence, the maximum number of columns is 8.

WORKSHEET – 4

1. Non-terminating repeating.

Hint: Denominator is not in the exact form of $2^m \times 5^n$, where m, n are non-negative integers.

2. $\sqrt{\frac{49}{147}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$, which is therefore an irrational number.

3. $(\sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5}) = (\sqrt{6})^2 - (\sqrt{5})^2$

$= 6 - 5 = 1$ = Rational number.

4. Terminating decimal form as denominator 4 of $\frac{107}{4}$ is of the form $2^n \times 5^m$.

Here $n = 2$, $m = 0$

5. (*i*) 1001 (*ii*) 91

Hint: 7 × 13 = (*ii*) and (*ii*) × 11 = (*i*).

6. Let us represent each of the numbers 30, 72 and 432 as a product of primes.

$30 = 2 \times 3 \times 5$

$72 = 2^3 \times 3^2$

$432 = 2^4 \times 3^3$

Now, HCF = $2 \times 3 = 6$

and LCM = $2^4 \times 3^3 \times 5 = 2160$.

7. Here, 396 > 82.

$\therefore$ $396 = 82 \times 4 + 68$

Further $82 = 68 \times 1 + 14$

Further $68 = 14 \times 4 + 12$

Further $14 = 12 \times 1 + 2$

Further $12 = 2 \times 6 + 0$

In the last equation, the remainder is zero and the divisor is 2.

Hence, the required HCF = 2.

8. Hint: Let $3 + 2\sqrt{5} = \frac{a}{b};\ b \neq 0$

$\Rightarrow$ $\frac{a - 3b}{2b} = \sqrt{5}$ = Rational

Which is a contradiction as $\sqrt{5}$ is an irrational number.

9. As: $4 = 2^2$

$12 = 2^2 \times 3$

$20 = 2^2 \times 5$

$\therefore$ LCM (4, 12, 20) $= 2^2 \times 3 \times 5$

$= 4 \times 3 \times 5$

$= 60$

∴ 60 is least number which is exactly divisible by 4, 12, 20.

Hence, all the alarm clocks will ring together after 60 min.

10. The required number of students will be the highest common factor (HCF) of 312, 260 and 156. Let us find out the HCF by the method of prime factorisation.

$$312 = 2^3 \times 3 \times 13$$
$$260 = 2^2 \times 5 \times 13$$
$$156 = 2^2 \times 3 \times 13$$
$$\therefore \quad \text{HCF} = 2^2 \times 13 = 52$$

Number of buses required

$$= \frac{\text{Total number of students}}{\text{Number of students in one bus}}$$
$$= \frac{312+260+156}{52} = 14$$

Thus, the maximum number of students in a bus and number of buses required are 52 and 14 respectively.

11. Hint: Let x = any positive integer

$x = 5m, 5m+1, 5m+2, 5m+3$ or $5m+4$

Now take square of all these form.

WORKSHEET – 5

1. Let the quotient is m when n^2-1 is divided by 8.

$\therefore \quad n^2-1 = 8\times m$

$\Rightarrow \quad n^2-1 = 0, 8, 16, 24, 32, \ldots$.

$\Rightarrow \quad n^2 = 1, 9, 17, 25, 33, \ldots$

$\Rightarrow \quad n = \pm 1, \pm 3, \pm\sqrt{17}\ \pm 5, \pm\sqrt{33}, \ldots$

$\therefore \quad n$ = An odd integer is the right Answer.

2. 2

Hint: HCF (65, 117) = 13

Now, $65m - 117 = 13$.

$\therefore \quad m = 2$ will satisfy this equation.

3. **Hint:** LCM of 18, 24, 30, 42 = 2520

∴ Required number = 2520 + 1 = 2521.

4. Prime factors of numbers 1 to 10 are:

$1 = 1$; $2 = 1 \times 2$; $3 = 1 \times 3$; $4 = 1 \times 2^2$

$5 = 1 \times 5$; $6 = 1 \times 2 \times 3$; $7 = 1 \times 7$;

$8 = 1 \times 2^3$; $9 = 1 \times 3^2$; $10 = 1 \times 2 \times 5$

Now,

$\text{LCM} = 1\times 2^3 \times 3^2 \times 5 \times 7$

$= 8\times 9 \times 5 \times 7 = 2520$ is required number.

5. 2.

Hint: $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 2x - \sqrt{15}$

$\Rightarrow \quad 4-\sqrt{15} = 2x - \sqrt{15}$

$\Rightarrow \quad x = 2$, which is a rational number.

6. Hint: Any odd positive integer will be type of $4q + 1$ or $4q + 3$

$$\therefore \quad (4q+1)^2 = 16q^2 + 8q + 1$$
$$= 8(2q^2+q)+1$$
$$= 8n + 1$$

$$\text{Also,} \quad (4q+3)^2 = 16q^2+24q+9$$
$$= 8(2q^2+3q+1)+1$$
$$= 8n+1.$$

7. 35 cm

Hint: Find HCF.

8. Hint: Let $\sqrt{5}-3\sqrt{2} = \dfrac{a}{b}$

where a, b are integers and $b \neq 0$

Squaring on both sides,

$$5 + 18 - 6\sqrt{10} = \frac{a^2}{b^2}$$
$$\Rightarrow \quad 23 - \frac{a^2}{b^2} = 6\sqrt{10}$$
$$\Rightarrow \quad \frac{23b^2-a^2}{6b^2} = \sqrt{10} \text{ ... a contradiction.}$$

9. (*i*) Terminating (*ii*) Terminating.

10. The required number of burfis will be the highest common factor of 420 and 130.

Let us find out the HCF using Euclid's division lemma.

It is clear that 420 > 130. We apply Division lemma to 420 and 130, to get

$$420 = 130 \times 3 + 30$$

Since the remainder $30 \neq 0$, so we apply Division lemma to 130 and 30, to get

$$130 = 30 \times 4 + 10$$

Again the remainder $10 \neq 0$, so we apply

Division lemma to 30 and 10, to get

$$30 = 10 \times 3 + 0$$

Now, the remainder is zero. So the HCF of 420 and 130 is the divisor at the last stage that is 10.

Hence, the required number of burfis is 10.

11. If possible let a is such an integer for which $\sqrt{a+1}+\sqrt{a-1}$ is rational

$$\therefore \quad \sqrt{a+1}+\sqrt{a-1} = \frac{p}{q};\ p, q \text{ are integers and } q \neq 0 \quad ...(i)$$

$$\Rightarrow \quad \frac{q}{p} = \frac{1}{\sqrt{a+1}+\sqrt{a-1}}$$

$$= \frac{\sqrt{a+1}-\sqrt{a-1}}{(\sqrt{a+1}+\sqrt{a-1})(\sqrt{a+1}-\sqrt{a-1})}$$

$$= \frac{\sqrt{a+1}-\sqrt{a-1}}{(a+1)-(a-1)}$$

$$= \frac{\sqrt{a+1}-\sqrt{a-1}}{2}$$

$$\Rightarrow \quad \frac{2q}{p} = \sqrt{a+1}-\sqrt{a-1} \quad ...(ii)$$

Adding (i) and (ii) $\Rightarrow \frac{p}{q}+\frac{2q}{p} = 2\sqrt{a+1}$

also $(i) - (ii)$ $\Rightarrow \frac{p}{q}-\frac{2q}{p} = 2\sqrt{a-1}$

$$\Rightarrow \quad \sqrt{a+1} = \frac{p^2+2q^2}{2qp} = \text{a rational number}$$

$$\text{and} \quad \sqrt{a-1} = \frac{p^2-2q^2}{2qp} = \text{a rational number}$$

$\Rightarrow$ $\sqrt{a+1}$ and $\sqrt{a-1}$ are both rational number

$\Rightarrow$ $a + 1$ and $a - 1$ are perfect square of positive integers.

This is not possible as any two perfect squares differs at least by 3.

Hence our assumption was wrong.

$\Rightarrow$ there doesn't exist any positive integer a for which $\sqrt{a+1}+\sqrt{a-1}$ is rational.

WORKSHEET – 6

1. HCF × LCM $= x \times 18$

$$\Rightarrow \quad 36 \times 2 = 18x$$

$$\Rightarrow \quad 72 = 18x$$

$$\Rightarrow \quad x = \frac{72}{18}$$

$$\therefore \quad x = 4.$$

2. As p and $p+1$ are two consecutive natural numbers, HCF = 1 and

LCM = $p\,(p + 1)$.

3. Hint: The given number is $\frac{51}{1500}$ or $\frac{17}{500}$

$\therefore$ Denominator = 500 = $2^2 \times 5^3$

Clearly, the denominator is exactly in the form $2^m \times 5^n$, where m and n are non-negative integers; so the given number has a terminating decimal expansion.

4. 1800

Hint: $\because$ $8 = 2^3$; $9 = 3^2$; $25 = 5^2$

$\therefore$ HCF (8, 9, 25) = 1

LCM (8, 9, 25) = 1800.

5. –19

Hint: HCF (210, 55) = 5

$$\therefore \quad 210 \times 5 + 55y = 5$$

$$\Rightarrow \quad 55y = 5 - 1050$$

$$\Rightarrow \quad y = \frac{-1045}{55} = -19.$$

6. Irrational

Hint: $\frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{x}{\sqrt{3}}$

$$\Rightarrow \quad 7 - 4\sqrt{3} = \frac{x}{\sqrt{3}}$$

$$\Rightarrow \quad 7\sqrt{3} - 12 = x = \text{Irrational.}$$

7. Rational Number = 0.55

Irrational number = 0.5477477747... .

8. 15

Hint: HCF (1380, 1455, 1620) = 15.

9. (i) 0.052. (ii) 5.8352.

10. We know that any positive integer is either of the form $3q$, $3q + 1$ or $3q + 2$ for some integer q.

Now, three cases arise.

Case I. When $p = 3q$,

$p+2 = 3q + 2$ and $p + 4 = 3q + 4$

Here, $p = 3q$ is exactly divisible by 3

$p + 2 = 3q + 2$ leaves 2 as remainder when it is divided by 3

$p + 4 = 3q + 4$ or $3(q + 1) + 1$ leaves 1 as remainder when it is divided by 3.

Case II. When $p = 3q + 1$,

$p+2 = 3q + 3$ and $p + 4 = 3q + 5$

Here, $p = 3q+1$ leaves 1 as remainder when it is divided by 3

$p + 2 = 3q + 3$ or $3(q+1)$ is exactly divisible by 3

$p + 4 = 3q + 5$ or $3(q + 1) + 2$ leaves 2 as remainder when it is divided by 3.

Case III. When $p = 3q + 2$, $p + 2 = 3q + 4$ and $p + 4 = 3q + 6$

Here, $p = 3q+2$ leaves 2 as remainder when it is divided by 3.

$p+2 = 3q+4$ or $3(q+1)+1$ leaves 1 as remainder when it is divided by 3

$p + 4 = 3q + 6$ or $3(q + 2)$ is exactly divisible by 3.

Hence, in all the cases, one and only one number out of p, $p+2$ and $p+4$ is divisible by 3, where p is any positive integer.

OR

Any positive odd integer is type of $2q+1$ where q is a whole number.

$\therefore \quad (2q+1)^2 = 4q^2 + 4q + 1 = 4q(q+1)+1$ ...(*i*)

Now, $q(q+1)$ is either 0 or even

So, it is $2m$, where m is some number.

$\therefore$ From (*i*) $\Rightarrow (2q+1)^2 = 8m+1$.

11. Since, height of each stack is the same, therefore, the number of books in each stack is equal to the HCF of 96, 240 and 336.

Let us find their HCF

$96 = 2^4 \times 2 \times 3$

$240 = 2^4 \times 3 \times 5$

$336 = 2^4 \times 3 \times 7$

So, HCF $= 2^4 \times 3 = 48$.

Now, number of stacks of English books

$$= \frac{96}{48} = 2$$

Number of stacks of Hindi books

$$= \frac{240}{48} = 5$$

Number of stacks of Mathematics books

$$= \frac{336}{48} = 7.$$

WORKSHEET–7

1. HCF × LCM = Product of the two numbers.

$\Rightarrow \quad 9 \times \text{LCM} = 306 \times 657$

$$\Rightarrow \quad \text{LCM} = \frac{306 \times 657}{9} = 22338.$$

2. As given number can be written as 2525 which is product of prime numbers: $5 \times 5 \times 101$. Hence it is a composite number.

3. HCF × LCM = Product of the two numbers

$\Rightarrow \quad 40 \times 252 \times p = 2520 \times 6600$

$$\Rightarrow \quad p = \frac{2520 \times 6600}{40 \times 252} = 1650.$$

4. No; because HCF must divide LCM and here HCF = 18 which doesn't divide LCM which is 380.

5. As n is odd integer

$\Rightarrow$ $2n$ is even integer

also $2n + 1$ is odd integer

and $4n + 2$ is even integer

$\therefore \quad (-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$

$= -1 + 1 - 1 + 1 = 0$

because $(-1)^{\text{odd integer}} = -1$

and $(-1)^{\text{even integer}} = 1$.

6. The required number would be the HCF of $967 - 7 = 960$ and $2060 - 12 = 2048$.

Let us find the HCF of 960 and 2048 by using Euclid's algorithm.

Since $2048 > 960$

$\therefore \quad 2048 = 960 \times 2 + 128$

$960 = 128 \times 7 + 64$

$128 = 64 \times 2 + 0$

Since the remainder becomes zero and the divisor at this stage is 64, the HCF of 960 and 2048 is 64.

Hence, the required number is 64.

7.

2	456
2	228
2	114
3	57
19	19
	1

2	360
2	180
2	90
3	45
3	15
5	5
	1

Clearly, $456 = 2^3 \times 3 \times 19$

and $360 = 2^3 \times 3^2 \times 5$

$\therefore$ HCF $= 2^3 \times 3 = 24$

Hence, $\text{LCM} = \dfrac{456 \times 360}{24} = 6840.$

8. (*i*) Time taken by Ram to complete one cycle = 180 seconds.

Time taken by Shyam to complete one cycle = 150 seconds.

$\therefore$ Consider LCM of 180 and 150.

$\therefore$ $180 = 2^2 \times 5 \times 3^2$

$150 = 2 \times 5^2 \times 3$

$\therefore$ LCM of 180 and 150 $= 2^2 \times 5^2 \times 3^2$

$= 4 \times 25 \times 9$

$= 900$ seconds

$= \dfrac{900}{60} = 15$ minutes

$\therefore$ They both will again meet after 15 minutes.

(*ii*) Since they started at 6 a.m. and they will be meeting again after 15 minutes.

$\therefore$ The time will be 6:15 a.m.

(*iii*) L.C.M. of real numbers.

(*iv*) Since Ram and Shyam go for morning walk daily. So, it depicts their **discipline and health consciousness.**

9. Let a be any odd positive integer. Then, it is of the form $6p + 1$, $6p + 3$ or $6p + 5$.

Here, three cases arise.

Case I: When $a = 6p + 1$,

$\therefore$ $a^2 = 36p^2 + 12p + 1$

$= 6p(6p + 2) + 1 = 6q + 1,$

where $q = p(6p + 2)$.

Case II: When $a = 6p + 3$,

$\therefore$ $a^2 = 36p^2 + 36p + 9$

$= 36p^2 + 36p + 6 + 3$

$= 6(6p^2 + 6p + 1) + 3$

$= 6q + 3,$

where $q = 6p^2 + 6p + 1$.

Case III: When $a = 6p + 5$,

$\therefore$ $a^2 = 36p^2 + 60p + 25$

$= 36p^2 + 60p + 24 + 1$

$= 6(6p^2 + 10p + 4) + 1$

$= 6q + 1,$

where $q = 6p^2 + 10p + 4$.

Hence, a is of the form $6q + 1$ or $6q + 3$.

CHAPTER TEST

1. We know, HCF × LCM = $a \times b$

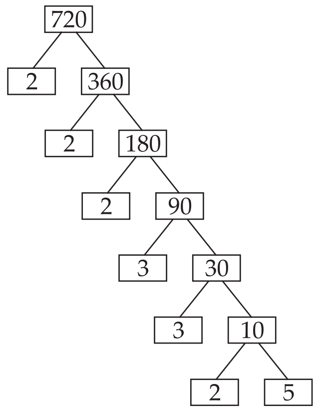
$15 \times \text{LCM} = 1800$

$\text{LCM} = \dfrac{1800}{15} = 120.$

2. Prime factorisation of 720

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 2^4 \times 3^2 \times 5.$



3. $\because$ $\text{LCM} = \dfrac{306 \times 1314}{18} = 22338.$

4. Yes.

$2 \times 3 \times 5 \times 13 \times 17 + 13$

$= 13 \times (2 \times 3 \times 5 \times 17 + 1)$

$= 13 \times 511$

= a composite number.

5. $\left(\sqrt{2} - \sqrt{9}\right)^2 = 2 - 2\sqrt{18} + 9$

$= 11 - 2\sqrt{18}$

= irrational.

6. No.

Hint: Prime factors of 9^n will be type of 3^{2n}, *i.e.*, $\underbrace{3 \times 3 \times \ldots \times 3}_{\text{Even no. of times.}}$

7. $\because \quad 0.56125 = \dfrac{56125}{100000} = \dfrac{449}{800}$

$$= \frac{449}{32 \times 25} = \frac{449}{2^5 \times 5^2}$$

$\because \quad 2^n \times 5^m = 2^5 \times 5^2$

$\therefore \quad n = 5, m = 2.$

8.
$$120 = 2^3 \times 3 \times 5$$
$$105 = 3 \times 5 \times 7$$
$$150 = 2 \times 3 \times 5^2$$

$\therefore \quad \text{HCF} = 3 \times 5 = 15$

And $\quad \text{LCM} = 2^3 \times 3 \times 5^2 \times 7$
$$= 8 \times 3 \times 25 \times 7$$
$$= 4200.$$

9. Hint:

Let $\quad \sqrt{2} - 3\sqrt{3} = x$, where x is rational.

$\Rightarrow \quad \left(\sqrt{2} - 3\sqrt{3}\right)^2 = x^2$

$\Rightarrow \quad 2 + 27 - 6\sqrt{6} = x^2$

$\Rightarrow \quad 29 - x^2 = 6\sqrt{6}$

$\Rightarrow \quad \dfrac{29 - x^2}{6} = \sqrt{6}.$

Since 6 is not a perfect square. So $\sqrt{6}$ is always irrational.

$\therefore$ It's a contradiction.

10. We know that any positive integer is of the form $3q$ or $3q + 1$ or $3q + 2$.

Case I: $\quad n = 3q$

$\Rightarrow \quad n^3 = (3q)^3 = 9 \times 3q^3 = 9m$

$\Rightarrow \quad n^3 + 1 = 9m + 1$, where $m = 3q^3$.

Case II: $\quad n = 3q + 1$

$\Rightarrow \quad n^3 = (3q+1)^3$
$$= 27q^3 + 1 + 27q^2 + 9q$$
$$= 9q\,(3q^2 + 3q + 1) + 1$$
$$= 9m + 1$$

$\Rightarrow \quad n^3 + 1 = 9m + 2$, where

$m = q(3q^2 + 3q + 1)$.

Case III: $\quad n = 3q + 2$

$\Rightarrow \quad n^3 = (3q+2)^3$
$$= 27q^3 + 8 + 54q^2 + 36q$$

$n^3 + 1 = 27q^3 + 54q^2 + 36q + 9$
$$= 9(3q^3 + 6q^2 + 4q + 1)$$
$$= 9m,$$

where $\quad m = 3q^3 + 6q^2 + 4q + 1$.

Hence, $n^3 + 1$ can be expressed in the form $9m$, $9m + 1$ or $9m + 2$, for some integer m.

11. (*i*) We will find HCF of 96 and 112 by using Euclid's lemma:

$\therefore \quad 112 = 96 \times 1 + 16$

and $\quad 96 = 16 \times 6 + 0$

$\Rightarrow$ the last divisor $= 16$

$\therefore \quad \text{HCF} = 16$

$\therefore$ The minimum number of boxes required

for apples $= \dfrac{96}{16} = 6$

and the minimum number of boxes

required for oranges $= \dfrac{112}{16} = 7$

$\therefore$ Total minimum number of boxes required $= 7 + 6 = 13$.

(*ii*) Concept used is HCF of two real numbers using Euclid's lemma.

(*iii*) **By distributing fruits in orphanage his kindness and concern towards the needful has been reflected.**

❑❑

Chapter 2

POLYNOMIALS

WORKSHEET – 9

1. $\because$ $D = b^2 - 4ac$

$= (-2)^2 - 4(-1)(3)$

$= 4 + 12 = 16 > 0$

$\Rightarrow$ $f(x)$ will have two distinct zeroes.

2. $\alpha + \beta = -\frac{7}{2}$

$\alpha\beta = \frac{5}{2}$

$\Rightarrow$ $\alpha + \beta + \alpha\beta = -\frac{7}{2} + \frac{5}{2} = \frac{-2}{2} = -1.$

3. -1

Hint: $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$.

4. Sum of zeroes of required polynomial (s)

$= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$= \frac{3+2}{3\cdot 2} = \frac{5}{6}.$

Product of zeroes (P) $= \frac{1}{\alpha}\cdot\frac{1}{\beta}$

$= \frac{1}{(3)(2)} = \frac{1}{6}$

$\therefore$ Required polynomial is

$P(x) = k.[x^2 - (S)x + P]$

$= k\left[x^2 - \frac{5}{6}x + \frac{1}{6}\right]$

where k is any non-zero real number

5. Sum of zeroes (S) $= -\frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{4}$

$= \frac{3-8}{4\sqrt{3}} = -\frac{5}{4\sqrt{3}}$

Product of zeroes (P) $= -\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{4} = -\frac{1}{2}$

Now, required polynomial will be

$x^2 - Sx + P$, *i.e.*, $x^2 + \frac{5}{4\sqrt{3}}x - \frac{1}{2}$

or $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$.

6. Let $f(x) = 2x^2 + 2ax + 5x + 10$

If $x + a$ is a factor of $f(x)$, then $f(-a) = 0$

Therefore, $2a^2 - 2a^2 - 5a + 10 = 0$

$\Rightarrow$ $a = 2.$

7. $x^3 - 4x^2 + x + 6$

Hint: If the zeroes are α, β and γ of a cubical polynomial, then the polynomial will be

$(x-\alpha)(x-\beta)(x-\gamma)$

$= (x-3)(x-2)(x+1) = x^3 - 4x^2 + x + 6.$

8. Solving $\alpha + \beta = 3$ and $\alpha - \beta = -1$,

we get $\alpha = 1, \beta = 2$

$\therefore$ Polynomial is $x^2 - (\alpha+\beta)x + \alpha\beta$

$\Rightarrow$ $p(x) = x^2 - 3x + 2.$

9. According to the division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow$ $x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$

(As given in question)

$\Rightarrow$ $g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$

To find $g(x)$, we proceed as given below.

$$\begin{array}{r|l} & x^2 - x + 1 \\ x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & \underline{-\quad +} \\ & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & \underline{+\quad -} \\ & x - 2 \\ & x - 2 \\ & \underline{-\quad +} \\ & 0 \end{array}$$

Thus, $g(x) = x^2 - x + 1.$

10. $-\frac{1}{3}$; $\frac{3}{2}$

Hint: $6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$

$= 3x(2x-3) + 1(2x-3)$

$= (2x-3)(3x+1)$

$2x - 3 = 0$ gives

$\therefore \quad x = \frac{3}{2}$

$3x + 1 = 0$ gives $x = -\frac{1}{3}$

$\therefore \quad \alpha + \beta = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-b}{a}$

$\therefore \quad \alpha \cdot \beta = \frac{-1}{3} \cdot \frac{3}{2} = \frac{-1}{2} = \frac{c}{a}.$

11. Let $p(x) = x^4 + x^3 - 34x^2 - 4x + 120$

Given zeroes of $p(x)$ are 2 and -2

$\therefore (x-2)(x+2) = x^4 - 4$ is a factor of $p(x)$.

We divide $p(x)$ by $x^2 - 4$,

$$
\begin{array}{r|l}
 & x^2 + x - 30 \\
x^2 - 4 & x^4 + x^3 - 34x^2 - 4x + 120 \\
 & \underline{\overset{-}{x^4} \qquad \overset{+}{-4x^2}} \\
 & x^3 - 30x^2 - 4x + 120 \\
 & \underline{\overset{-}{x^3} \qquad \overset{+}{-4x}} \\
 & -30x^2 + 120 \\
 & \underline{\overset{+}{-30x^2} \overset{-}{+ 120}} \\
 & 0
\end{array}
$$

$\therefore \quad p(x) = (x^2 - 4)(x^2 + x - 30)$

$\therefore$ Other zeroes of $p(x)$ are given by

$x^2 + x - 30 = 0$

$\Rightarrow \quad x^2 + 6x - 5x - 30 = 0$

$\Rightarrow \quad x(x+6) - 5(x+6) = 0$

$\Rightarrow \quad (x-5)(x+6) = 0$

$x = 5, -6$

Hence, all the zeroes are 2, -2, 5 and -6.

WORKSHEET – 10

1. Required quadratic polynomial

$= x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$

$= x^2 - 2\sqrt{3}x - 5\sqrt{3}$.

2. $f(x) = 3x^2 - 3 + 2x - 5$

$= 3x^2 + 2x - 8$

$\therefore \quad$ Sum of zeroes $= -\frac{b}{a} = \frac{-2}{3}$

Product of zeroes $= \frac{c}{a} = \frac{-8}{3}.$

3. Let $\alpha = 5$ and $\beta = -5$, then the quadratic polynomial will be $x^2 - (\alpha + \beta)x + \alpha\beta$ or $x^2 - 25$.

4. $p(x) = 4x^2 - 4x + 1$

$= 4x^2 - 2x - 2x + 1$

$= 2x(2x-1) - 1(2x-1)$

$= (2x-1)(2x-1)$

For zeroes, $2x - 1 = 0$ and $2x - 1 = 0$

$\therefore \quad x = \frac{1}{2}, \frac{1}{2}.$

5. $p = 2$

Hint: $(2)^3 - 3(2)^2 + 3(2) - p = 0$

$\Rightarrow \quad 8 - 12 + 6 - p = 0$

$\Rightarrow \quad 2 - p = 0$

$\therefore \quad p = 2.$

6. As -4 is a zero of polynomial.

$P(x) = x^2 - x - (2k+2)$

$\therefore \; P(-4) = 0 \Rightarrow (-4)^2 - (-4) - (2k+2) = 0$

$\Rightarrow 16 + 4 - 2k - 2 = 0$

$\Rightarrow 18 - 2k = 0$

$\Rightarrow k = 9.$

7. Let the third zero be α, then

sum of the zeroes $= -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$

$\Rightarrow \quad 2 + 3 + \alpha = -\frac{-6}{1}$

$\Rightarrow \quad \alpha = 1$

Hence, the third zero is 1.

8. Let us divide $6x^4 + 8x^3 + 17x^2 + 21x + 7$ by $3x^2 + 4x + 1$.

$$
\begin{array}{r|l}
 & 2x^2 + 5 \\
3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\
 & \underline{\underset{-}{6x^4} + \underset{-}{8x^3} + \underset{-}{2x^2}} \\
 & 15x^2 + 21x + 7 \\
 & \underline{\underset{-}{15x^2} + \underset{-}{20x} + \underset{-}{5}} \\
 & x + 2
\end{array}
$$

Clearly, the remainder is $x+2$.
Now, $ax+b = x+2$
Comparing the coefficients of like powers of x both the sides, we obtain

$$a = 1, b = 2.$$

9. Let $\alpha.\beta$ be zeroes of $(a + 2) x^2 + 6x + 5a$

Also Let $\beta = \frac{1}{\alpha}$.

Now product of zeroes $= \alpha.\beta = \frac{5a}{a+2}$

$$\Rightarrow \quad \alpha\left(\frac{1}{\alpha}\right) = \frac{5a}{a+2}$$

$$\Rightarrow \quad 1 = \frac{5a}{a+2} \Rightarrow a + 2 = 5a$$

$$\Rightarrow 2 = 4a$$

$$\Rightarrow a = \frac{1}{2}.$$

10. $\sqrt{3}$ and 1

Hint: $x^2 - \sqrt{3}\,x - x + \sqrt{3} = (x - \sqrt{3})\,(x-1)$

For zeroes, $x - \sqrt{3} = 0$ and $x - 1 = 0$

$$\Rightarrow \quad x = \sqrt{3}, 1$$

Now, sum of zeroes $= \sqrt{3} + 1$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

And product of zeroes $= \sqrt{3}$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

11. (*i*) To find the number of sweets which was distributed among the slum children, we divide the total number of sweets by number of children Mr. Vinod has. Remainder thus obtained is the required number of sweets.

$$\begin{array}{r|l} & x^2 + 6x + 8 \\ x^2 - 4x + 3 & x^4 + 2x^3 - 13x^2 - 12x + 21 \\ & \underline{\overset{-}{}x^4 \overset{+}{-} 4x^3 \overset{-}{+} 3x^2} \\ & \quad 6x^3 - 16x^2 - 12x + 21 \\ & \quad \underline{\overset{-}{}6x^3 \overset{+}{-} 24x^2 \overset{-}{+} 18x} \\ & \qquad 8x^2 - 30x + 21 \\ & \qquad \underline{\overset{-}{}8x^2 \overset{+}{-} 32x \overset{-}{+} 24} \\ & \qquad\qquad 2x - 3 \end{array}$$

Hence, the number of sweets which was distributed amoung the slum children was $2x - 3$.

(*ii*) Helping one another, fair division.

WORKSHEET – 11

1. Sum of zeroes $= \frac{-(-5)}{\left(\frac{1}{3}\right)} = 15$

Product of zeroes $= \frac{\frac{3}{2}}{\frac{1}{3}} = \frac{9}{2}$.

2. Let the zeroes be α, β, γ. Then $\alpha\beta\gamma = -\frac{c}{1}$

If $\gamma = -1$, then $\alpha\beta = c$ ...(*i*)

Further, $(-1)^3 + a(-1)^2 + b(-1) + c = 0$

$\Rightarrow -1 + a - b + c = 0$

$\Rightarrow c = b - a + 1$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$\alpha\beta = b - a + 1$.

3. Sum of zeroes = 6

$$\Rightarrow \quad 6 = -\frac{-3k}{1}$$

$$\therefore \quad k = \frac{6}{3} = 2.$$

4. Let one zero be α, then the other one will be $\frac{1}{\alpha}$.

So, $\alpha.\frac{1}{\alpha} = \frac{4a}{a^2+4}$

$a^2 - 4a + 4 = 0$

$\Rightarrow \quad (a-2)^2 = 0$

$\Rightarrow \quad a = 2.$

5. Given polynomial is:

$$f(x) = x^2 - px - 2p - c$$

$\therefore \quad \alpha + \beta = p$

and $\quad \alpha.\beta = -2p - c$

$\therefore (\alpha+2)(\beta+2) = \alpha\beta + 2(\alpha+\beta) + 4$

$= -2p - c + 2p + 4$

$= (4 - c)$.

6. $\lambda = 6$

Hint: $(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$.

7. $x = -1$ or 3; $f(x) = x^2 - 2x - 3$

Hint: $x = -1$ or 3,

$\therefore$ Sum of zeroes $= 2$

Product of zeroes $= -3$

$$\therefore \quad p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$$
$$= x^2 - 2x - 3.$$

8. $x^2 - x - \dfrac{47}{4}$

Hint: $f(x) = \{x^2 - (\text{sum of zeroes})\,x + (\text{product of zeroes})\}$

9. Let $P(x) = kx^2 + x - 6$

$\therefore$ α, β be its zeroes

$$\therefore \quad \alpha + \beta = -\frac{1}{k};\ \alpha\beta = \frac{-6}{k}$$

$$\therefore \quad \alpha^2 + \beta^2 = \frac{25}{4} \qquad \text{(Given)}$$

$$\Rightarrow \quad (\alpha + \beta)^2 - 2\alpha\beta = \frac{25}{4}$$

$$\Rightarrow \quad \left(\frac{-1}{k}\right)^2 - 2\left(\frac{-6}{k}\right) = \frac{25}{4}$$

$$\Rightarrow \quad \frac{1}{k^2} + \frac{12}{k} = \frac{25}{4}$$

$$\Rightarrow \quad \frac{1 + 12k}{k^2} = \frac{25}{4}$$

$$\Rightarrow \quad 4 + 48k = 25k^2$$
$$\Rightarrow \quad 25k^2 - 48k - 4 = 0$$
$$\Rightarrow \quad (k - 2)(25k + 2) = 0$$

$$\Rightarrow \quad k = 2 \text{ or } k = \frac{-2}{25}.$$

10. $g(x) = x^2 + 2x + 1$

Hint: $p(x) = g(x) \times q(x) + r(x)$

$$\Rightarrow \quad g(x) = \frac{p(x) - r(x)}{q(x)}$$

where, $p(x) = 3x^3 + x^2 + 2x + 5$

$q(x) = 3x - 5$

and $r(x) = 9x + 10$.

11. Since $x = \sqrt{\dfrac{5}{3}}$ and $x = -\sqrt{\dfrac{5}{3}}$ are zeroes of $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$, so $p(x)$ is divisible by $\left(x - \sqrt{\dfrac{5}{3}}\right)\left(x + \sqrt{\dfrac{5}{3}}\right)$, *i.e.*, $x^2 - \dfrac{5}{3}$.

$$\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
\hline
x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & \underset{-}{3x^4} \qquad \underset{+}{-5x^2} \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & \underset{-}{6x^3} \qquad \underset{+}{-10x} \\
\hline
 & 3x^2 \qquad -5 \\
 & \underset{-}{3x^2} \qquad \underset{+}{-5} \\
\hline
 & 0
\end{array}$$

Here, other two zeroes of $p(x)$ are the two zeroes of quotient $3x^2 + 6x + 3$

Put $3x^2 + 6x + 3 = 0$

$$\Rightarrow \quad 3(x + 1)^2 = 0$$
$$\Rightarrow \quad x = -1 \text{ and } x = -1$$

Hence, all the zeroes of $p(x)$ are $\sqrt{\dfrac{5}{3}}$, $-\sqrt{\dfrac{5}{3}}$, -1 and -1.

WORKSHEET – 12

1. Let the polynomial whose zeroes are -2 and $\dfrac{-1}{3}$ be $p(x)$.

$$\Rightarrow \quad p(x) = (x + 2)\left(x + \frac{1}{3}\right)$$

$$\Rightarrow \quad p(x) = \frac{1}{3}(x + 2)(3x + 1)$$

$$\text{or} \quad p(x) = x^2 + \frac{7}{3}x + \frac{2}{3}.$$

2. $-\dfrac{3}{2}, -\dfrac{11}{2}$

Hint: Given polynomial can be written as:

$p(x) = 2x^2 + 3x - 11$

Sum of zeroes = $\frac{-b}{a}$

Product of zeroes = $\frac{c}{a}$.

3. We know that the degree of the remainder is less than the degree of divisor or does't exist.
Here, degree of the divisor is 3, therefore, the possible degree of the remainder according to the options can be any out of 0, 1 and 2.

4. $k = 0$.
Hint: Substitute $x = -\sqrt{2}$ in $x^2 + \sqrt{2}x + k = 0$.

5. Since α, β are the zeroes of x^2+px+q, then

$\alpha + \beta = -p;\ \alpha\beta = q$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -\frac{p}{q}$

and $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{q}$

So the polynomial having zeroes $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ will be

$$q(x) = x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha} \times \frac{1}{\beta}\right)$$

$$= x^2 + \frac{p}{q}x + \frac{1}{q}$$

or $q(x) = qx^2 + px + 1$.

6. $g(x) = x^2 + 2x + 7$.
Hint: Divide $x^3 + 3x - 14$ by $x-2$.

7. One example is:

$p(x) = 3x^2 - 3x + 12$.
$g(x) = x^2 - x + 4$
∴ $q(x) = 3$
$r(x) = 0$.

8. $\sqrt{\frac{1}{7}}, -\sqrt{\frac{1}{7}}$

Hint: For zeroes: $21x^2 - 3 = 0$

$$x^2 = \frac{1}{7}$$

$$\therefore \quad x = \pm\sqrt{\frac{1}{7}}.$$

9. Since $a = 2$ is a zero of $a^3 - 3a^2 - 10a + 24$, therefore $a^3 - 3a^2 - 10a + 24$ is divisible by $a-2$. Further the obtained quotient will provide the other two zeroes.

$$
\begin{array}{r|l}
 & a^2 - a - 12 \\
a-2 & a^3 - 3a^2 - 10a + 24 \\
 & \underset{-\ \ +}{a^3 - 2a^2} \\
\hline
 & -a^2 - 10a + 24 \\
 & \underset{+\ \ -}{-a^2 + 2a} \\
\hline
 & -12a + 24 \\
 & \underset{+\ \ -}{-12a + 24} \\
\hline
 & 0
\end{array}
$$

$a^2 - a - 12 = (a - 4)(a + 3)$

For other zeroes, put $a-4 = 0$ and $a+3 = 0$

$a = -3, 4$

Thus, the other two zeroes are -3 and 4.

10. Let $P(x) = 2x^2 - 5x - (2k + 1)$

Let α, β be its zeroes such that β = 2α.

$$\therefore \quad \alpha + \beta = \frac{-(-5)}{2} = \frac{5}{2}$$

$$\Rightarrow \quad \alpha + 2\alpha = \frac{5}{2} \quad \Rightarrow \quad 3\alpha = \frac{5}{2}$$

$$\Rightarrow \quad \alpha = \frac{5}{3 \times 2} \quad \Rightarrow \quad \alpha = \frac{5}{6}$$

$$\therefore \quad \beta = 2\alpha = \frac{2}{5/6} = \frac{5}{3}.$$

∴ Zeroes are: $\frac{5}{6}$ and $\frac{5}{3}$.

Also $\alpha\beta = \frac{-(2k+1)}{2}$

$$\Rightarrow \quad \left(\frac{5}{6}\right)\left(\frac{5}{3}\right) = \frac{-(2k+1)}{2}$$

$$\Rightarrow \quad \frac{25}{18} \times 2 = -2k - 1$$

$$\Rightarrow \quad 2k = -1 - \frac{25}{9} = \frac{-9-25}{9} = \frac{-34}{9}$$

$$\Rightarrow \quad k = \frac{-17}{9}.$$

11. $\dfrac{b^2-2ac}{c^2}$

Hint: $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}=\dfrac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}=\frac{b^2-2ac}{c^2}.$$

OR

Let us divide $x^4+2x^3+8x^2+12x+18$ by x^2+5.

$$\begin{array}{r|l} & x^2+2x+3 \\ \hline x^2+5 & x^4+2x^3+8x^2+12x+18 \\ & \underline{\overset{-}{x^4} \quad \overset{-}{+5x^2}} \\ & \quad 2x^3+3x^2+12x+18 \\ & \quad \underline{\overset{-}{2x^3} \quad \overset{-}{+10x}} \\ & \qquad 3x^2+2x+18 \\ & \qquad \underline{\overset{-}{3x^2} \quad \overset{-}{+15}} \\ & \qquad\quad 2x+3 \end{array}$$

Clearly, the remainder is $2x+3$.
Now, $px+q=2x+3$
Comparing the coefficients of like powers of x both the sides, we get

$p=2, q=3.$

WORKSHEET–13

1. If –4 is zero of given polynomial $p(x)=x^2-x-(2k+2)$

$\Rightarrow \quad p(-4)=0$

$\Rightarrow \quad p(-4)=(4)^2-(-4)-(2k+2)=0$

$\Rightarrow \quad 16+4-2k-2=0$

$\Rightarrow \quad 18-2k=0$

$\Rightarrow \quad k=9.$

2. $\alpha+\beta=\dfrac{3}{2}, \alpha\beta=\dfrac{1}{2}$

$\therefore \quad (\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta=\dfrac{9}{4}-2=\dfrac{1}{4}$

$\Rightarrow \quad \alpha-\beta=\pm\dfrac{1}{2}$

$\therefore \quad \alpha=\dfrac{1}{2}, \beta=1$ or $\alpha=1, \beta=\dfrac{1}{2}$

$\therefore \alpha+2=\dfrac{5}{2}, \beta+2=3$ or $\alpha+2=3$,

$\beta+2=\dfrac{5}{2}.$

Hence, the required polynomial can be

$x^2-\left(\dfrac{5}{2}+3\right)x+\dfrac{5}{2}\times 3$, *i.e.*, $x^2-\dfrac{11}{2}x+\dfrac{15}{2}.$

3. $p(x)=x^2-$ (sum of zeroes) x + product of zeroes

$=x^2-(\dfrac{1}{2}+2)x+\dfrac{1}{2}\times 2$

$=x^2-\dfrac{5}{2}x+1$

4. Let zeroes be α and β.
$\alpha+\beta=6, \alpha\beta=4$
Using $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$, we get
$(\alpha-\beta)^2=6^2-4\times 4=20 \Rightarrow \alpha-\beta, =\pm 2\sqrt{5}$

Thus, the difference of zeroes is $\pm 2\sqrt{5}$.

5. $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}$

$=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$=\dfrac{25-12}{6}=\dfrac{13}{6}.$

6. $x^2-1=(x+1)(x-1)$

$\therefore \quad x=-1$ or 1, both will satisfy with the given polynomial.

$\therefore \quad$ We get, $p+q+r+s+t=0 \quad ...(i)$
and $\quad p-q+r-s+t=0 \quad ...(ii)$

From (*ii*),

$p+r+t=q+s$

$2(q+s)=0 \Rightarrow q+s=0$ [From (*i*)]

$\therefore \quad p+r+t=q+s=0.$

7. No.

Hint: Divide $q(x)$ by $g(x)$. If the remainder obtained is zero, then the $g(x)$ is a factor of $q(x)$ otherwise not.

8. $a = 1,\ b = 7$

Hint: Put remainder = 0 and equate coefficient of x in the remainder and constant term with zero.

9. According to division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

(*i*) $p(x) = 6x^2+3x+2,\ g(x) = 3$

$q(x) = 2x^2+x,\ r(x) = 2$

(*ii*) $p(x) = 8x^3 + 6x^2-x+7,\ g(x) = 2x^2+1$

$q(x) = 4x+3,\ r(x) = -5x+4$

(*iii*) $p(x) = 9x^2 + 6x + 5,\ g(x) = 3x+2,$

$q(x) = 3x,\ r(x) = 5.$

10. Given quadratic polynomial is

$$5\sqrt{5}\,x^2 + 30x + 8\sqrt{5}$$

$$= 5\sqrt{5}\,x^2 + 30x + 8\sqrt{5}$$

$$= 5\sqrt{5}\,x^2 + 20x + 10x + 8\sqrt{5}$$

$$= 5x\left(\sqrt{5}x+4\right) + 2\sqrt{5}\left(\sqrt{5}x+4\right)$$

$$= \left(5x+2\sqrt{5}\right)\left(\sqrt{5}x+4\right)$$

To find its zeroes, put $5x+2\sqrt{5} = 0$ and $\sqrt{5}\,x + 4 = 0.$

$$\Rightarrow \qquad x = -\frac{2}{\sqrt{5}} \text{ and } x = \frac{-4}{\sqrt{5}}$$

i.e.,
$$x = -\frac{2\sqrt{5}}{5} \text{ and } x = -\frac{4\sqrt{5}}{5}$$

So, sum of zeroes $= \dfrac{-2\sqrt{5}}{5} - \dfrac{4\sqrt{5}}{5} = -\dfrac{6\sqrt{5}}{5}$

And product of zeroes

$$= \left(-\frac{2\sqrt{5}}{5}\right) \times \left(-\frac{4\sqrt{5}}{5}\right) = \frac{8}{5}.$$

Also, sum of zeroes $= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\frac{30}{5\sqrt{5}} = -\frac{6\sqrt{5}}{5}$$

And product of zeroes $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$= \frac{8\sqrt{5}}{5\sqrt{5}} = \frac{8}{5}.$$

Hence verified.

OR

$q(x) = 3x^2-2x + 1$

Hint: Let $S = \dfrac{\alpha-1}{\alpha+1} + \dfrac{\beta-1}{\beta+1}$

$$P = \left(\frac{\alpha-1}{\alpha+1}\right)\left(\frac{\beta-1}{\beta+1}\right)$$

$\therefore$ Required polynomial $q(x) = x^2 - Sx + P.$

11. As $\sqrt{\dfrac{3}{2}}$ and $-\sqrt{\dfrac{3}{2}}$ are the zeroes of the given quadratic polynomial, so $\left(x-\sqrt{\dfrac{3}{2}}\right)$ and $\left(x+\sqrt{\dfrac{3}{2}}\right)$ will be the factors of that. Consequently, $\left(x-\sqrt{\dfrac{3}{2}}\right) \times \left(x+\sqrt{\dfrac{3}{2}}\right)$, *i.e.,* $\left(x^2 - \dfrac{3}{2}\right)$ must be the factor of that. Let us divide $2x^4-10x^3+5x^2+15x-12$ by $x^2 - \dfrac{3}{2}$.

$$\begin{array}{r|l}
 & 2x^2 - 10x + 8 \\ \hline
x^2 - \frac{3}{2} & 2x^4 - 10x^3 + 5x^2 + 15x - 12 \\
 & \underset{-}{2x^4} \qquad\qquad \underset{+}{-\,3x^2} \\ \hline
 & \quad -10x^3 + 8x^2 + 15x - 12 \\
 & \quad \underset{+}{-\,10x^3} \qquad \underset{-}{+\,15x} \\ \hline
 & \qquad\qquad 8x^2 \qquad - 12 \\
 & \qquad\qquad \underset{-}{8x^2} \qquad \underset{+}{-\,12} \\ \hline
 & \qquad\qquad\quad 0
\end{array}$$

Now, $2x^4-10x^3+5x^2+15x-12$

$$= \left(x^2 - \frac{3}{2}\right)(2x^2-10x+8)$$

By splitting $-10x$, we factorise $2x^2 - 10x + 8$ as $(x-4)\,(2x-2)$. So, its zeroes are given by $x = 4$ and $x = 1$.

Therefore, all zeroes of the given polynomial are $\sqrt{\dfrac{3}{2}}$, $-\sqrt{\dfrac{3}{2}}$, 1 and 4.

WORKSHEET–14

1. Let zeroes be α and β, then

$$(\alpha - \beta)^2 = 144$$

$\Rightarrow$ $\alpha - \beta = \pm 12$ ...(*i*)

$\alpha + \beta = -p$ ...(*ii*)

$\alpha\beta = 45$ ...(*iii*)

Also, we have

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$\Rightarrow$ $144 = p^2 - 180 \Rightarrow p = \pm 18$.

2. $1 - c$

Hint: $f(x) = x^2 - px - (p + c)$

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$.

3. $\dfrac{p}{r}$

Hint: $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\gamma\alpha} = \dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$.

4. Let the given linear polynomial be

$y = ax + b$ ...(*i*)

This passes through points (1,–1), (2,1) and $\left(\dfrac{3}{2}, 0\right)$

$-1 = a + b$ ...(*ii*)

$1 = 2a + b$ ...(*iii*)

$0 = \dfrac{3}{2}a + b$ ...(*iv*)

Solving equations (*ii*) and (*iii*), we get $a = 2$, $b = -3$ which satisfy to equation (*iv*).

Consequently, using equation (*i*), we get

$y = 2x - 3$

$\therefore$ Polynomial is $p(x) = 2x - 3$

Since $p(x) = 0$ if $x = \dfrac{3}{2}$

$\Rightarrow$ $x = \dfrac{3}{2}$ is zero of $p(x)$.

5. Let us divide $ax^3 + bx - c$ by $x^2 + bx + c$ by the long division method.

$$
\begin{array}{r|l}
 & ax - ab \\
x^2 + bx + c & ax^3 + bx - c \\
 & \underline{\overset{-}{ax^3} \overset{-}{+ abx^2} \overset{-}{+ acx}} \\
 & -abx^2 + (b - ac)x - c \\
 & \underline{\overset{+}{-abx^2} \overset{+}{-ab^2x} \overset{+}{-abc}} \\
 & (ab^2 + b - ac)x + abc - c
\end{array}
$$

Put remainder = 0

$\Rightarrow$ $(ab^2 + b - ac)x + (abc - c) = 0$

$\Rightarrow$ $ab^2 + b - ac = 0$ and $abc - c = 0$

Consider $abc - c = 0 \Rightarrow (ab - 1)c = 0$

$\Rightarrow$ $ab = 1$ or $c = 0$. Hence, $ab = 1$.

6. Hint: Let $f(x) = x^3 - mx^2 - 2npx + np^2$

$(x - p)$ is a factor of $p(x)$

$\Rightarrow$ $f(x) = 0$ at $x = p$.

$\Rightarrow$ $p^3 - p^2m - p^2n = 0$

$\Rightarrow$ $p^2[(p - (m + n)] = 0$

$\Rightarrow$ $p = m + n$ since $p \neq 0$.

7. $x^3 - 4x^2 + x + 6$

Hint: The required cubic polynomial is given by $(x-3)(x-2)(x + 1)$ or $x^3 - 4x^2 + x + 6$

This is the required polynomial.

8. $-2, 3, 4$

Hint: $\alpha + \beta + \gamma = 5$

$\alpha\beta + \beta\gamma + \alpha\gamma = -2$

$\alpha\beta\gamma = -24$

Let $\alpha\beta = 12$

$\therefore$ $\gamma = -2$

$\therefore$ $\alpha + \beta = 7$

$\Rightarrow$ $(\alpha - \beta)^2 = 1$

$\Rightarrow$ $\alpha - \beta = \pm 1$

$\therefore$ $\alpha - \beta = 1$ or $\alpha - \beta = -1$

Solving $\alpha + \beta = 7$ and $\alpha - \beta = 1$, we get

$\alpha = 4, \beta = 3$

And solving $\alpha + \beta = 7$ and $\alpha - \beta = -1$

we get $\alpha = 3, \beta = 4$.

9. $f(x)$ would become exactly divisible by $g(x)$ if the remainder is subtracted from $f(x)$.

Let us divide $f(x)$ by $g(x)$ to get the remainder.

$$
\begin{array}{r|l}
 & x^2 + 6x + 8 \\
x^2 - 4x + 3 & x^4 + 2x^3 - 13x^2 - 12x + 21 \\
 & \underline{\overset{-}{x^4} \overset{+}{- 4x^3} \overset{-}{+ 3x^2}} \\
 & 6x^3 - 16x^2 - 12x + 21 \\
 & \underline{\overset{-}{6x^3} \overset{+}{- 24x^2} \overset{-}{+ 18x}} \\
 & 8x^2 - 30x + 21 \\
 & \underline{\overset{-}{8x^2} \overset{+}{- 32x} \overset{-}{+ 24}} \\
 & 2x - 3
\end{array}
$$

Hence, we should subtract $2x - 3$ from $f(x)$.

10. If $2\pm\sqrt{3}$ are zeroes of $p(x)$, then $x-(2+\sqrt{3})$ and $x-(2-\sqrt{3})$ are factors of $p(x)$.

Consequently $\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\}$

i.e., $(x-2)^2-3$, *i.e.*, x^2-4x+1 is factor of $p(x)$.

Further,

$$\begin{array}{r|l} & x^2-2x-35 \\ \hline x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\ & \underset{-}{x^4}\underset{+}{-4x^3}\underset{-}{+x^2} \\ \hline & -2x^3-27x^2+138x-35 \\ & \underset{+}{-2x^3}\underset{-}{+8x^2}\underset{+}{-2x} \\ \hline & -35x^2+140x-35 \\ & \underset{+}{-35x^2}\underset{-}{+140x}\underset{+}{-35} \\ \hline & 0 \end{array}$$

Clearly $x^2-2x-35$ is a factor of $p(x)$

$\Rightarrow$ $(x-7)(x+5)$ is a factor of $p(x)$

$\Rightarrow$ $x-7$ and $x+5$ are factors of $p(x)$

$\Rightarrow$ $x-7=0$ and $x+5=0$ give other zeroes of $p(x)$

$\Rightarrow$ $x=7$ and $x=-5$ are other zeroes of $p(x)$.

Hence, 7 and -5 are required zeroes.

11. Hint: $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\alpha^4+\beta^4}{\alpha^2\beta^2}$

$$=\frac{\{(\alpha+\beta)^2-2\alpha\beta\}^2-2\alpha^2\beta^2}{\alpha^2\beta^2}.$$

OR

Given polynomial is:

$$\begin{aligned} f(x) &= pqx^2+(q^2-pr)x-qr \\ &= pqx^2+(q^2-pr)x-qr \\ &= pqx^2+q^2x-prx-qr \\ &= qx(px+q)-r(px+q) \\ &= (px+q)(qx-r) \end{aligned}$$

$px+q=0$ and $qx-r=0$ provide the zeroes of $f(x)$. So zeroes are $-\frac{q}{p}$ and $\frac{r}{q}$.

$$\text{Sum of zeroes} = -\frac{q}{p}+\frac{r}{q}=\frac{pr-q^2}{pq}$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = -\frac{q}{p}\times\frac{r}{q}=-\frac{qr}{pq}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

WORKSHEET– 15

1. Sum of zeroes $= -\frac{-3\sqrt{2}}{3}=\sqrt{2}$

Product of zeroes $=\frac{1}{3}$.

2. At $x=2$, $p(x)=0$, *i.e.*, $p(2)=0$

$\therefore$ $a(2)^2-3\times2(a-1)-1=0$

$\Rightarrow$ $4a-6a+6-1=0$

$\Rightarrow$ $a=\frac{5}{2}$.

3. Sum of zeroes $=\alpha+\beta=5$

Product of zeroes $=\alpha\beta=4$

Now, $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta=\frac{\alpha+\beta}{\alpha\beta}-2\alpha\beta$

$$=\frac{5}{4}-2\times4=-\frac{27}{4}.$$

4. Using division algorithm, we have

$g(x)\times(x-2)-2x+4=x^3-3x^2+x+2$

$\Rightarrow$ $g(x)=\frac{x^3-3x^2+3x-2}{x-2}$

Here, at $x=2$,

$x^3-3x^2+3x-2=8-12+6-2=0$

$\therefore$ $x^3-3x^2+3x-2=(x-2)(x^2-x+1)$

$\therefore$ $g(x)=\frac{(x-2)(x^2-x+1)}{(x-2)}$

$\Rightarrow$ $g(x)=x^2-x+1$.

5. Given $s=\sqrt{2}$ and $p=-\frac{3}{2}$

The required polynomial is given by $k[x^2-sx+p]$

i.e., $k\left(x^2-\sqrt{2}x-\frac{3}{2}\right)$, where k is any real number.

Let us find zeroes of this polynomial.

$$k(x^2-\sqrt{2}\,x-\frac{3}{2}) = \frac{k}{2}(2x^2-2\sqrt{2}\,x-3)$$

$$= \frac{k}{2}(\sqrt{2}\,x-3)(\sqrt{2}\,x+1)$$

$\sqrt{2}\,x-3 = 0$ and $\sqrt{2}\,x+1 = 0$ provides the zeroes.

Hence $\frac{3\sqrt{2}}{2}$ and $\frac{-\sqrt{2}}{2}$ are the required zeroes.

6. Let $f(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$$= 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$$

$$= 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$$

$$= (\sqrt{3}x + 2)(4x - \sqrt{3})$$

To find zeroes of $f(x)$, put

$$\sqrt{3}x+2=0 \quad \text{and} \quad 4x-\sqrt{3}=0$$

$$\Rightarrow \quad x = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3} \quad \text{and} \quad x = \frac{\sqrt{3}}{4}$$

Thus, the zeroes are $\alpha = -\frac{2\sqrt{3}}{3}$ and $\beta = \frac{\sqrt{3}}{4}$

Sum of zeroes $= \alpha + \beta$

$$= -\frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{4} = \frac{-5\sqrt{3}}{12}$$

$$= -\frac{5\sqrt{3}}{4\times3} = -\frac{5}{4\sqrt{3}}$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes $= \alpha\beta = -\frac{2\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{4}$

$$= -\frac{2\sqrt{3}}{4\sqrt{3}} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

Hence verified.

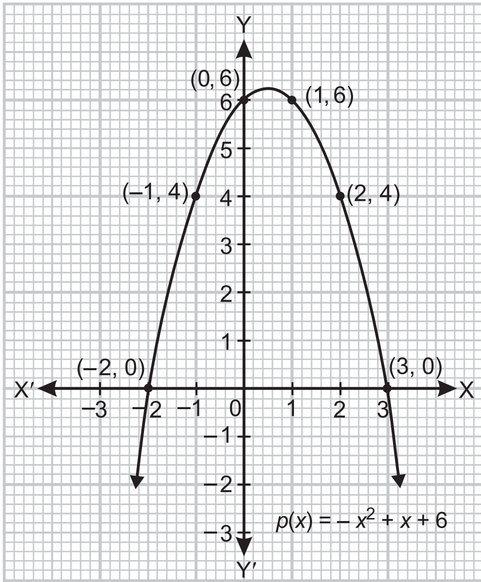
7. (*i*) Let $y = p(x)$

$\therefore$ $y = -x^2 + x + 6$

The table for some values of x and their corresponding values of y is given by

x	-2	-1	0	1	2	3
y	0	4	6	6	4	0

Let us draw the graph of $p(x)$ using this table.



From the graph, it is clear that the zeroes of $p(x)$ are -2 and 3.

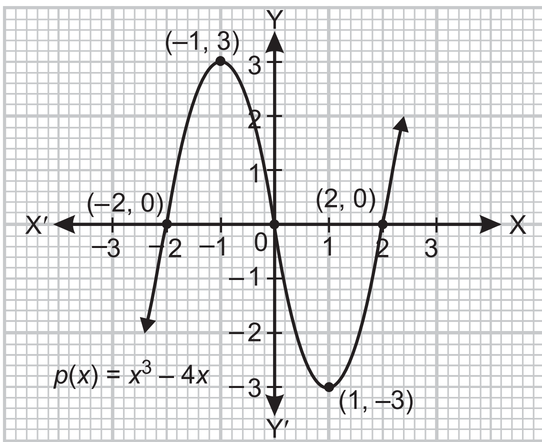
(*ii*) Let $y = p(x)$

$\therefore$ $y = x^3 - 4x$

The table for some values of x and their corresponding values of y is given by

x	-2	-1	0	1	2
y	0	3	0	-3	0

Let us draw the graph of $p(x)$ by using this table.



From the graph, it is clear that the zeroes of $p(x)$ are -2, 0 and 2 **.**

8. (*i*) First we divide $x^4 + x^3 + 8x^2 + ax + b$ by $x^2 + 1$ as follows:

$$\begin{array}{r} x^2 + x + 7 \\ x^2+1 \overline{\big) x^4 + x^3 + 8x^2 + ax + b} \\ x^4 + x^2 \qquad\qquad\qquad \\ - \quad - \qquad\qquad\qquad \\ \hline x^3 + 7x^2 + ax + b \\ x^3 + x \qquad\quad \\ - \quad - \qquad\quad \\ \hline 7x^2 + (a-1)x + b \\ 7x^2 + 7 \\ - \quad - \\ \hline (a-1)x + (b-7) \end{array}$$

Since, $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$, therefore remainder = 0

i.e., $(a - 1)x + (b - 7) = 0$ or $(a - 1)x + (b - 7) = 0.x + 0$

Equating the corresponding terms, we have

$a - 1 = 0$ and $b - 7 = 0$

i.e., $a = 1$ and $b = 7$

(*ii*) **Common good, Social responsibility.**

CHAPTER TEST

1. Let $p(x) = x\left(x + \frac{7}{2}\right)$

$\therefore$ Zeroes are given by

$x = 0$ and $\left(x + \frac{7}{2}\right) = 0.$

Hence zeroes are 0 and $-\frac{7}{2}$.

2. $\because$ $\alpha + \beta = \frac{-5}{2}, \alpha\beta = \frac{1}{2}$

$\therefore$ $\alpha + \beta + \alpha\beta = -2.$

3. $p(x) = x^2 - (\alpha+\beta)x + \alpha\beta = x^2 + x - 2.$

4. Let $p(x) = 2x^3 + 4x^2 + 5x + 7$

Now, $p(x) = g(x) \times 2x + (7 - 5x)$

$$g(x) = \frac{p(x) - (7-5x)}{2x}$$

$$= \frac{2x^3 + 4x^2 + 5x + 7 - 7 + 5x}{2x} = x^2 + 2x + 5.$$

5. $\frac{6\sqrt{5}}{5}, -\frac{9}{4}$

Hint: $\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

6. $\frac{-1}{3}$

Hint: $\alpha = -\beta$

$\alpha + \beta = 0 \quad \Rightarrow \quad \frac{-b}{a} = 0.$

7. $f(x) = ax^3 + bx^2 + cx + d$

$g(x) = ax^2 + bx + c$

$q(x) = x$

$r(x) = d.$

8. If α, β and γ are the zeroes of a cubic polynomial $f(x)$, then

$$f(x) = x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma$$

Here, $\alpha+\beta+\gamma = 4, \alpha\beta + \beta\gamma + \gamma\alpha = 1$

and $\alpha\beta\gamma = -6$

$\therefore$ $f(x) = x^3 - 4x^2 + x + 6.$

9. We have

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = \left(\sqrt{3}x + 2\right)\left(4x - \sqrt{3}\right)$$

So, the value of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ is zero when, $\sqrt{3}x + 2 = 0$ or $4x - \sqrt{3} = 0,$

i.e., when $x = \frac{-2}{\sqrt{3}}$ or $x = \frac{\sqrt{3}}{4}$.

Therefore, the zeroes of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are $\frac{-2}{\sqrt{3}}$ and $\frac{\sqrt{3}}{4}$.

Now, sum of zeroes $\frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4} = \frac{-5}{4\sqrt{3}}$

$$= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Product of zeroes $= \left(\frac{-2}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{4}\right) = \frac{-2\sqrt{3}}{4\sqrt{3}}$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

10. (*i*) Let $p(x)$ = Total Relief Fund

$g(x)$ = Number of families who received Relief Fund

$q(x)$ = Amount each family received

$r(x)$ = Amount left after distribution

When the polynomial $p(x)$ is divided by a polynomial $g(x)$ such that $q(x)$ and $r(x)$ are respectively the quotient and the remainder, the division algorithm is

$$p(x) = g(x) \,.\, q(x) + r(x) \qquad ...(i)$$

According to the question,

$$p(x) = 3x^3 + x^2 + 2x + 5$$
$$q(x) = 3x - 5$$
$$r(x) = 9x + 10$$

Substituting these values of $p(x)$, $q(x)$ and $r(x)$ in the equation (i), we get

$$3x^3 + x^2 + 2x + 5 = g(x) \,.\, (3x - 5) + 9x + 10$$
$$\Rightarrow \quad (3x - 5)\, g(x) = 3x^3 + x^2 + 2x + 5 - 9x - 10$$
$$= 3x^3 + x^2 - 7x - 5$$
$$\Rightarrow \quad g(x) = \frac{3x^3 + x^2 - 7x - 5}{3x - 5}$$

To find $g(x)$, we proceed as following:

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline 3x - 5 & 3x^3 + x^2 - 7x - 5 \\ & 3x^3 - 5x^2 \\ & - \quad + \\ \hline & \quad 6x^2 - 7x - 5 \\ & \quad 6x^2 - 10x \\ & \quad - \quad + \\ \hline & \qquad 3x - 5 \\ & \qquad 3x - 5 \\ & \qquad - \quad + \\ \hline & \qquad 0 \end{array}$$

Thus, $g(x) = x^2 + 2x + 1$.

(*ii*) **Common good, Accountability, social responsibility.**

11. Since $\frac{-1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$ are zeroes.

Therefore, $\left(x - \frac{1}{\sqrt{3}}\right)\left(x + \frac{1}{\sqrt{3}}\right)$ will be a factor of $p(x)$, *i.e.*, $x^2 - \frac{1}{3}$ is a factor of $p(x)$.

$$\begin{array}{r|l} & 3x^2 - 15x + 18 \\ \hline x^2 - \frac{1}{3} & 3x^4 - 15x^3 + 17x^2 + 5x - 6 \\ & 3x^4 \qquad\quad - x^2 \\ & - \qquad\qquad + \\ \hline & \quad - 15x^3 + 18x^2 + 5x - 6 \\ & \quad - 15x^3 \qquad\quad + 5x \\ & \quad + \qquad\qquad - \\ \hline & \qquad\quad 18x^2 - 6 \\ & \qquad\quad 18x^2 - 6 \\ & \qquad\quad - \qquad + \\ \hline & \qquad\qquad 0 \end{array}$$

Here, $3x^2 - 15x + 18 = x^2 - 5x + 6$

$= (x - 3)(x - 2)$

Other zeroes are given by $x-3 = 0$ and $x-2 = 0$.

So, other zeroes are 3 and 2.

Hence, all the zeroes are $-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$, 3 and 2.

❑❑

Chapter

3 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

WORKSHEET – 16

1. As $am \neq bl$

$\Rightarrow \quad \dfrac{a}{l} \neq \dfrac{b}{m}$

$\Rightarrow$ unique solution for given pair.

2. $k = 6$

Hint: $\dfrac{2}{k} = \dfrac{-3}{-9}$.

3. Let the two numbers be x and y

$$\Rightarrow \left.\begin{aligned} x + y &= 35 \\ x - y &= 13 \end{aligned}\right\}$$

Adding $\Rightarrow$ $2x = 48$ $\Rightarrow$ $x = 24$

Subtracting $\Rightarrow$ $2y = 22$ $\Rightarrow$ $y = 11$

Hence, two numbers are 24 and 11.

4. $a_1 = \dfrac{3}{2}, b_1 = \dfrac{5}{3}, c_1 = 7$

$a_2 = \dfrac{3}{2}, b_2 = \dfrac{2}{3}, c_3 = 6$

or $\dfrac{a_1}{a_2} = \dfrac{\frac{3}{2}}{\frac{3}{2}} = 1$; $\dfrac{b_1}{b_2} = \dfrac{\frac{5}{3}}{\frac{2}{3}} = \dfrac{5}{2}$.

Clearly $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ $\Rightarrow$ lines are intersecting.

5. $x = 3, y = 2$

Hint: Let $\dfrac{1}{x+y} = u, \dfrac{1}{x-y} = v$.

$\therefore$ Given equations become

$10u + 2v = 4$ and $15u - 5v = -2$.

6. False.

Let us substitute $c = 40$, The given equations become

$x - 2y = 8$

or $5x - 10y = 40$

Here, $\dfrac{1}{5} = \dfrac{-2}{-10} = \dfrac{8}{40}$

$\Rightarrow$ The equations represent a pair of coincident lines.

$\Rightarrow$ The equations have infinitely many solutions at $c = 40$ and no solutions at $c \neq 40$.

$\Rightarrow$ For no value of c, the given pair has a unique solution.

7. The given equations are

$4(2x + 3y) = 9 + 7y$

and $3x + 2y = 4$

or $8x + 5y - 9 = 0$

$3x + 2y - 4 = 0$

By cross-multiplication, we have

$$\frac{x}{-20+18} = \frac{-y}{-32+27} = \frac{1}{16-15}$$

$$\Rightarrow \quad \frac{x}{-2} = \frac{-y}{-5} = \frac{1}{1}$$

$x = -2$ and $y = 5$

Hence, $x = -2$, $y = 5$ is the solution of the given system of equations.

8. To draw a line, we need atleast two solutions of its corresponding equation.

$x+3y = 6$; at $x = 0$, $y = 2$ and $x = 3$, $y = 1$.

So, two solutions of $x + 3y = 6$ are:

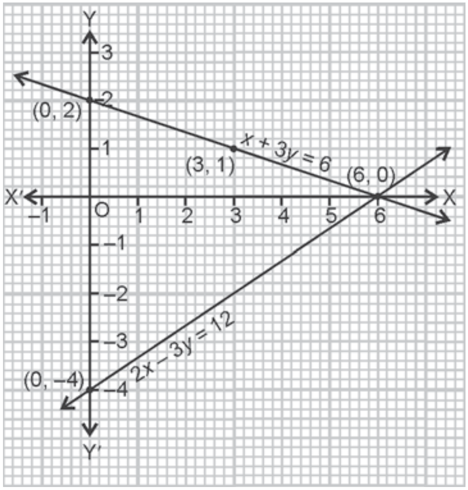
x	0	3
y	2	1

$2x - 3y = 12$; at $x = 0$, $y = -4$ and at $x = 6$, $y = 0$

So, two solutions of $2x - 3y = 12$ are:

x	0	6
y	−4	0

Now, we draw the graph of given system of equations by using their corresponding solutions obtained in the above tables.



From the graph, the two lines intersect the y-axis at (0, 2) and (0, −4).

9. Let the fixed charges and change per km be ` $x$ and ` y respectively.

$$x + 10y = 105 \quad ...(i)$$

$$x + 25y = 255 \quad ...(ii)$$

Subtracting equation (i) from equation (ii), we get

$$15y = 150$$

$$y = 10 \quad ...(iii)$$

From equations (i) and (iii), we get

$$x = 5$$

Now, the fare for travelling a distance of 35 km

$$= x + 35y$$

$$= 5 + 35 \times 10$$

$$= \text{` } 355.$$

Fixed charge = ` 5

Charge per km = ` 10

Total charge for 35 km = ` 355.

WORKSHEET – 17

1. As point of intersection of $y = x$, $x = 6$ is (6, 6)

$$\therefore \text{ area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 6 \times 6 = 18 \text{ sq. unit.}$$

2. $x - 5y = 5$.

(2, k) lies on it.

$\therefore \quad 2 - 5(k) = 5 \quad \Rightarrow \quad 5(k) = -3$

$\Rightarrow \quad k = -\frac{3}{5}$.

3. Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{-2}{k} \neq \frac{-3}{-1}$$

$$\Rightarrow \quad k = -6.$$

4. The given lines to be coincident, if

$$\underset{\text{I}}{\frac{k}{12}} = \underset{\text{II}}{\frac{3}{k}} = \underset{\text{III}}{\frac{-(k-3)}{-k}}$$

Taking I and II, we have

$k^2 = 36 \quad \Rightarrow \quad k = \pm 6. \quad ...(i)$

Taking II and III, we have

$k^2 - 3k = 3k \quad \Rightarrow \quad k(k - 6) = 0$

$\Rightarrow \quad k = 0$ or $6 \quad ...(ii)$

Using (i) and (ii), we obtain

$k = 6$.

5. $x = 5$, $y = 2$

Hint: Adding the given equations, we get $2x + y = 12 \quad ...(i)$

Subtracting the given equations, we get $3x + y = 17 \quad ...(ii)$

6. Yes.

Applying the condition

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

we have

$$\frac{1}{3} = \frac{2}{6} = \frac{-3}{-9}$$

That is true.

Therefore, the pair of equations is consistent with infinitely many solutions.

7. Let: $2x - y + 3 = 0 \quad \Rightarrow \quad y = 2x + 3 \quad ...(i)$

x	0	1
y	3	5

and $3x - 5y + 1 = 0 \Rightarrow y = \frac{3x+1}{5}$ ...(*ii*)

x	–2	3
y	–1	2

From (*i*) and (*ii*),

$$\frac{2x+3}{1} = \frac{3x+1}{5}$$

$\Rightarrow \quad 10x + 15 = 3x + 1$

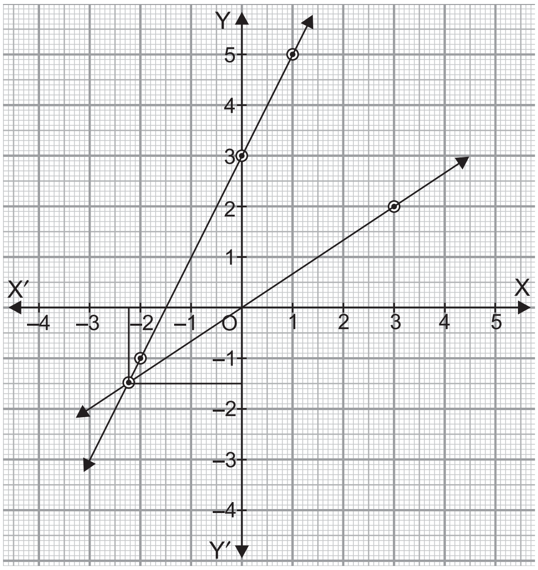
$\Rightarrow \quad 7x = -14 \quad \therefore \quad x = -2$

From equation (*i*),

$y = -4 + 3 \qquad (\because \; x = -2)$

$y = -1$

$\therefore \quad x = -2, y = -1.$



8. Let the cost price of the table be ` $x$ and the cost price of the chair be ` y.

The selling price of the table, when it is sold at a profit of 10%

$$= \text{`}\left(x + \frac{10}{100}x\right) = \text{`}\,\frac{110}{100}x$$

The selling price of the chair when it is sold at a profit of 25%

$$= \text{`}\left(y + \frac{25}{100}y\right) = \text{`}\,\frac{125}{100}y$$

So, $\frac{110}{100}x + \frac{125}{100}y = 1050$ ...(*i*)

When the table is sold at a profit of 25%,

its selling price $= \text{`}\left(x + \frac{25}{100}x\right) = \text{`}\,\frac{125}{100}x$

When the chair is sold at a profit of 10%,

its selling price $= \text{`}\left(y + \frac{10}{100}y\right) = \text{`}\,\frac{110}{100}y$

So, $\frac{125}{100}x + \frac{110}{100}y = 1065$ ...(*ii*)

From equations (*i*) and (*ii*), we get

$110x + 125y = 105000$

and $125x + 110y = 106500$

On adding and subtracting these equations, we get

$235x + 235y = 211500$

and $15x - 15y = 1500$

i.e., $x + y = 900$ ...(*iii*)

and $x - y = 100$ ...(*iv*)

Solving equations (*iii*) and (*iv*), we get

$x = 500, y = 400$

So, the cost price of the table is ` 500 and the cost price of the chair is ` 400.

9. Let the man's starting salary and fixed annually increment be x and y respectively.

According to the question,

$x + 4y = 15000$ ...(*i*)

$x + 10y = 18000$ ...(*ii*)

Equations (*i*) and (*ii*) form the required pair of linear equations. Let us solve this pair. Subtracting equation (*i*) from equation (*ii*), we get

$6y = 3000 \Rightarrow y = 500$

Substituting $y = 500$ in equation (*ii*), we get

$x = 13000$

Hence, starting salary was ` 13000 and annual increment was ` 500.

The value imbibe by the man are: consistency, hard work and sincerity

WORKSHEET – 18

1. Here, $\frac{2}{6} = \frac{-3}{-9} \neq \frac{9}{-5}$

$\therefore$ Lines are parallel.

2. As the lines are intersecting each other,

$$\frac{3}{a} \neq \frac{2}{-1} \quad \Rightarrow \quad a \neq \frac{-3}{2}.$$

3. $3x - y - 5 = 0$ and $6x - 2y - k = 0$ have no solution

$\Rightarrow$ These equations represent a pair of parallel lines.

$$\Rightarrow \quad \frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-k}$$

$$\Rightarrow \quad k \neq -10.$$

4. No.

For infinitely many solutions, the following condition must be satisfied.

$$\frac{\lambda}{2} = \frac{3}{6} = \frac{7}{-14}$$

But, here $\frac{3}{6} \neq \frac{-7}{14}$ as $\frac{1}{2} \neq -\frac{1}{2}$

Hence, no value of 'λ' provides the pair of infinitely many solutions.

5. As opposite sides are equal in rectangle

$$x + 3y = 13 \quad ...(i)$$

and $$3x + y = 7 \quad ...(ii)$$

From (*i*) and (*ii*),

$$x + 3y = 13 \times 3$$
$$3x + y = 7 \times 1$$
$$3x + 9y = 39$$
$$\underline{\begin{matrix} 3x + y = 7 \\ - \quad - \quad - \end{matrix}}$$
$$8y = 32$$
$$y = 4$$

From equation (*i*),

$$x + 3 \times 4 = 13$$
$$\Rightarrow \quad x + 12 = 13$$
$$\Rightarrow \quad x = 1$$
$$\therefore \quad x = 1, y = 4.$$

6. $x = 6, y = -4, m = 0$

Hint: Take $\frac{1}{x} = u$ and $\frac{1}{y} = v$.

7. No; (6, 0) , (4, 0)

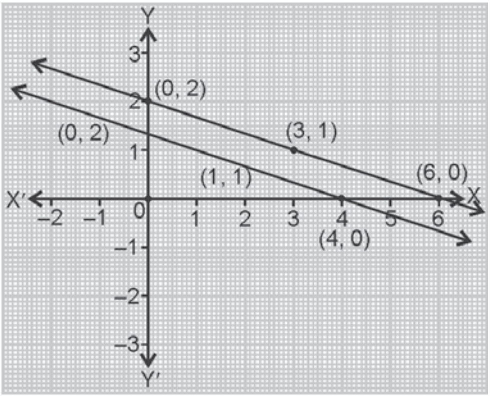
Hint: For $x + 3y = 6$

x	0	3
y	2	1

For $3x + 9y = 12$

x	1	4
y	1	0

Let us draw the graph of lines using the tables obtained above.



In the graph, lines are parallel. So, the pair of equations is not consistent.

The lines intersect the x–axis at (4, 0) and (6,0).

8. (*i*) Let l = length of the rectangle

b = breadth of the rectangle

According to question,

$$(l + 7)(b - 3) = lb \quad ...(i)$$
$$(l - 7)(b + 5) = lb \quad ...(ii)$$

From equation (*i*),

$$lb + 7b - 3l - 21 = lb$$
$$\Rightarrow \quad 7b - 3l = 21 \quad ...(iii)$$

From equation (*ii*),

$$lb - 7b + 5l - 35 = lb$$
$$\Rightarrow \quad -7b + 5l = 35 \quad ...(iv)$$

Adding equations (*iii*) and (*iv*), we get

$$2l = 56 \Rightarrow l = 28 \text{ m}$$

Putting the value of l in equation (*iii*), we get

$$b = 15 \text{ m}.$$

$\therefore$ $l = 28$ m, $b = 15$ m.

(*ii*) Solution of system of linear equations in two variables.

(*iii*) **Love for environment and human beings.**

WORKSHEET – 19

1. Let unit's and ten's digit be x and y respectively.

$$x + y = 9 \quad ...(i)$$

$$10y + x + 27 = 10x + y \quad ...(ii)$$

Solving equations (i) and (ii), we have

$$x = 6, y = 3$$

Hence, the required number is $3 \times 10 + 6$, that is, 36.

2. Given equation is $5(x - y) = 3$

$\Rightarrow 5x - 5y - 3 = 0$

Let $a_2 = 10$, $b_2 = 10$ and $c_2 = 6$

For coincident; $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence $\frac{a_1}{a_2} = \frac{5}{10} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{5}{10} = \frac{1}{2}$

and $\frac{c_1}{c_2} = \frac{3}{6} = \frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So required equation which can coincide is $10x - 10y - 6 = 0$

3. $p = 6$

Hint: $\frac{3}{p} = \frac{5}{10} \Rightarrow p = 6$

Note: At $p = 6$, the given system has both zero and non-zero solutions.

4. $a = 5, b = 1$

Hint: According to the condition of infinitely many solutions, we reaches at

$$\frac{a+b}{2} = \frac{2a-b}{3} = \frac{21}{7}.$$

5. $x = 1, y = 1$

Hint: Simplifying the given linear equations, we have

$$\frac{7}{y} - \frac{2}{x} = 5, \frac{8}{y} + \frac{7}{x} = 15$$

Now take $\frac{1}{x} = u$, $\frac{1}{y} = v$; and solve.

6. $x = \frac{4a-b}{5a}$, $y = \frac{-a+4b}{5b}$

Hint: $\frac{x}{-3b(2a+b)+2b(a+2b)}$

$$= \frac{-y}{-2a(2a+b)+3a(a+2b)}$$

$$= \frac{1}{2a \times 2b - 3a \times 3b}$$

Take first and third terms as well as second and third terms and solve.

7. $a = 7, b = 3$

Hint: For infinitely many solutions,

$$\frac{2}{4} = \frac{-(a-4)}{-(a-1)} = \frac{2b+1}{5b-1}$$

Take $\frac{1}{2} = \frac{a-4}{a-1}$ and $\frac{1}{2} = \frac{2b+1}{5b-1}$.

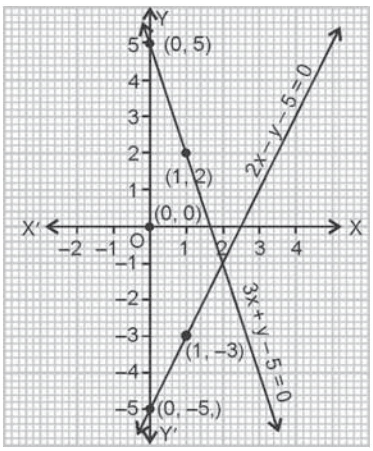
8. Table for values of x and y as regarding equation $3x+y-5 = 0$ is

x	0	1
y	5	2

Similarly table for equation $2x - y - 5 = 0$ is

x	0	1
y	–5	–3

Let us draw the graph of lines using the tables obtained above.



The lines intersect y-axis at (0, 5) and (0, –5).

9. (*i*) Total distance = 300 km
Let speed of train = x km/h
and speed of bus = y km/h

As $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$\therefore$ $\text{time} = \dfrac{\text{distance}}{\text{speed}}$

According to question,

$$\frac{60}{x} + \frac{240}{y} = 4 \text{ hours} \quad ...(i)$$

and $$\frac{100}{x} + \frac{200}{y} = 4 \text{ hours and } 10 \text{ minutes.} \quad ...(ii)$$

From equation (*i*),

$$\frac{15}{x} + \frac{60}{y} = 1 \quad ...(iii)$$

From equation (*ii*),

$$\frac{100}{x} + \frac{200}{y} = \frac{25}{6} \text{ hours}$$

$$\Rightarrow \quad \frac{4}{x} + \frac{8}{y} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{2}{y} = \frac{1}{24} \quad ...(iv)$$

$\therefore$ We will solve equations (*iii*) and (*iv*) by elimination method.
Applying (*iii*) – 15 × (*iv*), we get:

$$\frac{15}{x} + \frac{60}{y} = 1$$

$$\frac{15}{x} + \frac{30}{y} = \frac{15}{24}$$

$$- \quad - \quad -$$

$$\frac{30}{y} = 1 - \frac{15}{24}$$

$$\Rightarrow \quad \frac{30}{y} = \frac{9}{24}$$

$$\Rightarrow y = \frac{30 \times 24}{9} = 10 \times 8 = 80 \text{ km/h}$$

Putting the value of y in equation (*iv*), we get,

$$\frac{1}{x} + \frac{2}{80} = \frac{1}{24}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{24} - \frac{1}{40} = \frac{40-24}{24 \times 40} = \frac{16}{24 \times 40}$$

$$= \frac{1}{60} \Rightarrow x = 60 \text{ km/h}$$

(*ii*) Solution of system of linear equations in two variables.

(*iii*) By opting for public transport it depicts that she is a responsible citizen, so her **responsibility and rationality have been depicted here.**

WORKSHEET – 20

1. Since $x = a$ and $y = b$ is the solution of the given equation, then

$$a + b = 6 \quad ...(i)$$
$$a - b = 2 \quad ...(ii)$$

Adding (*i*) and (*ii*), we get

$$a + b + a - b = 8$$
$$2a = 8$$
$$a = 4$$

From equation (*i*), we get

$$4 + b = 6$$
$$b = 2$$

Thus the values of a and b are 4 and 2 respectively.

2. For no solutions,

$$\frac{k}{12} = \frac{3}{k} \neq \frac{-(k-2)}{-k} \quad \Rightarrow \quad k = \pm 6$$

If $k = 6$

$$\frac{6}{12} = \frac{3}{6} \neq \frac{6-2}{6} = \frac{4}{6} = \frac{2}{3} \quad \text{(True)}$$

If $k = -6$

$$\frac{-6}{12} = \frac{3}{-6} \neq \frac{-8}{-6} = \frac{4}{3} \quad \text{(True)}$$

$\therefore$ Required value of k, can be 6 or –6.

3. Let the required equation be $ax + by + c = 0$.

Then, $\dfrac{a}{\sqrt{2}} = \dfrac{b}{-\sqrt{3}} \neq \dfrac{c}{-5}$

$$\Rightarrow \quad \frac{a}{\sqrt{2}} = \frac{b}{-\sqrt{3}} = k \text{ (say)}$$

$\Rightarrow\ a = \sqrt{2}\,k,\ b = -\sqrt{3}\,k,\ k \neq -\frac{c}{5}$ any real number

Then, $\sqrt{2}\,kx - \sqrt{3}\,ky + c = 0$

$$\Rightarrow \quad \sqrt{2}\,x - \sqrt{3}\,y + \frac{c}{k} = 0$$

Putting $k = -c$, we have

$$\Rightarrow \quad \sqrt{2}\,x - \sqrt{3}\,y = 1.$$

4. For infinite number of solutions, we have

$$\frac{2}{p+q} = \frac{-3}{-(p+q-3)} = \frac{-7}{-(4p+q)}$$

On solving, $\frac{2}{p+q} = \frac{-3}{-(p+q-3)}$ and

$$\frac{-3}{-(p+q-3)} = \frac{-7}{-(4p+q)},$$

we obtain $p = -5,\ q = -1$.

5. $x = 1,\ y = 2$

Hint: Adding and subtracting the given two equations, we have

$$x + y = 3 \quad ...(i)$$

and $$x - y = -1 \quad ...(ii)$$

Now, solve equations (*i*) and (*ii*).

6. $x = a^2,\ y = b^2$

Hint: Given system of linear equations may be written as

$$bx + ay - ab\,(a + b) = 0$$
$$b^2x + a^2y - 2\,a^2b^2 = 0$$

Solve these two equations by the method of cross-multiplication.

7. Let the two digits number be $10x + y$.

Since ten's digit exceeds twice the unit's digit by 2

$$\therefore \quad x = 2y + 2$$
$$\Rightarrow \quad x - 2y - 2 = 0 \quad ...(i)$$

Since the number obtained by inter-changing the digits, *i.e.*, $10y + x$ is 5 more than three times the sum of the digits.

$$\therefore \quad 10y + x = 3\,(x + y) + 5$$
$$\Rightarrow \quad 2x - 7y + 5 = 0 \quad ...(ii)$$

On solving equations (*i*) and (*ii*), we obtain

$x = 8$ and $y = 3$

$$\therefore \quad 10x + y = 83$$

Hence, the required two-digit number is 83.

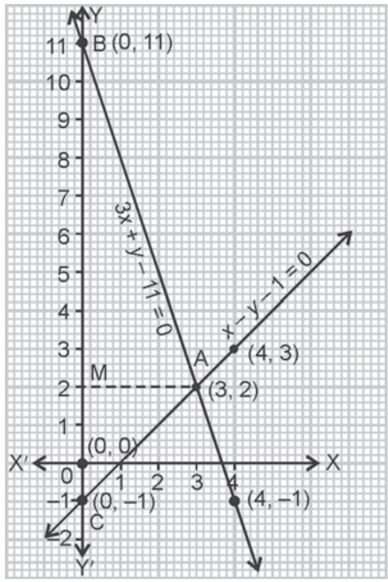
8. Tables for equations $3x + y - 11 = 0$ and $x - y - 1 = 0$ are respectively

x	3	4
y	2	-1

and

x	0	4
y	-1	3

Let us draw the graph.



From the graph, it is clear that the lines intersect each other at a point A(3, 2). So the solution is $x = 3,\ y = 2$.

The line $3x + y - 11 = 0$ intersects the y-axis at B(0, 11) and the line $x - y - 1 = 0$ intersects the y-axis at C (0, −1). Draw the perpendicular AM from A on the y-axis to intersect it at M.

Now, in ΔABC,

base BC = 11 + 1 = 12 units,

height AM = 3 units.

$$\therefore \quad ar(\Delta ABC) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 12 \times 3 = 18 \text{ sq. units}$$

Hence, $x = 3,\ y = 2$; area = 18 sq. units.

9. Speed of boat = 6 km/hr,

Speed of stream = 2 km/hr

Hint: Let the speed of boat in still water = x km/h and the speed of stream = y km/h

$$\frac{12}{x-y} + \frac{40}{x+y} = 8 \quad ...(i)$$

$$\left[\text{Using Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\frac{16}{x-y} + \frac{32}{x+y} = 8 \quad ...(ii)$$

Put $x - y = u$, $x + y = v$ and solve further to find x and y.

OR

Let each boy receives ` $x$ and the number of boys be $y$. Then sum of money which is distributed is ` xy.

Had there been 10 boys more, each would have received a rupee less,

$\therefore \quad (y + 10)(x - 1) = xy$

$\Rightarrow \quad 10x - y = 10 \quad ...(i)$

Had there been 15 boys fewer, each would have received ` 3 more,

$\therefore \quad (y - 15)(x + 3) = xy$

$\Rightarrow \quad 5x - y = -15 \quad ...(ii)$

Solving (i) and (ii), we get

$x = 5$ and $y = 40$

$\therefore \quad xy = 200$

Hence, sum of money = ` 200

And number of boys = 40.

WORKSHEET – 21

1. In the case of no solution,

$$\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-k} \Rightarrow k \neq 10.$$

2. $x = 80$, $y = 30$

Hint: $x + 2y = 140$, $3x + 4y = 360$.

3. For unique solution,

$$\frac{4}{2} \neq \frac{p}{2} \Rightarrow p \neq 4.$$

4. **True.**

According to the conditions of consistency,

$$\text{either } \frac{\frac{2}{3}}{\frac{3}{2}} \neq \frac{-5}{-5} \text{ or } \frac{\frac{2}{3}}{\frac{3}{2}} = \frac{-5}{-5} = \frac{1}{3}$$

Clearly, the first condition holds. Hence, the system of equations is consistent with a unique solution.

5. For infinitely many solutions,

$$\frac{p+q}{3} = \frac{2(p-q)}{4} = \frac{-(5p-1)}{-12}$$

$\Rightarrow \quad 4p + 4q = 6p - 6q$ and

$-12p - 12q = -15p+3$

$\Rightarrow \quad 2p - 10q = 0$ and $3p - 12q = 3$

$\Rightarrow \quad p = 5, q = 1.$

6. $x = 1$, $y = 1$,

Hint: Take $\frac{1}{3x+y} = u$, $\frac{1}{3x-y} = v$

$\therefore$ Given equation can be written as:

$$u + v = \frac{3}{4}$$

$\Rightarrow \quad 4u + 4v = 3$

and $\quad \frac{1}{2}u - \frac{1}{2}v = -\frac{1}{8}$

$\Rightarrow \quad 4u - 4v = -1.$

7. $x = \frac{-1}{2}$, $y = \frac{1}{3}$

Hint: Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$.

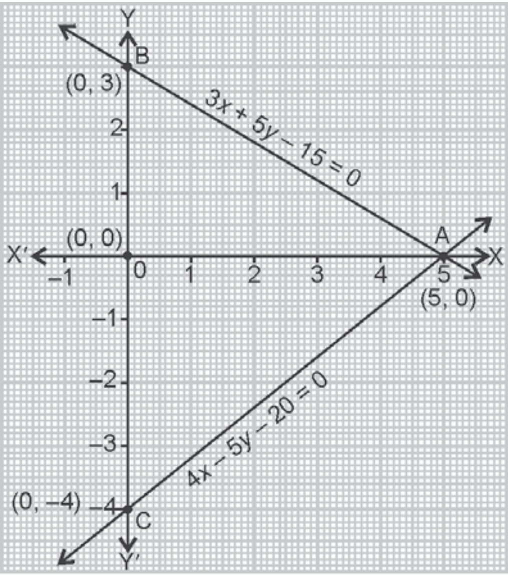
8. Table for values of x and y corresponding to equation $4x - 5y - 20 = 0$ is

x	5	0
y	0	–4

Similarly for the equation $3x+5y-15 = 0$

x	5	0
y	0	3

Let us draw the graphs for the two equations.



As the graphs of the two lines intersect each other at the point A(5, 0), the required solution is $x = 5$, $y = 0$.

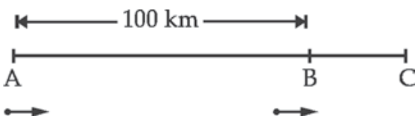
The graphs intersect the y-axis at B (0, 3) and C(0, −4). Therefore, the coordinates of vertices of the triangle ABC are A(5, 0), B(0, 3) and C(0, −4).

Hence, the answer: $x = 5$, $y = 0$ and (5, 0), (0, 3), (0, −4).

9. Let speeds of two cars that start from places A and B be x km/hr and y km/hr respectively.

Case I: When two cars travel in same direction.

Let the cars meet at C



Distance travelled by the car that starts from A

$$AC = 5 \times x$$

Similarly distance for other car

$$BC = 5 \times y$$

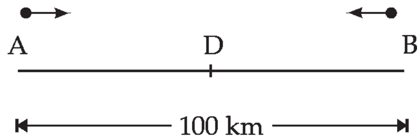
$\therefore \quad AC - BC = 5x - 5y$

$\Rightarrow \quad 5x - 5y = 100$

$\Rightarrow \quad x - y = 20 \quad ...(i)$

Case II: When two cars travel in opposite directions.

Let the cars meet at D



Distance travelled by the car that starts from A

$$AD = 1 \times x$$

Similarly distance for other car

$$BD = 1 \times y$$

$\therefore \quad AD + BD = x + y$

$\Rightarrow \quad x + y = 100 \quad ...(ii)$

Solving equations (i) and (ii), we get

$$x = 60 \text{ and } y = 40$$

Hence, speeds of two cars that start from places A and B are 60 km/h and 40 km/h respectively.

WORKSHEET – 22

1. $x - y = 0 \quad ...(i)$

$2x - y = 2 \quad ...(ii)$

$\quad - \quad + \quad -$

$-x = -2 \quad$ (Subtracting)

$\therefore \quad x = 2.$

Further $y = x = 2$.

2. The given equations represent to be parallel lines if

$$\frac{2(k-1)}{3} = \frac{1}{-1} \neq \frac{-1}{-1}$$

$$\Rightarrow \quad k - 1 = -\frac{3}{2}$$

$$\Rightarrow \quad k = -\frac{1}{2}.$$

3. $m \neq 4$

Hint: $\frac{m}{2} \neq \frac{-2}{-1}$.

4. For the point of intersection of any line with x-axis, put $y = 0$

$\therefore \quad -3x + 7(0) = 3$

$\Rightarrow \qquad x = -1$

So the required point is (–1, 0).

5. For inconsistency,

$$\frac{k+2}{2} = \frac{6}{3} \neq \frac{-(3k+2)^2}{-4}$$

$\Rightarrow \qquad k + 2 = 4$ and $(3k + 2)^2 \neq 8$

$\Rightarrow \qquad k = 2$ and $k \neq \frac{1}{3}\left(\pm 2\sqrt{2}-2\right)$

Hence, $\qquad k = 2$.

6. Given system of equations can be written as

$$2x + 3y - 18 = 0 \qquad ...(i)$$
$$x - 2y - 2 = 0 \qquad ...(ii)$$

Now, $\qquad \frac{2}{1} \neq \frac{3}{-2}$

Hence the system has unique solution. Now, by cross-multiplication on (*i*) and (*ii*), we get

$$\frac{x}{-6-36} = \frac{-y}{-4+18} = \frac{1}{-4-3}$$

$\Rightarrow \qquad x = 6, y = 2$

Thus, the solution of given system is

$$x = 6, y = 2.$$

7. $x = 5, y = -1$

Hint: Take $\frac{1}{x+y} = u, \frac{1}{x-y} = v$ and solve.

8. Let Meena received x notes of ` 50 and $y$ notes of ` 100

Since total number of notes is 25

$\therefore \qquad x + y = 25 \qquad ...(i)$

Since the value of both types of notes is ` 2000.

$\therefore \qquad 50x+100y = 2000$

$\Rightarrow \qquad x+2y = 40 \qquad ...(ii)$

Solving equations (*i*) and (*ii*), we get

$$x = 10, y = 15$$

Hence, Meena received 10 notes of ` 50 and 15 notes of ` 100.

OR

Let the length and breadth of rectangle be x units and y units respectively.

Then area of rectangle = xy sq. units

Case I. The length is increased and breadth is reduced by 2 units.

$\therefore \qquad (x + 2)(y - 2) = xy - 28$

$\Rightarrow \qquad xy - 4 - 2x + 2y = xy - 28$

$\Rightarrow \qquad x - y = 12 \qquad ...(i)$

Case II. The length is reduced by 1 unit and breadth increased by 2 units.

$\therefore \qquad (x - 1)(y + 2) = xy + 33$

$\Rightarrow \qquad xy - 2 - y + 2x = xy + 33$

$2x - y = 35 \qquad ...(ii)$

Solving equations (*i*) and (*ii*), we get

$$x = 23 \text{ and } y = 11$$

Hence, the length of the rectangle is 23 units and the breadth is 11 units.

9. We have:

$x + 3y = 6 \quad \Rightarrow \quad x = 6 - 3y$

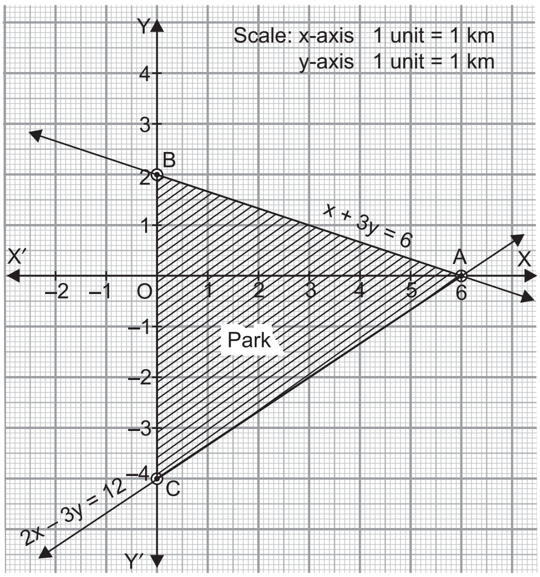
x	6	0
y	0	2

and $2x - 3y = 12 \quad \Rightarrow \quad x = \frac{12+3y}{2}$

x	6	0
y	0	–4

$\therefore$ Also $x = 0$ mean y-axis.

$\therefore$ Graph gives:



Points of intersection are:
A(6, 0) ; B(0, 2), C(0, –4).

Area of ΔABC = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times BC \times AO = \frac{1}{2} \times 6 \times 6$

= 18 sq. km.

General public should keep the park clean and should maintain the greenary.

WORKSHEET – 23

1. For coincident lines,

$$\frac{1}{2} = \frac{2}{k} = \frac{7}{14} \quad \Rightarrow \quad k = 4.$$

2. Given pair of linear equations are

$x - y = 3$ ...(*i*)

and $4x + 2y = 0$ ...(*ii*)

From equation (*i*), we get $x = y + 3$ ...(*iii*)

Now, from equation (*ii*) and (*iii*), we get

$\Rightarrow 4(y + 3) + 2y = 0$

$\Rightarrow 4y + 12 + 2y = 0$

$\Rightarrow 6y = -12$

$\Rightarrow y = -2$

From equation (*iii*), we get

$x = -2 + 3 \Rightarrow x = 1$

Thus, the values of x and y are 1 and –2 respectively.

3. Adding the given equations, we have

$3x = 0 \Rightarrow x = 0$

Substituting $x = 0$ in any of the given equations, we get $y = 0$

Hence, the required solution is $x = 0$, $y = 0$.

4. **False**.

As $\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{5}{10}, \frac{c_1}{c_2} = 6$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

⇒ They are parallel.

5. $a = -1, b = \frac{5}{2}$

Hint: $\frac{2}{2b+1} = \frac{-(2a+5)}{-9} = \frac{5}{15}$.

6. Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$ in given system of equations.

$u + v - 7 = 0$ ...(*i*)

$2u + 3v - 17 = 0$ ...(*ii*)

By cross-multiplication,

$$\frac{u}{-17+21} = \frac{-v}{-17+14} = \frac{1}{3-2}$$

$\Rightarrow u = 4, v = 3$

$\Rightarrow x = \frac{1}{4}, y = \frac{1}{3}$

Hence, $x = \frac{1}{4}, y = \frac{1}{3}$ is the solution of the given system of equations.

7. $x = -2$, $y = 5$ and $m = -1$

Hint: $2x + 3y = 11 \Rightarrow y = \frac{11-2x}{3}$

Substitute this value of y in $2x - 4y = -24$ and solve for x.

8. The given system of linear equations is

$2x - y - 5 = 0$ ...(*i*)

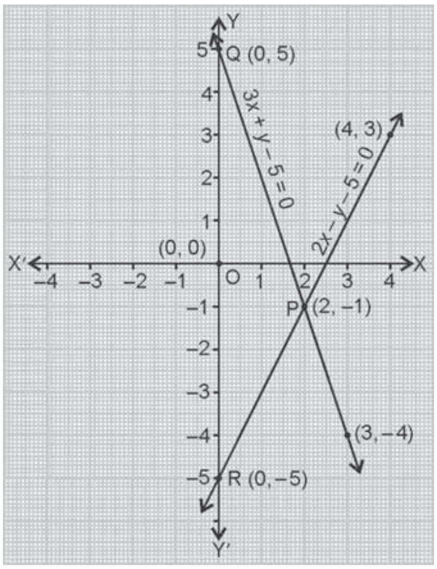
$3x + y - 5 = 0$ ...(*ii*)

To draw the graph of equations (*i*) and (*ii*), we need atleast two solutions of each of the equations, which are given below:

x	0	4
$y = 2x - 5$	–5	3

x	0	3
$y = -3x + 5$	5	–4

Using these points, we are drawing the graphs of lines as shown below:



From the graph, the lines intersect each other at the point P(2, −1). Therefore, the solution is $x = 2$, $y = -1$.

The lines meet the y-axis at the points Q(0, 5) and R(0, −5).

9. Let the fixed charge and additional charge for each day be ` $x$ and ` y respectively. Since Saritha paid ` 27 for a book kept for 7 days

$\therefore$ $x + 4y = 27$ ...(*i*)

Also, Susy paid ` 21 for the book kept for 5 days

$\therefore$ $x + 2y = 21$ ...(*ii*)

Subtracting equation (*ii*) from (*i*), we get

$$2y = 6 \Rightarrow y = 3$$

Again substituting $y = 3$ in equation (*ii*), we get

$$x = 15$$

Hence, the fixed charge is ` 15 and additional charge for each day is ` 3.

OR

Son's age = 10 years, father's age = 40 years.

Hint: Let the present age of father and son be x and y years respectively.

$\therefore$ $x + 5 = 3(y + 5)$

And $x - 5 = 7(y - 5)$.

WORKSHEET – 24

1. $\angle A = 70°$, $\angle B = 53°$, $\angle C = 110°$, $\angle D = 127°$.

Hint: In a cyclic quadrilateral ABCD, $\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$.

2. $x = 0$, $y = 0$

Hint: Both lines are passing through the origin.

3. For infinite number of solutions,

$$\frac{p+q}{2} = \frac{2p-q}{3} = \frac{-21}{-7}$$

$\Rightarrow$ $p + q = 6$ and $2p - q = 9$

$\Rightarrow$ $p = 5$, $q = 1$.

4. **False**.

Hint: As $a + 5b = -10$.

5. **False**, $x = 4$, $y = 1$ does not satisfy the second equation.

6. No solution

Hint: $2x + 3y = 7$, $6x + 9y = 11$.

Here, $\frac{2}{6} = \frac{3}{9} \neq \frac{7}{11}$ Parallel lines.

7. The given system of linear equations can be written as

$$px + qy - (p - q) = 0$$
$$qx - py - (p + q) = 0$$

To solve the system for x and y, using the method of cross-multiplication, we have

$$\frac{x}{-q(p+q)-p(p-q)} = \frac{-y}{-p(p+q)+q(p-q)} = \frac{1}{-p^2-q^2}$$

$$\Rightarrow \quad \frac{x}{-p^2-q^2} = \frac{-y}{-p^2-q^2} = \frac{1}{-p^2-q^2}$$

$\Rightarrow$ $x = 1$, $y = -1$.

8. The given system of equations can be written as

$3x - 4y - 1 = 0$ ...(*i*)

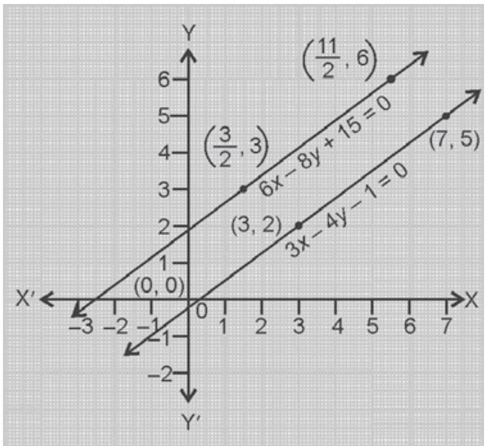
$6x - 8y + 15 = 0$ ...(*ii*)

To draw the graph, we need atleast two solutions for each of the equations (*i*) and (*ii*), which are respectively given below:

x	3	7
$y = \frac{3x-1}{4}$	2	5

x	$\frac{3}{2}$	$\frac{11}{2}$
$y = \frac{6x+15}{8}$	3	6

Let us draw the graph by using these points.



From the graph, it is clear that the lines are parallel. Hence, the given system of equations is inconsistent.

9. Let the fraction be $\frac{x}{y}$

On adding 1 to each of numerator and denominator, the fraction becomes $\frac{6}{5}$

$$\therefore \quad \frac{x+1}{y+1} = \frac{6}{5}$$

$\Rightarrow \quad 5x + 5 = 6y + 6$

$\Rightarrow \quad 5x - 6y = 1$ ...(*i*)

Further, on subtracting 5 from each of numerator and denominator, the fraction becomes $\frac{3}{2}$

$$\therefore \quad \frac{x-5}{y-5} = \frac{3}{2}$$

$\Rightarrow \quad 2x - 10 = 3y - 15$

$\Rightarrow \quad 2x - 3y = -5$ ...(*ii*)

Solving equations (*i*) and (*ii*), we get

$x = 11, y = 9$

Hence, the required fraction is $\frac{11}{9}$.

OR

` 6000, ` 5250

Hint: Let incomes of X and Y be $8x$ and $7x$ respectively; and expenditures of them be $19y$ and $16y$ respectively.

$\therefore \quad 8x - 19y = 1250$ ...(*i*)

$7x - 16y = 1250$ ...(*ii*)

WORKSHEET – 25

1. For no solution, we know that,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{2}{m} \neq \frac{5}{-15}$$

$\Rightarrow \quad m = 6.$

2. Multiplying first equation by 2 and the other one by 3 and adding, we get

$$21.8x = 10.9 \Rightarrow x = \frac{1}{2}$$

Substituting $x = \frac{1}{2}$ in any of the given equations, we have $y = \frac{1}{3}$.

$$\therefore \quad x = \frac{1}{2}, \; y = \frac{1}{3}.$$

3. $k = 6$

Hint: $\frac{k-3}{k} = \frac{3}{k} = \frac{k}{12}$.

4. The condition that the given system of equations represents parallel lines is

$$\frac{p^2+1}{3p+1} = \frac{p-2}{3} \neq \frac{5}{2}$$

$\Rightarrow \quad 5p = -5 \Rightarrow p = -1.$

5. True.

The condition for parallel lines is

$$\frac{2}{6} = \frac{-2}{-6} \neq \frac{-3}{5}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1}{3} \neq \frac{-3}{5}$$

The condition holds. The lines are parallel.

6. $x = a^2$, $y = b^2$

Hint: Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$.

7. Given system of linear equations can be written as:

$(a - b)\,x + (a + b)\,y - (a^2 - 2ab - b^2) = 0$

$(a + b)\,x + (a + b)\,y - (a^2 + b^2) = 0$

By cross-multiplication,

$$\frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}$$

$$= \frac{-y}{-(a-b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}$$

$$= \frac{1}{(a-b)(a+b)-(a+b)(a+b)}$$

$$\Rightarrow \quad \frac{x}{-2b(a+b)^2} = \frac{-y}{-4ab^2} = \frac{1}{-2b(a+b)}$$

Hence, the solution of given system of equations is

$$x = a + b, \; y = -\frac{2ab}{a+b}.$$

8. To draw graph of the equation, we need atleast two solutions.

Two solutions of the equation

$4x + 3y - 24 = 0$ are mentioned in the following table:

x	0	6
y	8	0

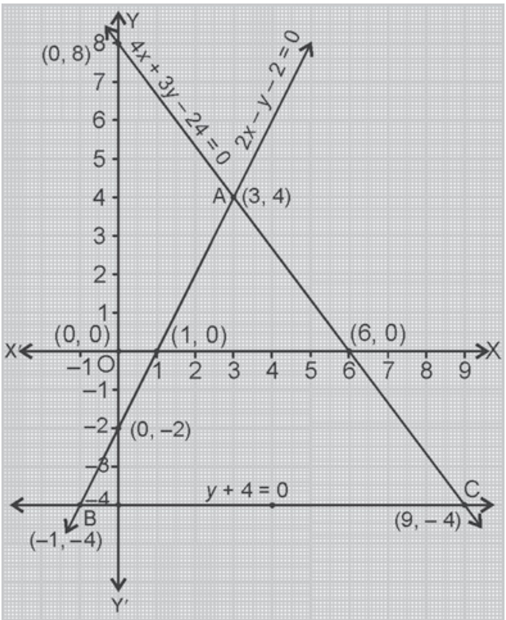
Similarly, two solutions of each of the equations $2x - y - 2 = 0$ and $y + 4 = 0$ are respectively

x	0	1
y	−2	0

and

x	0	4
y	−4	−4

Using the tables obtained above, let us draw the graph.



Observing the graph, we get the lines meet each other pairwise in A(3, 4), B(–1, – 4) and C(9, –4).

Hence, the vertices of the triangle ABC so obtained are A(3, 4), B(–1, –4) and C(9, –4).

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 10 \times 8 = 40$ sq. units.

9. ` 600, ` 700

Hint: Let cost price of trouser be ` $x$ and that of shirt ` y. Then

$$\left.\begin{aligned}\frac{125}{100}x + \frac{110}{100}y = 1520\\ \frac{110}{100}x + \frac{125}{100}y = 1535\end{aligned}\right\}$$

$$\Rightarrow \quad \left.\begin{aligned}25x + 22y = 30400\\ 22x + 25y = 30700\end{aligned}\right\}$$

OR

6 l of 50% and 4 l of 25%.

Hint: Let x litres of 50% acid and y litres of 25% acid should be mixed.

$$\left.\begin{aligned}\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100}(x+y)\\ x + y = 10\end{aligned}\right\}$$

$$\Rightarrow \quad \left.\begin{aligned}2x = 3y\\ x + y = 10\end{aligned}\right\}$$

WORKSHEET – 26

1. $x = 9, y = 6$

Hint: $x - y = 3$ and $2x + 3y = 36$.

2. Solving $3x - 2y = 4$ and $2x + y = 5$, we have $x = 2, y = 1$.

Now, substituting these values of x and y in $y = 2x + m$, we have $1 = 2 \times 2 + m$

$\therefore \ m = -3$.

3. $$\frac{3p}{\sqrt{18}} = \frac{6}{\sqrt{24}} \neq \frac{\sqrt{50}}{\sqrt{75}}$$

$$\Rightarrow \quad \frac{p}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \neq \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore \quad p = \sqrt{3}.$$

4. For inconsistency,

$$\frac{\alpha}{12} = \frac{3}{\alpha} \neq \frac{\alpha-3}{\alpha}$$

$\Rightarrow$ $\alpha^2 = 36$ and $3\alpha \neq \alpha^2 - 3\alpha$

$\Rightarrow$ $\alpha = \pm 6$ and $\alpha \neq 0$ or $\alpha \neq 6 \Rightarrow \alpha = -6$.

5. $x = b, y = -a$

Hint: $a^2x - b^2y = ab(a+b)$, $ax - by = 2ab$

Solving the equations, we get $x = b, y = -a$.

6. $x = \frac{22a}{5}, y = \frac{-26b}{5}$

Hint: $4bx + 3ay - 2ab = 0$

$3bx + ay - 8ab = 0$.

7. $3x + 2y = 800$,

$12x + 8y = 3000$;

Not possible

Hint: Let cost of 1 chair be ` $x$ and that of 1 table be ` y.

$\therefore$ $3x + 2y = 800$, $12x + 8y = 3000$.

8. Let the actual prices of tea-set and lemon-set be ` $x$ and ` y respectively

According to the question,

Case I. Selling price – Cost price = Profit

$\Rightarrow$ $0.95x + 1.15y - (x + y) = 7$

$\Rightarrow$ $-0.05x + 0.15y = 7$ ...(*i*)

Case II. Selling price – Cost price = Profit

$\Rightarrow$ $1.05x + 1.10y - (x + y) = 13$

$\Rightarrow$ $0.05x + 0.10y = 13$ ...(*ii*)

Solving equations (*i*) and (*ii*), we get

$x = 100$, $y = 80$

Hence, actual prices of tea-set and lemon-set are ` 100 and ` 80 respectively.

OR

The person invested ` 500 at the rate of 12% per year and ` 700 at the rate of 10% per year.

Hint: Let the person invested ` $x$ at the rate of 12% per year and ` y at the rate of 10 % per year

$\therefore$ $\frac{12x}{100} + \frac{10y}{100} = 130$

$\Rightarrow$ $6x + 5y = 6500$ ...(*i*)

and $\frac{12y}{100} + \frac{10x}{100} = 134$

$\Rightarrow$ $5x + 6y = 6700$ ...(*ii*)

Adding and subtracting (*i*) and (*ii*), we get

$x + y = 1200$ ...(*iii*)

$x - y = -200$ ...(*iv*)

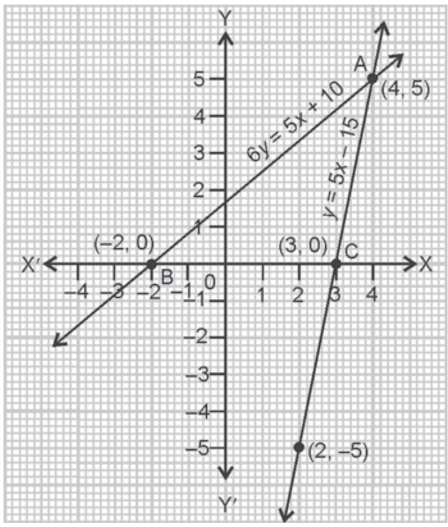
9. Two solutions of $6y = 5x + 10$ are:

x	-2	4
y	0	5

Two solutions of $y = 5x - 15$ are

x	3	2
y	0	-5

Now, we draw the graph of the system on the same coordinate axes.



(*i*) From the graph, we look that the two lines intersect each other at A(4, 5).

(*ii*) The vertices of the triangle: A(4, 5); B(–2, 0); C(3, 0).

Height of ΔABC corresponding to the base BC,

$h = 5$ units

and base, $b = BC = 5$ units

Now, $ar(\Delta ABC) = \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 5 \times 5$$

= 12.5 square units.

WORKSHEET – 27

1. For no solution,

$$\frac{3}{12} = \frac{7}{2k} \neq \frac{k}{4k+1}$$

$$\therefore \quad \frac{3}{12} = \frac{7}{2k} \Rightarrow k = 14.$$

2. $4^{x-y} = 4^2 \Rightarrow x - y = 2$ ...(*i*)

$$\begin{array}{r} x - 2y = 8 \\ - \quad + \quad - \\ \hline y = -6 \end{array}$$ ...(*ii*)

(Subtracting)

$\therefore$ From (*i*) $\Rightarrow x = -4 \quad \therefore \; x + y = -10.$

3. For coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{a+b} = \frac{3}{a+b-3} = \frac{7}{4a+b}$$

$$\Rightarrow \quad a - 5b = 0.$$

4. **False**, because the given pair of equations has infinitely many solutions at $k = 40$ and no solutions at $k \neq 40$.

5. Given equations are

$$2^{y-x}.(x + y) = 1$$

$$\Rightarrow \quad x + y = \frac{1}{2^{y-x}} \quad ...(i)$$

and $(x + y)^{x-y} = 2$ ...(*ii*)

Substituting the value of $x + y$ from equation (*i*) in equation (*ii*), we get

$$\left(\frac{1}{2^{y-x}}\right)^{x-y} = 2$$

$$\Rightarrow \quad (2^{x-y})^{x-y} = 2^1$$

$$\Rightarrow \quad (x - y)^2 = 1$$

$$\Rightarrow \quad x - y = \pm 1$$

$$\Rightarrow \quad x - y = 1 \quad ...(iii)$$

$$\text{or} \quad x - y = -1 \quad ...(iv)$$

Substituting $x - y = 1$ and $x - y = -1$ in equation (*ii*), we get respectively

$$x + y = 2 \quad ...(v)$$

$$\text{and} \quad x + y = \frac{1}{2} \quad ...(vi)$$

Solving equations (*iii*) and (*v*), we have

$$x = \frac{3}{2}\,;\; y = \frac{1}{2}.$$

Therefore, $xy = \frac{3}{4}$

Solving equations (*iv*) and (*vi*), we have

$$x = -\frac{1}{4}\,;\; y = \frac{3}{4}$$

Therefore, $xy = -\frac{3}{16}$.

Hence, $xy = \frac{3}{4}$ or $-\frac{3}{16}$.

6. Given equations can be written as

$$\frac{x}{a} + \frac{y}{b} - (a + b) = 0$$

$$\frac{x}{a^2} + \frac{y}{b^2} - 2 = 0$$

Let us apply cross-multiplication method to solve these equations.

$$\frac{x}{-\frac{2}{b} + \frac{a}{b^2} + \frac{1}{b}} = \frac{-y}{-\frac{2}{a} + \frac{1}{a} + \frac{b}{a^2}} = \frac{1}{\frac{1}{ab^2} - \frac{1}{ba^2}}$$

$$\Rightarrow \quad \frac{b^2x}{-b + a} = \frac{-a^2y}{-a + b} = \frac{a^2b^2}{a - b}$$

Taking $\frac{b^2x}{-b + a} = \frac{a^2b^2}{a - b}$

and $\frac{-a^2y}{-a + b} = \frac{a^2b^2}{a - b}$

$$\Rightarrow \quad x = \frac{a^2b^2(a - b)}{b^2(a - b)} \text{ and } y = \frac{(a - b)\,a^2b^2}{a^2(a - b)}$$

$\Rightarrow \; x = a^2$ and $y = b^2$.

7. Given equations of lines are:

$$3x + y + 4 = 0 \quad ...(i)$$

$$\text{and} \quad 6x - 2y + 4 = 0 \quad ...(ii)$$

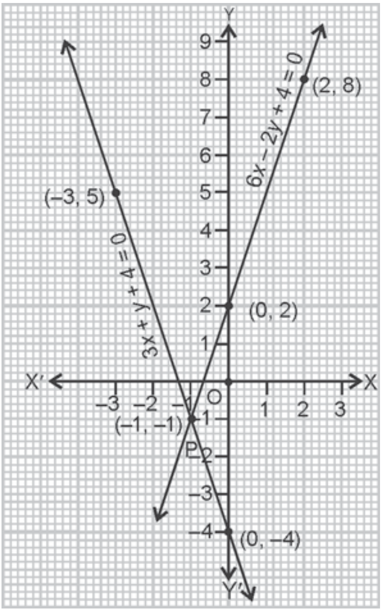
To draw the graphs of lines (*i*) and (*ii*), we need atleast two solutions of each equation. For equation (*i*), two solutions are:

x	0	−3
y	−4	5

For equation (*ii*), two solutions are:

x	0	2
y	2	8

Let us draw the graphs of the lines (*i*) and (*ii*).



From the graph it is clear that the two lines intersect each other at a point, P(–1, –1), therefore, the pair of equations consistent. The solution is $x = -1$, $y = -1$.

8. Let the cost price of the saree and the list price of the sweater be ` $x$ and ` y respectively. Now two cases arise.

Case I.

$$\text{Sale price of the saree} = x + x \times \frac{8}{100} = \frac{108}{100}x$$

$$\text{Sale price of the sweater} = y - y \times \frac{10}{100} = \frac{90}{100}y$$

$$\therefore \quad \frac{108}{100}x + \frac{90}{100}y = 1008$$

$$\Rightarrow \quad 108x + 90y = 100800 \quad ...(i)$$

Case II.

$$\text{Sale price of the saree} = x + x \times \frac{10}{100} = \frac{110x}{100}$$

$$\text{Sale price of the sweater} = y - y \times \frac{8}{100} = \frac{92}{100}y$$

$$\therefore \quad \frac{110}{100}x + \frac{92}{100}y = 1028$$

$$\Rightarrow \quad 110x + 92y = 102800 \quad ...(ii)$$

Adding equations (*i*) and (*ii*), we get

$$218x + 182y = 203600 \quad ...(iii)$$

Subtracting equation (*i*) from (*ii*), we get

$$2x + 2y = 2000$$

or

$$218x + 218y = 218000 \quad ...(iv)$$

(Multiplying by 109)

Solving equations (*iii*) and (*iv*), we have

$$x = 600 \text{ and } y = 400$$

Hence, the cost price of the saree is ` 600 and the list price of the sweater is ` 400.

CHAPTER TEST

1. Let the two numbers of x and y, such that $x > y$.

$\therefore \quad x + y = 35 \quad ...(i)$

and $\quad x - y = 9 \quad ...(ii)$

On solving (*i*) and (*ii*), we get $x = 22$, and $y = 13$

Hence, the two numbers are 22 and 13.

2. 6, 36

Hint: Let the son's age = x,

And father's age = y

$\therefore \quad y = 6x$

and $y + 4 = 4(x + 4)$

Solve yourself.

3. The lines are coincident

$\Rightarrow \quad \frac{3}{6} = \frac{-1}{-k} = \frac{8}{16} \Rightarrow k = 2.$

4. Yes.

For consistency,

either $\frac{2a}{4a} \neq \frac{b}{2b}$ or $\frac{2a}{4a} = \frac{b}{2b} = \frac{-a}{-2a}$

Here only the relation $\frac{2a}{4a} = \frac{b}{2b} = \frac{-a}{-2a}$,

i.e., $\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ holds.

$\Rightarrow$ The pair is consistent.

5.
$$21x + 47y = 110$$
$$47x + 21y = 162$$
$$68x + 68y = 272 \quad \text{(Adding)}$$

$\Rightarrow \quad x + y = 4 \quad ...(i)$

Subtracting the given equations from one another, we get

$$-26x + 26y = -52$$

$\Rightarrow \quad x - y = 2 \quad ...(ii)$

Solve equations (*i*) and (*ii*) to get

$$x = 3, y = 1.$$

6. We are given

$$\frac{2xy}{x+y} = \frac{3}{2} \quad ...(i)$$

and $\frac{xy}{2x-y} = \frac{-3}{10} \quad ...(ii)$

Taking equation (*i*),

$$\frac{2xy}{x+y} = \frac{3}{2}$$

$\Rightarrow \quad 3x + 3y = 4xy \quad ...(iii)$

Now, taking equation (*ii*),

$$\frac{xy}{2x-y} = \frac{-3}{10}$$

$\Rightarrow \quad -6x + 3y = 10xy \quad ...(iv)$

Multiplying equation (*iii*) by 2 and adding its result to (*iv*), we get

$$9y = 18xy$$

$\therefore \quad x = \frac{1}{2}$

Putting $x = \frac{1}{2}$ in equation (*iv*), we get

$\Rightarrow \quad -3 + 3y = 5y$

$\therefore \quad y = \frac{-3}{2}$

Thus, $x = \frac{1}{2}$ and $y = \frac{-3}{2}$.

7. The given system of equations will have infinite number of solutions if

$$\frac{1}{a-b} = \frac{2}{a+b} = \frac{1}{a+b-2}$$

$\Rightarrow \quad \frac{1}{a-b} = \frac{1}{a+b-2}$

and $\frac{2}{a+b} = \frac{1}{a+b-2}$

$\Rightarrow \quad a + b - 2 = a - b$

and $2a + 2b - 4 = a + b$

$\Rightarrow \quad a + b - a + b = 2$ and $a + b = 4$

$\Rightarrow \quad b = 1$ and $a = 3$

Hence, the given system of equations will have infinite number of solutions, if

$$a = 3, b = 1.$$

8. (*i*) Let fixed charge = ` x

and charges for a distance of 1 km = ` y

Now, According to question,

$x + 12y = 89 \quad ...(i)$

$x + 20y = 145 \quad ...(ii)$

We will solve equations (*i*) and (*ii*) by elimination method.

Subtract equation (*ii*) from equation (*i*):

$$-8y = -56 \quad \Rightarrow y = \frac{56}{8} = 7$$

Putting value of y in equation (*i*), we get

$x + 12(7) = 89$

$x = 89 - 84 = 5$

$\therefore \quad x = 5; y = 7$

∴ For a journey of 30 km charge paid = $x + 30y = 5 + 30(7) = 5 + 210 =$ ` 215.

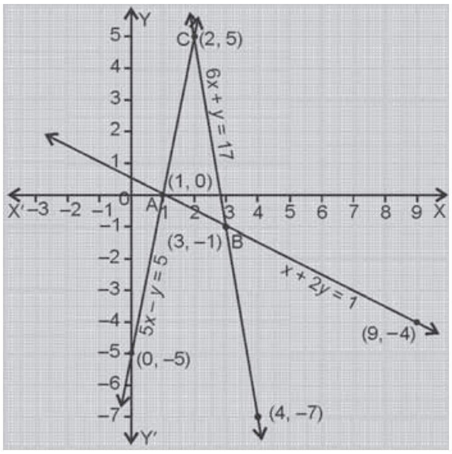
(*ii*) Solution of pair of linear equations in two variables.

(*iii*) **Love towards environment.**

9. To draw the graph of a line, we are required atleast two solutions of its corresponding equation.

At $x = 0$, $5x - y = 5$ gives $y = -5$

At $x = 1$, $5x - y = 5$ gives $y = 0$



Thus, two solutions of $5x - y = 5$ are given in the following table:

x	0	1
y	–5	0

Similarly, we can find the solution of each remaining equation as given in the following tables:

$x + 2y = 1$:

x	9	1
y	–4	0

$6x + y = 17$:

x	2	4
y	5	–7

Now, we will draw the graphs of the three lines on the same coordinate axes.

From the graph, it is clear that the lines form a triangle ABC with vertices A(1, 0), B(3, –1) and C(2, 5)

❑❑

Chapter 4

QUADRATIC EQUATIONS

WORKSHEET – 29

1. Since 1 is a root of $ay^2 + ay + 3 = 0$

$\Rightarrow \quad a(1)^2 + a(1) + 3 = 0$

$\Rightarrow \quad a + a + 3 = 0 \quad \Rightarrow \quad 2a = -3$

$\Rightarrow \quad a = -\dfrac{3}{2}$

Also as 1 is a root of $y^2 + y + b = 0$

$\Rightarrow \quad (1)^2 + (1) + b = 0$

$\Rightarrow \quad 2 + b = 0 \Rightarrow b = -2$

$\therefore \quad ab = \left(-\dfrac{3}{2}\right)(-2) = 3.$

2. $p(1)^2 + p(1) + 3 = 0$

$\Rightarrow \quad 2p = -3 \Rightarrow p = -\dfrac{3}{2}$

and $\quad (1)^2 + 1 + q = 0 \quad \Rightarrow \quad q = -2$

$\therefore \quad pq = \left(-\dfrac{3}{2}\right)(-2) = 3.$

3. $\dfrac{(-5)^2}{-5} + 2(-5 - k) = 0$

$\Rightarrow \quad -10 - 2k = 5$

$\Rightarrow \quad -2k = 15$

$\Rightarrow \quad k = \dfrac{-15}{2}.$

4. We have $x^2 - 4kx + k = 0$

This equation will have equal roots if

$b^2 - 4ac = 0$

$\Rightarrow \quad 16k^2 - 4k = 0$

$\Rightarrow \quad 4k(4k - 1) = 0$

$\Rightarrow \quad k = 0 \text{ or } k = \dfrac{1}{4}.$

5. True

Reason: The value of t for which given equation has real and equal roots are $\pm 2\sqrt{21}$ which are irrational.

6. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \quad \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow \quad x\left(\sqrt{2}x + 5\right) + \sqrt{2}\left(\sqrt{2}x + 5\right) = 0$

$\Rightarrow \quad \left(x + \sqrt{2}\right)\left(\sqrt{2}x + 5\right) = 0$

$\Rightarrow \quad x + \sqrt{2} = 0 \text{ or } \sqrt{2}x + 5 = 0$

$\Rightarrow \quad x = -\sqrt{2} \text{ or } x = \dfrac{-5}{\sqrt{2}}.$

7. $2x^2 - 5x + 3 = 0$

$\Rightarrow \quad x^2 - \dfrac{5}{2}x + \dfrac{3}{2} = 0$

$\Rightarrow \quad x^2 - \dfrac{5}{2}x = -\dfrac{3}{2}$

Adding both sides $\left(\dfrac{5}{4}\right)^2$, we have

$\Rightarrow \quad x^2 - \dfrac{5}{2}x + \left(\dfrac{5}{4}\right)^2 = -\dfrac{3}{2} + \left(\dfrac{5}{4}\right)^2$

$\Rightarrow \quad \left(x - \dfrac{5}{4}\right)^2 = \dfrac{-24 + 25}{16} = \dfrac{1}{16}$

$\Rightarrow \quad x - \dfrac{5}{4} = \pm \dfrac{1}{4}$

$\Rightarrow \quad x = \dfrac{5}{4} + \dfrac{1}{4} \text{ or } \dfrac{5}{4} - \dfrac{1}{4}$

$\Rightarrow \quad x = \dfrac{3}{2} \text{ or } 1.$

8. As $ax^2 + bx + 6 = 0$ doesn't have 2 distinct real roots

$\Rightarrow \quad D \le 0$

$\Rightarrow \quad b^2 - 4ac \le 0$

$\Rightarrow \quad b^2 - 24a \le 0$

$\Rightarrow \quad b^2 \le 24a$

$\Rightarrow \quad b^2 + 8b \le 24a + 8b$

$\Rightarrow \quad b^2 + 8b \le 8(3a + b)$

$\Rightarrow \quad \dfrac{1}{8}(b^2 + 8b) \le 3a + b$

$\Rightarrow \quad \frac{1}{8}[(b+4)^2 - 16] \leq 3a + b$

$\Rightarrow \quad \frac{1}{8}(b+4)^2 - 2 \leq 3a + b$

$\therefore$ Minimum value of $3a + b = -2$.

9. Let speed of stream $= x$ km/h

$\therefore$ upstream speed $= (18 - x)$ km/h

downstream speed $= (18 + x)$ km/h

Time taken to cover upstream distance of

$$24 \text{ km} = \frac{24}{18-x}$$

Time taken to cover downstream distance of

$$24 \text{ km} = \frac{24}{18-x}$$

According to question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\Rightarrow \quad 24\left[\frac{18+x-18+x}{324-x^2}\right] = 1$$

$\Rightarrow \quad 48x = 324 - x^2$

$\Rightarrow \quad x^2 + 48x - 324 = 0$

$x^2 + 54x - 6x - 324 = 0$

$\Rightarrow \quad x(x+54) - 6(x+54) = 0$

$\Rightarrow \quad (x-6)(x+54) = 0$

$\Rightarrow \quad x = 6$ or $x = -54$ (Reject)

$\therefore \quad x = 6$ km/h.

OR

Let size of square be x

$\therefore$ No. of students in square $= x^2$

$\therefore$ According to question,

Case I: Total students $= x^2 + 24$

Case II: Also total students $= (x+1)^2 - 25$

$\therefore \quad x^2 + 24 = (x+1)^2 - 25$

$\Rightarrow \quad x^2 + 24 = x^2 + 2x + 1 - 25$

$\Rightarrow \quad 2x = 48 \quad \Rightarrow \quad x = 24$

Number of students $= (24)^2 + 24$

$= 576 + 24 = 600.$

WORKSHEET – 30

1. S = 3; P = –5

$\therefore$ Required equation can be: $x^2 - Sx + P = 0$

i.e., $x^2 - 3x - 5 = 0$.

2. Let $\alpha = 2 + \sqrt{3}$; $\beta = 2 - \sqrt{3}$ be roots.

$\therefore \quad \alpha + \beta = 4; \alpha\beta = (2)^2 - (\sqrt{3})^2 = 4 - 3 = 1$

$\therefore$ Quadratic equation can be:

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e., $x^2 - 4x + 1 = 0$

3. Since $\frac{2}{3}$ and -3 are roots of equation

Sum of roots $= \frac{2}{3} - 3 = \frac{-7}{m} \Rightarrow m = 3$

Product of roots $= \frac{2}{3}(-3) = \frac{n}{m} \Rightarrow n = -6$.

4. No, as given equation is:

$x^2 + x + 8 = x^2 - 4$

$\Rightarrow \quad x - 12 = 0$

$\therefore$ It is a linear equation.

5. False.

$\because \quad x^2 - 3x + 1 = 0$

is an equation with integral coefficients but its roots are not integers.

6. Given equation can be written as:

$x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$

$\Rightarrow \quad x^2 + 5x - [a(a+3) - 2(a+3)] = 0$

$\Rightarrow \quad x^2 + 5x - (a-2)(a+3) = 0$

$\Rightarrow \quad x^2 + (a+3)x - (a-2)x - (a-2)(a+3) = 0$

{$\because$ By splitting the middle term}

$\Rightarrow x\{x + (a+3)\} - (a-2)\{x + (a+3)\} = 0$

$\Rightarrow \quad (x + a + 3)(x - a + 2) = 0$

$\Rightarrow \quad x + a + 3 = 0$ or $x - a + 2 = 0$

$\Rightarrow \quad x = -(a+3)$ or $x = a - 2$.

7. $4x^2 - 4ax + (a^2 - b^2) = 0$

$\Rightarrow 4x^2 - [(2a - 2b) + (2a + 2b)]x + (a^2 - b^2) = 0$

$\Rightarrow 4x^2 - (2a - 2b)x - (2a + 2b)x + (a^2 - b^2) = 0$

$\Rightarrow \quad 2x[2x - a + b] - (a + b)[2x - a + b] = 0$

$\Rightarrow \quad [2x - (a + b)][2x - a + b] = 0$

$\Rightarrow \quad 2x - (a + b) = 0$ or $2x - a + b = 0$

$\Rightarrow \quad x = \frac{a+b}{2}$ or $x = \frac{a-b}{2}$.

OR

$$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$\Rightarrow \quad \sqrt{3}x\left(\sqrt{3}x - \sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}x - \sqrt{2}\right) = 0$$

$$\Rightarrow \quad \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$$

$\Rightarrow \quad \sqrt{3}x - \sqrt{2} = 0$ or $\sqrt{3}x - \sqrt{2} = 0$

$\Rightarrow \; x = \frac{\sqrt{2}}{\sqrt{3}} \; ; \; \frac{\sqrt{2}}{\sqrt{3}} \;\Rightarrow\; x = \frac{\sqrt{6}}{3} \; ; \; \frac{\sqrt{6}}{3}$.

8. $x = 0; 2(a+b)$

Hint: Given equation is:

$$x^2 - x\,(2a + 2b) + 4ab = 4ab$$

$$\Rightarrow \quad x^2 - 2x\,(a + b) = 0.$$

9. Let Zeba's present age = x yrs.

$\therefore$ Zeba's age 5 yrs ago = $(x - 5)$ yrs.

According to question $(x - 5)^2 = 5x + 11$

$\Rightarrow \quad x^2 + 25 - 10x = 5x + 11$

$\Rightarrow \quad x^2 - 15x + 14 = 0$

$\Rightarrow \quad x^2 - 14x - x + 14 = 0$

$\Rightarrow \quad x(x - 14) - 1(x - 14) = 0$

$x = 1$ or $x = 14$

$x = 1$ is not possible

$\therefore$ Present age of Zeba = 14 yrs.

OR

Let Ist part = x (larger)

$\therefore$ 2nd part = $16 - x$ (smaller)

According to question,

$(16 - x)^2 + 164 = 2x^2$

$256 + x^2 - 32x + 164 = 2x^2$

$\Rightarrow \quad x^2 + 32x - 420 = 0$

$\Rightarrow \quad x^2 + 42x - 10x - 420 = 0$

$\Rightarrow \quad x(x + 42) - 10\,(x + 42) = 0$

$\Rightarrow \quad (x - 10)\,(x + 42) = 0$

$\Rightarrow \quad x = 10$ or $x = -42$ (Reject)

$\therefore$ Larger part = 10

Smaller part = 6.

WORKSHEET – 31

1. $2(x^2 - x) = 3 \;\Rightarrow\; 2x^2 - 2x - 3 = 0$

Here, $D = 4 + 4 \times 2 \times 3 = 28 > 0$.

$\Rightarrow$ So, roots are real and distinct.

2. For real and equal roots,

$D = 0 \;\Rightarrow\; 9k^2 - 4 \times 4 \times 1 = 0 \;\Rightarrow\; k = \pm\frac{4}{3}$.

3. No.

We have, $x^2 - 2x + 8 = 0$

Put $x = -2$, we get

$\Rightarrow \quad (-2)^2 - 2(-2) + 8 = 0$

$16 = 0$ which is wrong.

Therefore, $x = -2$ does not satisfy given equation.

4. Given equation is $2x^2 - 14x - 1 = 0$

$\therefore \quad D = b^2 - 4ac$

$D = (-14)^2 - 4(2)\,(-1)$

$= 196 + 8 = 204$

$\therefore \quad x = \frac{14 \pm \sqrt{204}}{4} = \frac{7 \pm \sqrt{51}}{2}$.

5. True.

$D = b^2 - 4ac$

Put $b = 0$ and $a = 1$

$D = -4c$

As $c < 0 \;\Rightarrow\; -4c > 0 \;\Rightarrow\; D > 0$

$\Rightarrow$ Roots are real

Also, sum of roots = $-\frac{b}{a} = 0$

$\Rightarrow$ Roots are numerically equal and opposite in sign.

6. Quadratic equation written as:

$$(2x)^2 + 2 \times (2x) \times \frac{3}{4} + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + 5 = 0$$

$$\Rightarrow \left(2x + \frac{3}{4}\right)^2 - \frac{9}{16} + 5 = 0$$

$$\Rightarrow \quad \left(2x + \frac{3}{4}\right)^2 = \frac{-71}{16} < 0$$

But $\left(2x + \frac{3}{4}\right)^2$ cannot be negative for any real value of x. So there is no real value of x satisfying the given equation. Therefore, the given equation has no real roots.

7. Let another root be α.

Product of roots $= 2\alpha = \frac{-6}{2} \;\Rightarrow\; \alpha = -\frac{3}{2}$

Sum of roots $= -\frac{k}{2} \;\Rightarrow\; -\frac{3}{2} + 2 = -\frac{k}{2}$

$\Rightarrow \quad k = -1$

Thus, $k = -1$ and another root $= -\frac{3}{2}$.

8. Let $y = \frac{x-1}{2x-1}$

∴ Given equation can be written as:

$$\Rightarrow \quad y + \frac{1}{y} = \frac{5}{2}$$
$$\Rightarrow \quad 2y^2 + 2 = 5y$$
$$\Rightarrow \quad 2y^2 - 5y + 2 = 0$$
$$\Rightarrow \quad 2y^2 - 4y - y + 2 = 0$$
$$\Rightarrow \quad 2y(y-2) - 1(y-2) = 0$$
$$\Rightarrow \quad (2y-1)(y-2) = 0$$
$$\Rightarrow \quad y = \frac{1}{2} \text{ or } y = 2$$
$$\Rightarrow \quad \frac{x-1}{2x-1} = \frac{1}{2} \text{ or } \frac{x-1}{2x-1} = 2$$
$$\Rightarrow \quad 2x - 2 = 2x - 1 \text{ Not possible}$$
$$\text{or } x - 1 = 4x - 2$$
$$\Rightarrow \quad 1 = 3x$$
$$\Rightarrow \quad x = \frac{1}{3}.$$

9. 15 hrs or 25 hrs

Hint: Let smaller tap takes x hrs to fill the tank itself.

∴ Larger tap will take $(x-10)$ hrs to fill the tank itself.

∴ Given situation can be expressed as:

$$\frac{1}{x-10} + \frac{1}{x} = \frac{8}{75}$$
$$\Rightarrow \quad 4x^2 - 115x + 375 = 0$$
$$\Rightarrow \quad x = \frac{15}{4} \text{ (cannot be taken) or } 25.$$

OR

Let one natural number is x.

∴ Second natural number is $x + 5$.

According to question,

$$\frac{1}{x} - \frac{1}{x+5} = \frac{1}{10}$$
$$\Rightarrow \quad \frac{x+5-x}{x(x+5)} = \frac{1}{10}$$
$$\Rightarrow \quad 50 = x(x+5)$$
$$\Rightarrow \quad x^2 + 5x - 50 = 0$$
$$\Rightarrow \quad x^2 + 10x - 5x - 50 = 0$$
$$\Rightarrow \quad x(x+10) - 5(x+10) = 0$$
$$\Rightarrow \quad (x-5)(x+10) = 0$$
$$\Rightarrow \quad x = 5 \text{ or } x = -10$$

(Reject as x is natural)

∴ Required numbers are 5 and 10.

WORKSHEET – 32

1. $D = \left(4\sqrt{3}\right)^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

Since, the discriminant is zero, therefore, the given equation has real and equal roots.

2. $x^2 - 4x + p = 0$

For real roots, $D \geq 0$

$\Rightarrow (-4)^2 - 4 \times 1 \times p \geq 0$

$\Rightarrow 16 - 4p \geq 0 \quad \Rightarrow \quad 4p \leq 16 \quad \Rightarrow \quad p \leq 4.$

3. For equal roots, $D = 0$

$\Rightarrow (6k)^2 - 4 \times 9 \times 4 = 0$

$\Rightarrow 36k^2 = 4 \times 36 \Rightarrow k = \pm 2.$

4. -5 must satisfy $2x^2 + px - 15 = 0$,

i.e., $2 \times 25 - 5p - 15 = 0 \Rightarrow p = 7$

As $p(x^2+x) + k = 0$, *i.e.*, $px^2+px+k = 0$ has equal roots,

$$D = 0 \quad \Rightarrow \quad p^2 - 4pk = 0$$

But $\quad p = 7, \quad \therefore (7)^2 - 4(7)k = 0.$

$$\Rightarrow 4 \times 7k = 7 \times 7 \quad \Rightarrow \quad k = \frac{7}{4}.$$

5.
$$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$
$$\Rightarrow \quad \sqrt{3}x\left(\sqrt{3}x - \sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}x - \sqrt{2}\right) = 0$$
$$\Rightarrow \quad \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$$
$$\Rightarrow \quad \sqrt{3}x - \sqrt{2} = 0 \text{ or } \sqrt{3}x - \sqrt{2} = 0$$
$$\Rightarrow \quad x = \frac{\sqrt{2}}{\sqrt{3}} \; ; \; \frac{\sqrt{2}}{\sqrt{3}} \quad \Rightarrow \quad x = \frac{\sqrt{6}}{3} \; ; \; \frac{\sqrt{6}}{3}.$$

6.
$$2\sqrt{2}x^2 + \sqrt{15}x + \sqrt{2} = 0$$
$$\Rightarrow \quad x^2 + \frac{\sqrt{15}}{2\sqrt{2}}x + \frac{1}{2} = 0$$
$$\Rightarrow \quad x^2 + \frac{\sqrt{15}}{2\sqrt{2}}x = -\frac{1}{2}.$$

Add $\left(\frac{15}{4\sqrt{2}}\right)^2$ to both sides,

$$\Rightarrow x^2 + \frac{\sqrt{15}}{2\sqrt{2}}x + \left(\frac{\sqrt{15}}{4\sqrt{2}}\right)^2 = -\frac{1}{2} + \left(\frac{\sqrt{15}}{4\sqrt{2}}\right)^2$$

$$\Rightarrow \left(x + \frac{\sqrt{15}}{4\sqrt{2}}\right)^2 = -\frac{1}{2} + \frac{15}{32}$$

$$= \frac{-16+15}{32}$$

$$\therefore \left(x + \frac{\sqrt{15}}{4\sqrt{2}}\right)^2 = -\frac{1}{32}$$

which is not possible as square of any real number can't be negative.

∴ No real roots possible.

7. $x = \frac{1}{2}$ or $\frac{4}{3}$

Hint: Let $y = \frac{2x-3}{x-1}$ $\therefore$ $\frac{1}{y} = \frac{x-1}{2x-3}$

Given equation becomes: $y - \frac{4}{y} = 3$

Now solve.

8. Let the numbers are : x, $x + 1$, $x + 2$.

According to question,

$$(x + 1)^2 = (x + 2)^2 - x^2 + 60$$

$$\Rightarrow x^2 + 2x + 1 = x^2 + 4x + 4 - x^2 + 60$$

$$\Rightarrow x^2 + 2x - 4x + 1 - 64 = 0$$

$$\Rightarrow x^2 - 2x - 63 = 0$$

$$\Rightarrow x^2 - 9x + 7x - 63 = 0$$

$$\Rightarrow x(x - 9) + 7(x - 9) = 0$$

$$\Rightarrow (x - 9)(x + 7) = 0$$

$$\Rightarrow x - 9 = 0 \text{ or } x + 7 = 0$$

$$\Rightarrow x = 9 \text{ or } x = -7$$

as x has to be a natural number ∴ Reject $x = -7$

⇒ Required numbers are : 9, 10, 11.

9. (*i*) Let the cost price of each glass plate be ` x and the total number of glass plates bought be n.

Then, total CP = nx = 900

This gives, $x = \frac{900}{n}$ …(*i*)

∴ Selling price of each glass plate = ` $(x + 2)$

And number of glass plates sold = $n - 5$

∴ Total SP = $(n - 5)(x + 2)$

Now, SP – CP = Profit

$$\therefore (n - 5)(x + 2) - nx = 80$$

$$\Rightarrow nx + 2n - 5x - 10 - nx = 80$$

$$\Rightarrow 2n - 5x = 90$$

$$\Rightarrow 2n - \frac{5 \times 900}{n} = 90 \Rightarrow 2n^2 - 4500 = 90n$$

$$\Rightarrow n^2 - 45n - 2250 = 0$$

$$\Rightarrow n^2 - 75n + 30n - 2250 = 0$$

$$\Rightarrow n(n - 75) + 30(n + 75) = 0$$

$$\Rightarrow (n - 75)(n + 30) = 0$$

$$\Rightarrow n = 75 \text{ or } n = -30$$

$n = -30$ is not possible ∴ $n = 75$.

Thus the trader bought 75 glass plates.

(*ii*) Self-reliance being industrious and rationality.

WORKSHEET – 33

1. For equal roots, D = 0

$$\therefore b^2 - 4ac = 0 \Rightarrow c = \frac{b^2}{4a}.$$

2. Sum of roots = $-\frac{-l}{1} \Rightarrow l + m = l$

⇒ $m = 0$ and l can take any real value, *e.g.*, $m = 0$, $l = -2$.

Product of roots = $\frac{m}{1} \Rightarrow lm = m$

⇒ $m(l - 1) = 0$ ⇒ $m = 0$, $l = 1$.

3. For real roots: D ≥ 0

$$\Rightarrow b^2 - 4ac \geq 0$$

$$\Rightarrow k^2 - 4(5)(5) \geq 0$$

$$\Rightarrow k^2 - 100 \geq 0$$

$$\Rightarrow (k - 10)(k + 10) \geq 0$$

$$\Rightarrow k \leq -10 \text{ or } k \geq 10.$$

4. $px(x - 2) + 6 = 0$

$$\Rightarrow px^2 - 2px + 6 = 0$$

For equal roots: D = 0

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (-2p)^2 - 4(p)(6) = 0$$

$$\Rightarrow 4p^2 - 24p = 0$$

$$\Rightarrow 4p(p - 6) = 0$$

$$\Rightarrow p = 0 \text{ or } p = 6 \text{ as } p \neq 0$$

$$\Rightarrow p = 6.$$

5. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$$\Rightarrow x^2 + \frac{10}{\sqrt{3}}x + 7 = 0$$

$$\Rightarrow \quad \left(x+\frac{5}{\sqrt{3}}\right)^2-\frac{25}{3}+7=0$$

$$\Rightarrow \quad \left(x+\frac{5}{\sqrt{3}}\right)^2-\left(\frac{2}{\sqrt{3}}\right)^2=0$$

$$\Rightarrow \left(x+\frac{5}{\sqrt{3}}+\frac{2}{\sqrt{3}}\right)\left(x+\frac{5}{\sqrt{3}}-\frac{2}{\sqrt{3}}\right)=0$$

$$\Rightarrow \; x=-\frac{7}{\sqrt{3}},-\sqrt{3}.$$

6. $D = (8ab)^2 - 4(3a^2)(4b^2)$

$= 64a^2b^2 - 48a^2b^2 = 16a^2b^2 = (4ab)^2 \geq 0$

$$\therefore x = \frac{-b\pm\sqrt{D}}{2a}=\frac{-8ab\pm 4ab}{2\times 3a^2}$$

$$= \frac{-12ab}{6a^2} \text{ or } \frac{-4ab}{6a^2}$$

$$\Rightarrow x = \frac{-2b}{a} \text{ or } x = \frac{-2b}{3a}.$$

7. $p = 14$

Hint: For real and equal roots

$D = 0$

$\therefore$ Take $a = p - 12$

$b = 2(p - 12)$

$c = 2$

$b^2 - 4ac = 0.$

8.
$$\frac{4}{x}-3=\frac{5}{2x+3}$$

$$\Rightarrow \frac{4-3x}{x}=\frac{5}{2x+3}$$

$\Rightarrow (4-3x)(2x+3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x + 2) - (x + 2) = 0$

$\Rightarrow (x - 1)(x + 2) = 0$

$\Rightarrow x = 1$ or $x = -2.$

9. Son = 2 years; Father = 22 years

Hint: Let boy's present age $= x$

$\therefore$ Father's present age $= 24 - x$

According to question,

$$\frac{1}{4}x.(24-x) = x+9$$

Now solve.

OR

(*i*) Let the number of arrows Arjun used be n.

Number of arrows to cut down arrow used by Bheeshma $= \frac{n}{2}$

Number of arrows used to kill the rath driver = 6

Number of arrows to knock down the rath, flag and bow of Bheeshma

= 1 + 1 + 1 = 3

Number of arrow used in arrow bed

$= 4\sqrt{n} + 1$

Hence number of arrows Arjun used in all

$$= \frac{n}{2}+6+3+4\sqrt{n}+1 = 10+\frac{n}{2}+4\sqrt{n}$$

This must be equal to the number of arrows we supposed

$$\therefore \; 10+\frac{n}{2}+4\sqrt{n}=n$$

$\Rightarrow 20 + n + 8\sqrt{n} = 2n \Rightarrow 8\sqrt{n} = n - 20$

$\Rightarrow 64n = n^2 - 40n + 400$

$\Rightarrow n^2 - 104n + 400 = 0$

$\Rightarrow n^2 - 4n - 100n + 400 = 0$

$\Rightarrow n(n - 4) - 100(n - 4) = 0$

$\Rightarrow (n - 4)(n - 100) = 0 \Rightarrow n = 4, 100$

$n \neq 4$ as 6 arrows have already been used to kill the rath driver.

$\therefore n = 100$

Hence, the required number of arrows is 100.

(*ii*) Forming and solving a quadratic equation.

(*iii*) Courage and mutual respect.

WORKSHEET – 34

1. We have, $kx^2 - 2kx + 6 = 0$

For real, equal roots $D = 0$

$\Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k - 6) = 0$

$\Rightarrow \quad k = 0 \text{ or } k = 6$

$\therefore \quad k = 6.$

2. We have $3x^2 - 2x + \frac{1}{3} = 0$

$a = 3, b = -2, c = \frac{1}{3}$

$D = b^2 - 4ac$

$D = (-2)^2 - 4 \times 3 \times \frac{1}{3}$

$D = 4 - 4 = 0$

3. If $x = -2$ is a root of the equation, then

$k(-2)^2 + 5(-2) - 3k = 0 \Rightarrow k - 10 = 0$

$\Rightarrow k = 10.$

4. Yes.

At $x = \frac{2}{3}$, $9x^2 - 3x - 2 = 9\left(\frac{2}{3}\right)^2 - 3\left(\frac{2}{3}\right) - 2$

$= 4 - 4 = 0$

At $x = -\frac{1}{3}$, $9x^2 - 3x - 2 = 9\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) - 2$

$= 2 - 2 = 0$

Clearly, both the values of $x = \left(\frac{2}{3}, -\frac{1}{3}\right)$ satisfy the equation $9x^2 - 3x - 2 = 0$, so,

$x = \left(\frac{2}{3}, -\frac{1}{3}\right)$ are the roots of it.

5. Consider, $\quad \alpha + \beta = 4$

$\Rightarrow \quad 2 - \sqrt{3} + \beta = 4$

$\Rightarrow \quad \beta = 2 + \sqrt{3}$

$\therefore$ Equation is: $(x - 2 + \sqrt{3})(x - 2 - \sqrt{3}) = 0$

$\Rightarrow \quad (x - 2)^2 - \left(\sqrt{3}\right)^2 = 0$

$\Rightarrow \quad x^2 + 4 - 4x - 3 = 0$

$\Rightarrow \quad x^2 - 4x + 1 = 0.$

6. $\quad \frac{x - 1 + 2x - 4}{(x-2)(x-1)} = \frac{6}{x}$

$\Rightarrow \quad (3x - 5)x = 6(x^2 - 3x + 2)$

$\Rightarrow \quad 3x^2 - 5x = 6x^2 - 18x + 12$

$\Rightarrow \quad 3x^2 - 13x + 12 = 0$

$\Rightarrow \quad 3x^2 - 9x - 4x + 12 = 0$

$\Rightarrow \quad 3x(x-3) - 4(x-3) = 0$

$\Rightarrow \quad (3x-4)(x-3) = 0$

$\Rightarrow \quad x = \frac{4}{3} \text{ or } x = 3.$

7. Discriminant for $\sqrt{3}\, x^2 + 10x - 8\sqrt{3} = 0$ is given by

$D = 10^2 - 4 \times \sqrt{3} \times (-8\sqrt{3})$

$= 100 + 96 = 196$

$\Rightarrow \quad D > 0$

As D > 0, the given equation has real roots.

Now, $\quad x = \frac{-10 \pm \sqrt{196}}{2\sqrt{3}} = \frac{-10 \pm 14}{2\sqrt{3}}$

$\Rightarrow \quad x = -4\sqrt{3}, \frac{2}{\sqrt{3}}$

Thus, the given equation has real roots which are $-4\sqrt{3}$ and $\frac{2}{\sqrt{3}}$.

8. $\quad x^2 - 4ax - b^2 + 4a^2 = 0$

$\Rightarrow \quad x^2 - (2a + b)x - (2a - b)x - b^2 + 4a^2 = 0$

$\Rightarrow x[x - (2a + b)] - (2a - b)[x - (2a + b)] = 0$

$\Rightarrow \quad [x - (2a + b)][x - (2a - b)] = 0$

$\Rightarrow \quad x = 2a + b \text{ or } x = 2a - b.$

OR

Let first number = x

$\therefore$ 2nd number = $8 - x$

$\therefore$ According to questions, $x(8 - x) = 15$

$\Rightarrow \quad 8x - x^2 = 15$

$\Rightarrow \quad x^2 - 8x + 15 = 0$

$\Rightarrow \quad x^2 - 5x - 3x + 15 = 0$

$\Rightarrow \quad x(x - 5) - 3(x - 5) = 0$

$\Rightarrow \quad (x - 5)(x - 3) = 0$

$\Rightarrow \quad x = 5 \text{ or } x = 3$

$\therefore$ two numbers are: 3 and 5.

9. (*i*) Let usual speed = x km/hr

As distance = Time × Speed

$\therefore \quad \text{Usual time} = \frac{1500}{x}$

$\text{New speed} = (x + 250)$

$\therefore \quad \text{New time} = \frac{1500}{(x+250)}$

According to question

Usual time of flight – New time of flight

$= 30 \text{ minutes} = \frac{1}{2}\text{h}$

$\Rightarrow \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

$\Rightarrow 1500\left[\frac{x + 250 - x}{x(x+250)}\right] = \frac{1}{2}$

$\Rightarrow\ 3000 \times 250 = x(x + 250)$

$\Rightarrow\ x^2 + 250x - 750000 = 0$

$\Rightarrow\ x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow\ x(x + 1000) - 750(x + 1000) = 0$

$\Rightarrow\ (x - 750)(x + 1000) = 0$

$\Rightarrow\ x = 750$ or $x = -1000$ (Rejected)

$\therefore$ Usual speed = 750 km/h

(*ii*) Formation and solving a quadratic equation by splitting the middle term.

(*iii*) Punctuality of pilot is reflected in this problem.

WORKSHEET – 35

1. For real roots, D ≥ 0

$\therefore\ (-3p)^2 - 4\times4\times9 \geq 0 \Rightarrow 9p^2 \geq 4\times4\times9$

$\Rightarrow p \geq 4$ or $p \leq -4$.

2. $x = \frac{5}{2}$ must satisfy $2x^2 - 8x - m = 0$

$\therefore\ 2\left(\frac{5}{2}\right)^2 - 8\left(\frac{5}{2}\right) - m = 0 \Rightarrow m = -\frac{15}{2}$.

3. Let $y = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+....}}}}$.

$\therefore\ y = \sqrt{6+y} \Rightarrow y^2 - y - 6 = 0$

$\Rightarrow (y-3)(y+2) = 0$

$\Rightarrow y = 3$ or -2 (Reject).

4. $4x^2 + 4\sqrt{3}x + 3 = 0 \Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = 0$

$\Rightarrow\ x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$

$\Rightarrow\ \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0 \Rightarrow x + \frac{\sqrt{3}}{2} = 0$

$\Rightarrow\ x = -\frac{\sqrt{3}}{2}$.

5. $D = b^2 - 4ac$

$= (\sqrt{3} + 1)^2 - 4(1)(\sqrt{3})$

$= 3 + 1 + 2\sqrt{3} - 4\sqrt{3}$

$= 4 - 2\sqrt{3} = (\sqrt{3} - 1)^2 > 0$

$\therefore\ x = \frac{-b \pm \sqrt{D}}{2a} \Rightarrow x = \frac{\sqrt{3} + 1 \pm (\sqrt{3} - 1)}{2}$

$x = \sqrt{3}, 1$.

6. Given equation is

$$\sqrt{\frac{x^2+2}{x^2-2}} + 6\sqrt{\frac{x^2-2}{x^2+2}} = 5 \qquad ...(i)$$

Putting $y = \sqrt{\frac{x^2+2}{x^2-2}}$ so that $\frac{1}{y} = \sqrt{\frac{x^2-2}{x^2+2}}$, equation (*i*) reduces to

$$y + \frac{6}{y} = 5 \Rightarrow y^2 - 5y + 6 = 0$$

$\Rightarrow\ (y - 3)(y - 2) = 0 \Rightarrow y = 2$ or 3

Case I. If $y = 2$

$\sqrt{\frac{x^2+2}{x^2-2}} = 2 \Rightarrow x^2 + 2 = 4x^2 - 8$

$\Rightarrow 3x^2 = 10 \Rightarrow x = \pm\sqrt{\frac{10}{3}}$

Case II. If $y = 3$

$\sqrt{\frac{x^2+2}{x^2-2}} = 3 \Rightarrow x^2 + 2 = 9x^2 - 18$

$\Rightarrow\ 8x^2 = 20 \Rightarrow x = \pm\sqrt{\frac{5}{2}}$

Hence, $x = \pm\sqrt{\frac{10}{3}}, \pm\sqrt{\frac{5}{2}}$.

7. $(k + 4)x^2 + (k + 1)x + 1 = 0$ will have equal roots if

$D = 0$

$\Rightarrow\ b^2 - 4ac = 0$

$\Rightarrow\ (k + 1)^2 - 4(k + 4)(1) = 0$

$\Rightarrow\ k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow\ k^2 - 2k - 15 = 0$

$\Rightarrow\ k^2 - 5k + 3k - 15 = 0$

$\Rightarrow\ k(k - 5) + 3(k - 5) = 0$

$\Rightarrow\ (k + 3)(k - 5) = 0$

$\Rightarrow\ k = -3$ or $k = 5$

8. 25 min and 20 min.

Hint: Use: $\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100}$.

OR

750 km/h

Hint: Use: $= \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

where x = usual speed.

WORKSHEET – 36

1. For equal roots, D = 0

$\therefore\ 4(a^2c^2+b^2d^2+2abcd) - 4(a^2c^2+a^2d^2+b^2c^2 + b^2d^2) = 0$

$\Rightarrow 8abcd = 4(a^2d^2 + b^2c^2)$

$\Rightarrow a^2d^2 + b^2c^2 - 2abcd = 0$

$\Rightarrow (ad - bc)^2 = 0 \Rightarrow ad = bc.$

2. Let us find the discriminant of equation $x^2 - 4x + 3\sqrt{2} = 0$.

$D = (-4)^2 - 4 \times 1 \times 3\sqrt{2} = 16 - 12\sqrt{2}$

$= 16 - 16.97$

$\Rightarrow D < 0.$

Therefore, $x^2 - 4x + 3\sqrt{2}$ has no real roots.

3. Given equation is:

$x^2 + ax - 4 = 0$

$D = b^2 - 4ac$

$= a^2 - 4(-4)$

$= a^2 + 16 > 0$

As D > 0, two real and distinct roots exist.

4. For no real roots, D < 0

$\therefore\ k^2 - 4 \times 1 \times 1 < 0 \Rightarrow k^2 - 2^2 < 0$

$\Rightarrow (k-2)(k+2) < 0 \Rightarrow -2 < k < 2.$

5. Let the required whole number be x.

$\therefore \quad x - 20 = 69 \times \frac{1}{x}$

$\Rightarrow \quad x^2 - 20x = 69$

$\Rightarrow \quad x^2 - 20x - 69 = 0$

$\Rightarrow \quad x^2 - (23-3)x - 23 \times 3 = 0$

$\Rightarrow \quad (x-23)(x+3) = 0$

$\Rightarrow \quad x = 23 \text{ or } x = -3$

But -3 is not a whole number

$\therefore \quad x = 23.$

6. $x = \frac{2a+b}{3}, \frac{a+2b}{3}$

Hint: ***See solved example 5(ii)***.

7. $(x-5)(x-6) = \frac{25}{(24)^2}$

$\Rightarrow \quad x^2 - 11x = \frac{25}{(24)^2} - 30$

Add $\left(\frac{11}{2}\right)^2$ to both sides.

$\Rightarrow \quad x^2 - 11x + \left(\frac{11}{2}\right)^2 = \frac{25}{(24)^2} - 30 + \left(\frac{11}{2}\right)^2$

$\Rightarrow \quad \left(x - \frac{11}{2}\right)^2 = \frac{25}{(24)^2} - 30 + \frac{121}{4}$

$= \frac{25}{(24)^2} + \frac{1}{4}$

$= \frac{25+144}{576} = \left(\frac{13}{24}\right)^2$

$\Rightarrow \quad x - \frac{11}{2} = \pm\frac{13}{24}. \Rightarrow x = \frac{11}{2} \pm \frac{13}{24}$

$\Rightarrow \quad x = \frac{132 \pm 13}{24} \Rightarrow x = \frac{145}{24}; \frac{119}{24}$

$\Rightarrow \quad x = 6\frac{1}{24};\ 4\frac{23}{24}.$

8. Let the tap of larger diameter takes x hours to fill the tank. Therefore, the other tap will take $(x+10)$ hours to fill the same tank. The tap of larger diameter will fill the tank $\frac{1}{x}$ part in one hour and the other one will fill $\frac{1}{x+10}$ part in the same time.

According to the question,

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{9\frac{3}{8}}$$

$\Rightarrow \quad \frac{2x+10}{x(x+10)} = \frac{8}{75}$

$\Rightarrow \quad 4x^2 - 35x - 375 = 0$

$\Rightarrow \quad 4x^2 - 60x + 25x - 375 = 0$

$\Rightarrow \quad 4x(x-15) + 25(x-15) = 0$

$\Rightarrow \quad x = 15, -\frac{25}{4}$

Rejecting $x = -\frac{25}{4}$ hours due to negative time, we have

$x = 15$ hours and $x + 10 = 25$ hours.

Hence the tap of larger diameter and of smaller diameter can separately fill the tank in 15 hrs and 25 hrs respectively.

OR

Let the breadth of the rectangular park be b metres.
Then its length = $(b + 3)$ metres
Area of the rectangular park = $b(b+3)$ sq. m
Area of the triangular park

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times b \times 12 = 6b$$

Now,
area of rectangular park − area of triangular park = 4

$$b(b + 3) - 6b = 4$$

$$\Rightarrow \quad b^2 + 3b - 6b - 4 = 0$$

$$\Rightarrow \quad b^2 - 3b - 4 = 0$$

$$\Rightarrow \quad (b - 4)(b + 1) = 0 \Rightarrow b = -1, 4$$

Reject $b = -1$ as breadth is not possible in negative.

$\therefore$ $b = 4$ m and $b + 3 = 7$ m

Hence, length = 7 m and breadth = 4 m.

9. Let Denominator = x

$\therefore$ Numerator = $x - 3$

$$\therefore \quad \text{Fraction} = \frac{x-3}{x}$$

A.T.Q. $$\frac{x-3}{x+1} = \frac{x-3}{x} - \frac{1}{15}$$

$$\Rightarrow \quad \frac{x-3}{x} - \frac{x-3}{x+1} = \frac{1}{15}$$

$$\Rightarrow (x-3)\left[\frac{x+1-x}{x(x+1)}\right] = \frac{1}{15}$$

$$\Rightarrow \quad (x-3) \times 15 = x^2 + x$$

$$x^2 - 14x + 45 = 0$$

$$\Rightarrow \quad x^2 - 9x - 5x + 45 = 0$$

$$\Rightarrow x(x-9) - 5(x-9) = 0$$

$$(x-5)(x-9) = 0$$

$$x = 5 \text{ or } x = 9 \quad \{\because \text{Reject } x = 9\}$$

$\therefore$ Fraction is $\frac{2}{5}$

($x = 9$ doesn't satisfy given criteria as if $x = 9$ then fraction is $\frac{6}{9} = \frac{2}{3}$

$\therefore$ Numerator is not less than denominator by 3).

OR

3 hr 30 min.

Hint: Let average speed = x km/h

$\therefore$ Distance = 2800 km

$\therefore$ Original time (duration) = $\frac{2800}{x}$

$\therefore$ New time = $\frac{2800}{x-100}$

$$\therefore \quad \frac{2800}{x-100} - \frac{2800}{x} = \frac{1}{2}$$

Now solve.

WORKSHEET – 37

1. For real roots, D ≥ 0

$\therefore$ $(-k)^2 - 4 \times 5 \times 1 \geq 0 \quad \Rightarrow \quad k^2 \geq 20$

$\Rightarrow k \leq -\sqrt{20}$ or $k \geq \sqrt{20}$.

2. $$D = \left(4\sqrt{3}\right)^2 - 4 \times 3 \times 4.$$

$$= 48 - 48 = 0$$

$\Rightarrow$ Two roots are real and equal.

3. $3(2)^2 - 2p(2) + 2q = 0$

and $3(3)^2 - 2p(3) + 2q = 0$

$\Rightarrow 4p - 2q = 12$ and $6p - 2q = 27$

$\Rightarrow p = \frac{15}{2}$, $q = 9$.

4. False.

There can be quadratic equation which have no real roots, *e.g.* $x^2+2x+7=0$; This equation has no real roots because D = – 24 < 0.

5. No.

Let their ages be x years and y years.

Then $\quad x + y = 20 \quad$ ...(*i*)

And $\quad (x-4)(y-4) = 48 \quad$ ...(*ii*)

Consider equation (*ii*).

$\quad xy = 112 \quad$ ...(*iii*)

From equations (*i*) and (*iii*), we have

$$x^2 - 20x + 112 = 0$$

Here, $\quad D < 0$

Hence, the given situation is not possible.

6. $D = b^2 - 4ac$

$= 16a^4 - 4(4)(a^4 - b^4)$

$= 16a^4 - 16a^4 + 16b^4 = (4b^2)^2 \geq 0$

$\therefore \quad x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{4a^2 \pm 4b^2}{2 \times 4}$

$\Rightarrow \quad x = \dfrac{a^2 + b^2}{2}; \ \dfrac{a^2 - b^2}{2}.$

7. $(\alpha - 3)x^2 + 4(\alpha - 3) = 4$

$\Rightarrow \quad (\alpha - 3)x^2 + 4(\alpha - 3) - 4 = 0 \quad \ldots (i)$

Since equation (*i*) has real and equal roots,

$\therefore$ Discriminant (D) = 0

$\Rightarrow \quad b^2 - 4ac = 0$

$\therefore \quad a = \alpha - 3, \ b = 0,$

$c = 4(\alpha - 3) - 4 = 4(\alpha - 4)$

$\therefore \quad D = 0 - 4(\alpha - 3) \times 4(\alpha - 4) = 0$

$\Rightarrow \quad \alpha = 3$ or $\alpha = 4$

But $\alpha \neq 3$, *i.e.*, $\alpha - 3 \neq 0$, as $(\alpha - 3)$ is the constant of the leading term.

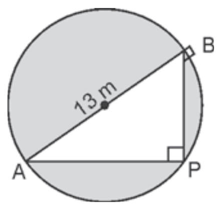
Hence, $\alpha = 4$.

8. Yes; 5 m and 12 m

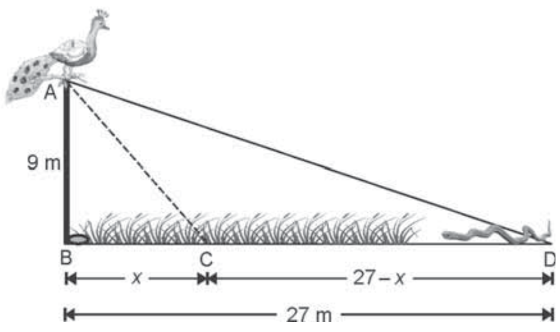
Hint: Let distance of pole P from gate B be x m and from A, $(x + 7)$ m.

Therefore, $x^2 + (x + 7)^2 = 13^2$

Now solve.



OR



Let the snake is caught at a distance of x m from the pillar base

$\therefore$ From figure, $AC^2 = 9^2 + x^2$

(Using Pythagoras Theorem)

and $\quad CD = 27 - x.$

Since their speed are same so,

$AC = CD$ ($\because$ Distance covered will be equal in equal time)

$\Rightarrow \quad AC^2 = CD^2$

$\Rightarrow \quad 81 + x^2 = (27 - x)^2$

$81 + x^2 = 729 + x^2 - 54x$

$54x = 648$

$\therefore \quad x = 12\,\text{m}.$

9. $x^2 + \dfrac{a}{a+b}x + \dfrac{a+b}{a}x + 1 = 0$

$\Rightarrow \quad x\left(x + \dfrac{a}{a+b}\right) + \dfrac{a+b}{a}\left(x + \dfrac{a}{a+b}\right) = 0$

$\Rightarrow \quad \left(x + \dfrac{a+b}{a}\right)\left(x + \dfrac{a}{a+b}\right) = 0$

$\therefore \quad x = \dfrac{-a}{a+b}$ or $-\dfrac{a+b}{a}$

OR

$4x^2 + 4bx = a^2 - b^2$

$\Rightarrow \quad x^2 + bx = \dfrac{a^2 - b^2}{4}$

$\Rightarrow \quad x^2 + bx + \left(\dfrac{b}{2}\right)^2 = \dfrac{a^2 - b^2}{4} + \dfrac{b^2}{4}$

$\Rightarrow \quad \left(x + \dfrac{b}{2}\right)^2 = \dfrac{a^2}{4}$

$\Rightarrow \quad x + \dfrac{b}{2} = \pm \dfrac{a}{2}$

$x = \dfrac{-b \pm a}{2}.$

WORKSHEET – 38

1. For real and equal roots, D = 0.

$\therefore \quad (4k)^2 - 4 \times 12 \times 3 = 0$

$\Rightarrow \quad 16(k^2 - 3^2) = 0$

$\Rightarrow \quad (k - 3)(k + 3) = 0$

$\Rightarrow \quad k = \pm 3.$

2. $\alpha + \beta = \dfrac{2}{1}$ and $\alpha\beta = -3$

$\Rightarrow \quad (\alpha + 2) + (\beta + 2) = 2 + 4$

and $(\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4$

$= -3 + 2(2) + 4$

$\Rightarrow$ S = 6 and P = 5

Required equation: $x^2 - Sx + P = 0$,

i.e., $x^2 - 6x + 5 = 0$.

3. $(b)^2 - (a + b)\, b + p = 0$

$\Rightarrow b^2 - ab - b^2 + p = 0$

$\Rightarrow p = ab$.

4. Yes.

$(x - 1)^3 = x^3 - 2x + 1$

$\Rightarrow x^3 - 1 + 3x(-1)(x - 1) = x^3 - 2x + 1$

$\Rightarrow x^3 - 1 - 3x^2 + 3x = x^3 - 2x + 1$

$\Rightarrow 3x^2 - 5x + 2 = 0$

That is a quadratic equation.

5. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow 4x\left(\sqrt{3}x+2\right) - \sqrt{3}\left(\sqrt{3}x+2\right) = 0$

$\Rightarrow \left(4x-\sqrt{3}\right)\left(\sqrt{3}x+2\right) = 0$

$\Rightarrow 4x - \sqrt{3} = 0$ or $\sqrt{3}x + 2 = 0$

$\Rightarrow x = \dfrac{\sqrt{3}}{4}$ or $x = -\dfrac{2}{\sqrt{3}} = \dfrac{-2\sqrt{3}}{3}$.

6. $4x^2 - 2(a^2 + b^2) + a^2b^2 = 0$

Here, $A = 4$, $B = -2(a^2 + b^2)$ and $C = a^2b^2$

Now, $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \dfrac{2(a^2+b^2) \pm \sqrt{4(a^2+b^2)^2 - 4\times 4\times a^2b^2}}{2\times 4}$

$= \dfrac{2(a^2+b^2) \pm 2\sqrt{a^4+b^4+2a^2b^2-4a^2b^2}}{2\times 4}$

$= \dfrac{a^2+b^2 \pm \sqrt{(a^2-b^2)^2}}{4}$

$= \dfrac{a^2+b^2+a^2-b^2}{4}, \dfrac{a^2+b^2-a^2+b^2}{4}$

$\therefore x = \dfrac{a^2}{2}, \dfrac{b^2}{2}$.

7. $k = -\dfrac{10}{9}$ or $k = 2$

Hint: Put $D = b^2 - 4ac = 0$

Take $a = 1$, $b = -2(1 + 3k)$

$c = 7(3 + 2k)$.

8. Let the required number has x as ten's digit of the number.

Given: Product of the digit = 8

$\Rightarrow$ Unit's digit $= \dfrac{8}{x}$

$\Rightarrow$ Number $= 10x + \dfrac{8}{x}$

If 63 is subtracted from the number the digit interchange their places.

$\Rightarrow 10x + \dfrac{8}{x} - 63 = 10 \times \dfrac{8}{x} + x$

$\Rightarrow 10x + \dfrac{8}{x} - 63 = \dfrac{80}{x} + x$

$\Rightarrow 9x - \dfrac{72}{x} - 63 = 0$

$\Rightarrow 9x^2 - 63x - 72 = 0$

$\Rightarrow x^2 - 7x - 8 = 0$

$\Rightarrow (x + 1)(x - 8) = 0$

$\Rightarrow x + 1 = 0,\ x - 8 = 0$

$\Rightarrow x = -1,\ \ x = 8$

Reject $x = -1 \Rightarrow x = 8$

$\Rightarrow$ Required number $= 10 \times 8 + \dfrac{8}{8} = 81$.

OR

25 students

Hint: Let the number of students attended picnic $= x$

$\therefore$ Per head contribution $= \dfrac{500}{x}$

According to question,

$\dfrac{500}{x-5} - \dfrac{500}{x} = 5$

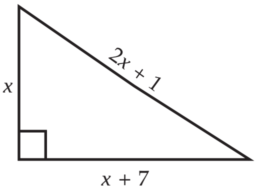
$\therefore 500\left[\dfrac{x-x+5}{x(x-5)}\right] = 5$

$\Rightarrow 500 = x^2 - 5x$

$\Rightarrow x^2 - 5x - 500 = 0$.

9. Let shortest side = x

$\therefore$ Hypotenuse = $(2x + 1)$



and Third side = $x + 7$

$\therefore$ Using Pythagoras theorem, we get

$$x^2 + (x + 7)^2 = (2x + 1)^2$$

$\Rightarrow$ $x^2 + x^2 + 14x + 49 = 4x^2 + 4x + 1$

$\Rightarrow$ $2x^2 - 10x - 48 = 0$

$\Rightarrow$ $2x^2 - 16x + 6x - 48 = 0$

$\Rightarrow$ $2x(x - 8) + 6(x - 8) = 0$

$\Rightarrow$ $(2x + 6)(x - 8) = 0$

$\Rightarrow$ $2x + 6 = 0$ or $x - 8 = 0$

$\Rightarrow$ $x = -3$ or $x = 8$

Since, side can't be negative

$\therefore$ $x = -3$ is not possible

$\therefore$ $x = 8$

$\Rightarrow$ Sides of grassy lands are: 8 m, 15 m and 17 m.

OR

The given quadratic equation is

$$x^2 - (\sqrt{2}+1)x + \sqrt{2} = 0$$

Here, $a = 1$, $b = -(\sqrt{2}+1)$, $c = \sqrt{2}$

Now, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{\sqrt{2}+1 \pm \sqrt{2+1+2\sqrt{2}-4\sqrt{2}}}{2}$$

$$= \frac{\sqrt{2}+1 \pm \sqrt{2-2\sqrt{2}+1}}{2}$$

$$= \frac{\sqrt{2}+1 \pm \sqrt{(\sqrt{2}-1)^2}}{2}$$

$$= \frac{\sqrt{2}+1+\sqrt{2}-1}{2}, \frac{\sqrt{2}+1-\sqrt{2}+1}{2}$$

$$= \frac{2\sqrt{2}}{2}, \frac{2}{2} \qquad \therefore \ x = \sqrt{2}, 1$$

Hence, the required roots are $\sqrt{2}$ and 1.

WORKSHEET – 39

1. $S = 8 + 2 \Rightarrow 10 = -a \Rightarrow a = -10$ (For 1st eqn.)

$P = 3 \times 3 \Rightarrow 9 = b \Rightarrow b = 9$ (For 2nd eqn.)

$\therefore x^2 - 10x + 9 = 0 \Rightarrow x = 9, 1.$

2. For equal roots, D = 0.

$\therefore 64k^2 - 4 \times 9 \times 16 = 0 \Rightarrow k = \pm 3.$

3. No.

At $x = 1$, $x^2 + x + 1 = 1^2 + 1 + 1 = 3 \neq 0$

At $x = -1$, $x^2 + x + 1 = (-1)^2 - 1 + 1 = 1 \neq 0$

Hence, neither $x = 1$ nor $x = -1$ is a solution of the equation $x^2 + x + 1 = 0$.

4. $\dfrac{a}{x-a} + \dfrac{b}{x-b} = \dfrac{2c}{x-c}$

$\Rightarrow \dfrac{ax-ab+bx-ab}{(x-a)(x-b)} = \dfrac{2c}{x-c}$

$\Rightarrow (ax + bx - 2ab)(x - c)$

$= 2c(x^2 - ax - bx + ab)$

$\Rightarrow (a + b - 2c)x^2 - 2abx + bcx + cax = 0$

$\Rightarrow x[(a + b - 2c)x - (2ab - ca - bc)] = 0$

$\Rightarrow x = 0$ or $x = \dfrac{2ab-ac-bc}{a+b-2c}$.

5. $x = \pm\sqrt{2}, \pm 2$

Hint: Let $y = (5 + 2\sqrt{6})^{x^2-3}$

$\therefore \quad \dfrac{1}{y} = (5 - 2\sqrt{6})^{x^2-3}$

$\therefore \quad y + \dfrac{1}{y} = 10$

$\therefore \quad y = \dfrac{10 \pm \sqrt{96}}{2 \times 1}$

(Using: $D = b^2 - 4ac$)

$\Rightarrow \quad y = 5 \pm 2\sqrt{6}$.

Now compare the exponent.

6. $x = \dfrac{1 \pm \sqrt{5}}{2}$

Hint: Use $a^2 + b^2 = (a - b)^2 + 2ab$

$$\left(x - \frac{x}{x+1}\right)^2 + 2x\left(\frac{x}{x+1}\right) = 3$$

$$\Rightarrow \quad \left(\frac{x^2+x-x}{x+1}\right)^2 + 2\ \frac{x^2}{x+1} = 3$$

$$\Rightarrow \quad \left(\frac{x^2}{x+1}\right)^2 + 2\left(\frac{x^2}{x+1}\right) = 3$$

Let $\quad y = \dfrac{x^2}{x+1}$

$\Rightarrow \quad y^2 + 2y - 3 = 0$

$\Rightarrow \quad y = 1$ or $y = -3$

$$\Rightarrow \quad \frac{x^2}{x+1} = 1 \text{ or } \frac{x^2}{x+1} = -3$$

$x^2 - x - 1 = 0$ or $x^2 + 3x + 3 = 0$

Now solve.

7. $\quad kx(x-2) + 6 = 0$

$\Rightarrow \quad kx^2 - 2kx + 6 = 0$

$\therefore \quad D = b^2 - 4ac$

$\quad = 4k^2 - 4(k)6 = 4k^2 - 24k$

$\therefore$ root will be equal if D = 0

$\Rightarrow \quad 4k^2 - 24k = 0$

$\quad 4k(k-6) = 0$

$\Rightarrow \quad k = 0$ or $k = 6$

or $\quad k \neq 0 \Rightarrow k = 6$

8. Hint: Let roots of

$Ax^2 + 2Bx + C = 0$ be α' and β'

$\therefore \alpha' = \alpha + \delta;$

$\quad \beta' = \beta + \delta$

$\therefore \alpha' - \beta' = \alpha - \beta;$

$\therefore (\alpha' - \beta')^2 = (\alpha - \beta)^2$

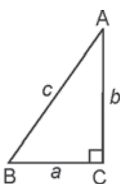
$\therefore (\alpha' + \beta')^2 - 4\alpha'\beta' = (\alpha + \beta)^2 - 4\alpha\beta$

$$\Rightarrow \quad \frac{4B^2}{A^2} - 4\cdot\frac{C}{A} = \frac{4b^2}{a^2} - 4\cdot\frac{c}{a}$$

$$\Rightarrow \quad \frac{b^2 - ac}{a^2} = \frac{B^2 - AC}{A^2}$$

$$\Rightarrow \quad \frac{b^2 - ac}{B^2 - AC} = \left(\frac{a}{A}\right)^2.$$

9. Let ΔABC is a right-angled triangle such that $\angle C = 90°$ and $b > a$.



$\therefore \quad c^2 = a^2 + b^2$

$\Rightarrow \quad a^2 + b^2 = (3\sqrt{5})^2 = 45$

$\Rightarrow \quad 4a^2 + 4b^2 = 180 \quad \ldots(i)$

Let the new corresponding sides be a', b' and c' such that

$a' = 3a$, $b' = 2b$ and $c' = 15$ cm

Then, $(3a)^2 + (2b)^2 = (15)^2$

$\Rightarrow 9a^2 + 4b^2 = 225 \quad \ldots(ii)$

Subtracting equation (*i*) from equation (*ii*), we have

$5a^2 = 45 \Rightarrow a = 3$

Substituting $a = 3$ in equation (*ii*), we have

$9 \times 9 + 4b^2 = 225$

$\Rightarrow \quad 4b^2 = 225 - 81 = 144 \Rightarrow b = 6$

Hence, the original length of sides are 3 cm, 5 cm and $3\sqrt{5}$ cm.

OR

According to the question, the two times:

(*i*) t minutes past 2 p.m. and

(*ii*) $60 - \left(\dfrac{t^2}{4} - 3\right)$ minutes past 2 pm are equal. It means

$$t = 60 - \left(\frac{t^2}{4} - 3\right)$$

$$\Rightarrow \quad t + \frac{t^2}{4} - 3 = 60$$

$\Rightarrow \quad t^2 + 4t - 252 = 0$

$\Rightarrow \quad t^2 + 18t - 14t - 252 = 0$

$\Rightarrow \quad t(t + 18) - 14(t + 18) = 0$

$\Rightarrow \quad (t + 18)(t - 14) = 0$

$\Rightarrow \quad t = -18$ (rejected), $t = 14$.

WORKSHEET – 40

1. The wrong equation is $x^2 + 17x + q = 0$

$\therefore \ q = (-2) \times (-15) = 30$

Now, the original equation will be $x^2 + 13x + 30 = 0$. Its roots are $-10, -3$.

2. $x = \dfrac{2}{3}$ must satisfy $kx^2 - x - 2 = 0$

$$\therefore \ k \times \frac{4}{9} - \frac{2}{3} - 2 = 0 \Rightarrow k = 6.$$

3. Hint: For no real root D < 0.

4. $\quad x^2 + p(2x + 4) + 12 = 0$

$\Rightarrow \quad x^2 + 2px + 4p + 12 = 0$

For real and equal roots, D = 0

$\therefore \quad 4p^2 - 4 \times (4p + 12) = 0$

$\Rightarrow \quad 4p^2 - 16p - 48 = 0$

$\Rightarrow \quad 4(p-6)(p+2) = 0$

$\Rightarrow \quad p - 6 = 0 \text{ or } p + 2 = 0$

$\Rightarrow \quad p = 6 \text{ or } -2.$

5. Product of roots $= \dfrac{c}{a}$

$\Rightarrow \quad \dfrac{1}{2} \times (-2) = \dfrac{-q}{p+1}$

$\Rightarrow \quad -p - 1 = -q$

$\Rightarrow \quad q - p = 1 \quad \ldots(i)$

Also sum of roots $= \dfrac{1}{2} - 2 = \dfrac{3}{p+1}$

$\Rightarrow \quad -\dfrac{1}{2} = \dfrac{1}{p+1} \Rightarrow p = -3 \quad \ldots(ii)$

$\therefore$ From (*i*), $\quad q = -2$

$\therefore \quad p + q + 5 = -3 - 2 + 5 = 0.$

6. Hint: Use $D \geq 0$ for both the equation.

7. Hint: Use $\sin\alpha + \cos\alpha = -\dfrac{b}{a} \quad \ldots(i)$

and $\quad \sin\alpha \cdot \cos\alpha = \dfrac{c}{a} \quad \ldots(ii)$

Squaring both sides of (*i*) and using (*ii*) you will get the result.

8. The given equation is

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$$

$$\Rightarrow \quad \frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{x^2-5x+5}{x^2-6x+8} = \frac{5}{3}$$

$\Rightarrow \quad 5x^2 - 30x + 40 = 3x^2 - 15x + 15$

$\Rightarrow \quad 2x^2 - 15x + 25 = 0$

Let us use quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{15 \pm \sqrt{225 - 4\times 2\times 25}}{2\times 2}$$

$$\Rightarrow x = \frac{15 \pm 5}{4} \Rightarrow x = 5, \frac{5}{2}.$$

9. Yes, 25 m and 16 m

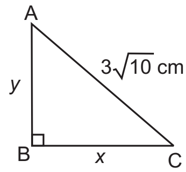
Hint: Let the two adjacent sides of the field be *a* and *b*.

Then $\quad 2(a + b) = 82 \Rightarrow a + b = 41$

And $\quad ab = 400.$

OR

3 cm and 9 cm



Hint: Let smaller leg = x

From figure,

$x^2 + y^2 = (3\sqrt{10})^2 = 90$

$\Rightarrow \quad y^2 = 90 - x^2 \quad \ldots(i)$

Also $\quad (3x)^2 + (2y)^2 = (9\sqrt{5})^2$

$\Rightarrow \quad 9x^2 + 4y^2 = 405 \quad \ldots(ii)$

Use (*i*) and (*ii*) and then solve.

WORKSHEET – 41

1. Let us find the discriminant of:

$x^2 + 2\sqrt{3}x - 1 = 0$

$D = \left(2\sqrt{3}x\right)^2 - 4 \times 1 \times (-1) = 12 + 4 = 16 > 0$

$\Rightarrow x^2 + 2\sqrt{3}x - 1 = 0$ has real roots.

2. $\quad 7\left(\dfrac{2}{3}\right)^2 + t\left(\dfrac{2}{3}\right) - 3 = 0$

$\Rightarrow \quad \dfrac{28}{9} + \dfrac{2}{3}t - 3 = 0$

$\Rightarrow \; t = \dfrac{3}{2}\left(3 - \dfrac{28}{9}\right) \;\Rightarrow\; t = -\dfrac{3}{2} \times \dfrac{1}{9} = -\dfrac{1}{6}.$

3. Consider $x^2 + 5px + 16$ has no real roots.

$\Rightarrow D < 0$

$\Rightarrow (5p)^2 - 4 \times 1 \times 16 < 0$

$\Rightarrow 25p^2 < 64 \;\Rightarrow\; p < \pm\sqrt{\dfrac{64}{25}}$

$\Rightarrow -\dfrac{8}{5} < p < \dfrac{8}{5}.$

4. True.

Let equation is $ax^2 + bx + c = 0$

Case: I $a > 0$ and $c < 0 \Rightarrow ac < 0 \Rightarrow -ac > 0$

$\therefore D = b^2 - 4ac > 0 \quad \therefore b^2 \geq 0$

Case: II $a < 0$ and $c > 0 \Rightarrow ac < 0 \Rightarrow -ac > 0$

$\therefore D = b^2 - 4ac > 0.$

5. $(a-b)x^2 + (b-c)x + (c-a) = 0$

As this equation has equal roots, the discriminant of it vanishes.

i.e., $\quad D = 0$

$\Rightarrow \quad (b-c)^2 - 4 \times (a-b) \times (c-a) = 0$

$$\Rightarrow \quad b^2-2bc+c^2-4ac+4a^2 +4bc - 4ab = 0$$
$$\Rightarrow \quad 4a^2+b^2+c^2-4ab+2bc-4ac = 0$$
$$\Rightarrow \quad (2a-b-c)^2 = 0$$
$$[\because (x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx]$$
$$\Rightarrow \quad 2a-b-c = 0$$
$$\Rightarrow \quad 2a = b+c.$$

Hence proved.

6. $\frac{14}{x+3} - 1 = \frac{5}{x+1}$

$$\Rightarrow \quad \frac{14}{x+3}-\frac{5}{x+1} = 1$$
$$\Rightarrow \quad \frac{14x+14-5x-15}{(x+3)(x+1)} = 1$$
$$\Rightarrow \quad 9x-1 = (x+3)(x+1)$$
$$\Rightarrow \quad 9x-1 = x^2+4x+3$$
$$\Rightarrow \quad x^2-5x+4 = 0$$
$$\Rightarrow \quad (x-4)(x-1) = 0$$
$$\Rightarrow \quad x = 4 \text{ or } x = 1$$

7. $$\frac{1}{2a+b+2x} = \frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

$$\Rightarrow \quad \frac{1}{2a+b+2x}-\frac{1}{2x} = \frac{1}{2a}+\frac{1}{b}$$
$$\Rightarrow \quad \frac{2x-2a-b-2x}{2x(2a+b+2x)} = \frac{b+2a}{2ab}$$
$$\Rightarrow \quad \frac{-(2a+b)}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$$
$$\Rightarrow \quad -ab = 2ax+bx+2x^2$$
$$\Rightarrow \quad 2x^2+2ax+bx+ab = 0$$
$$2x(x+a)+b(x+a) = 0$$
$$(2x+b)(x+a) = 0$$
$$\Rightarrow \quad x = -\frac{b}{2a} \quad \text{or} \quad x = -a$$

8. Let first natural number = x

$\therefore$ second natural number = $x + 3$

$\therefore$ According to question,

$$\frac{1}{x}-\frac{1}{x+3} = \frac{3}{28}$$
$$\Rightarrow \quad \frac{x+3-x}{x(x+3)} = \frac{3}{28}$$
$$\Rightarrow \quad 28 = x(x+3)$$
$$\Rightarrow \quad x^2+3x-28 = 0$$
$$\Rightarrow \quad x^2+7x-4x-28 = 0$$
$$\Rightarrow \quad x(x+7)-4(x+7) = 0$$
$$\Rightarrow \quad (x-4)(x+7) = 0$$
$$\Rightarrow \quad x = 4 \text{ or } x = -7$$

[Reject as x is natural]

$$\Rightarrow \quad x = 4$$
$$\Rightarrow \quad x+3 = 7$$

CHAPTER TEST

1. We have,

$$2x^2+3+2\sqrt{6}x+x^2 = 3x^2-5x$$
$$\Rightarrow \quad (5+2\sqrt{6})x+3 = 0$$

which is not a quadratic equation.

2. The given equation can be written as

$$x^4+x^2+1 = 0$$

Here, $D = 1^2 - 4 \times 1 \times 1 = -3 < 0$

As $D < 0$, there is no real root.

3. $$9x^2+\frac{3}{4}x-\sqrt{2} = 0$$

Let us add and subtract $\frac{1}{64}$.

$$9x^2+\frac{3}{4}x+\frac{1}{64}-\frac{1}{64}-\sqrt{2} = 0$$
$$\Rightarrow \quad \left(3x+\frac{1}{8}\right)^2-\left(\frac{\sqrt{1+64\sqrt{2}}}{8}\right)^2 = 0$$

Clearly, the required number is $\frac{1}{64}$.

4. $2x^2-kx+k=0$ has equal roots, if discriminant $= 0$.

$\Rightarrow (-k)^2 - 4 \times 2 \times k = 0 \Rightarrow k(k-8) = 0$

$\Rightarrow k = 0$ or 8.

5. **True.**

Let us consider a quadratic equation

$$\sqrt{3}x^2-7\sqrt{3}x+12\sqrt{3} = 0$$

Here, $D = \left(-7\sqrt{3}\right)^2 - 4 \times \sqrt{3} \times 12\sqrt{3}$

$\Rightarrow \quad D = 147 - 144 = 3 \Rightarrow D > 0$

$\Rightarrow$ Roots are real and distinct.

So, $x = \dfrac{7\sqrt{3} \pm \sqrt{3}}{2\sqrt{3}}$ $\therefore$ $x = 4, 3$ which are rationals.

OR

No.

$$(x-1)^2 + (2x+1) = 0$$
$$\Rightarrow \quad x^2 - 2x + 1 + 2x + 1 = 0$$
$$\Rightarrow \quad x^2 + 2 = 0$$

Here, $D = 0^2 - 4 \times 1 \times 2 = -8 < 0$.

Hence, the given equation has no real root.

6. $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

Here, $a = \frac{1}{2}$, $b = -\sqrt{11}$, $c = 1$

$$D = b^2 - 4ac$$
$$= \left(-\sqrt{11}\right)^2 - 4 \times \frac{1}{2} \times 1 = 9$$
$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-\left(-\sqrt{11}\right) \pm 3}{2 \times 1}$$
$$\alpha = \frac{\sqrt{11}+3}{2}, \quad \beta = \frac{\sqrt{11}-3}{2}.$$

7. Let the required natural number be N.

$N^2 - 84 = (N + 8) \times 3 \quad \Rightarrow N^2 - 3N - 108 = 0$

$\Rightarrow (N - 12)(N + 9) = 0 \Rightarrow N = 12$ or -9

But -9 is not a natural number.

So, N = 12 is the required natural number.

8. $(b-c)x^2 + (c-a)x + (a-b) = 0$

A quadratic equation is a perfect square, if its discriminant (D) is equal to zero.

Here, $A = b - c$, $B = c - a$ and $C = a - b$

Now, $D = 0$

$\Rightarrow D = B^2 - 4AC = (c-a)^2 - 4(b-c)(a-b)$

$\Rightarrow c^2 + a^2 - 2ca - 4(ab - b^2 - ca + bc) = 0$

$\Rightarrow c^2 + a^2 + 4b^2 + 2ca - 4bc - 4ab = 0$

$\Rightarrow (c + a - 2b)^2 = 0$

$[\because (x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx]$

$\Rightarrow c + a - 2b = 0 \Rightarrow b = \frac{a+c}{2}$.

Hence proved.

9. (*i*) Let width of grass paths = x m

$\therefore$ Length of rectangular pond = $(50 - 2x)$ m

breadth of rectangular pond = $(40 - 2x)$ m

$\therefore$ Area of grass path = Area of lawn – Area of rectangular pond

$= 50 \times 40 - (50 - 2x)(40 - 2x)$

$= 2000 - 2000 + 100x + 80x - 4x^2$

$= 180x - 4x^2$

According to question

$$180x - 4x^2 = 1184$$
$$\Rightarrow \quad 4x^2 - 180x + 1184 = 0$$
$$\Rightarrow \quad x^2 - 45x + 296 = 0$$
$$\Rightarrow \quad x^2 - 8x - 37x + 296 = 0$$
$$\Rightarrow \quad x(x-8) - 37(x-8) = 0$$
$$\Rightarrow \quad (x-8)(x-37) = 0$$
$$\Rightarrow \quad x = 8 \text{ or } x = 37$$

Reject $x = 37$ as it is not possible

$\therefore x = 8$ m

(*ii*) $\therefore$ Length of pond = 34 m; breadth of pond = 24 m.

(*iii*) Concept of quadratic equation is used in solving this problem.

(*iv*) Love for environment.

❑❑

Chapter 5 ARITHMETIC PROGRESSIONS

WORKSHEET – 43

1. $S_{15} = \frac{15}{2}\left[2 \times \frac{3}{\sqrt{5}} + (15-1) \times \left(\sqrt{5} - \frac{3}{\sqrt{5}}\right)\right]$

$= \frac{15}{2} \times \left(\frac{6}{\sqrt{5}} + \frac{28}{\sqrt{5}}\right) = \frac{17 \times 15}{\sqrt{5}}$

$= 51\sqrt{5}$.

2. $2k - 1 - k = 2k + 1 - (2k - 1)$

$\Rightarrow \quad k - 1 = 2$

$\Rightarrow \quad k = 3.$

3. Common difference $= -2-1 = -5 - (-2)$

$= -3.$

4. $p = 4$

Hint: Use: if a, b, c are in A.P. $\Rightarrow 2b = a + c$.

5. $a_4 = 0 \Rightarrow a + 3d = 0 \Rightarrow a = -3d \quad \ldots(i)$

$\therefore\ a_{25} = a + 24d = -3d + 24d \quad \{\because$ Using $(i)\}$

$= 21d$

Also $\quad a_{11} = a + 10d = -3d + 10d = 7d$

Clearly $\quad 21d = 3 \times 7d$

$\Rightarrow \quad a_{25} = 3\,(a_{11})$ **Hence Proved**

6. $a_{12} = -13$

$\Rightarrow \quad a + 11d = -13 \quad \ldots(i)$

Also, $a + a + d + a + 2d + a + 3d = 24$

$\Rightarrow \quad 4a + 6d = 24$

$\Rightarrow \quad 2a + 3d = 12 \quad \ldots(ii)$

Multiplying equation (i) by 2, we get

$2a + 22d = -26 \quad \ldots(iii)$

Subtracting (iii) from (ii), we get

$-19d = 38 \Rightarrow d = -2$

$\therefore$ From (i), $\quad \Rightarrow a - 22 = -13$

$\Rightarrow \quad a = 9$

$\therefore \quad S_{10} = \frac{10}{2}\{2 \times 9 + (10 - 1).(-2)\}$

$= 5\{18 - 18\} = 0.$

OR

$a_5 + a_9 = 72 \Rightarrow 2a + 12d = 72 \quad \ldots(i)$

also $\quad a_7 + a_{12} = 97 \Rightarrow 2a + 17d = 97 \quad \ldots(ii)$

$(i) - (ii) \quad \Rightarrow -5d = -25$

$\Rightarrow \quad d = 5$

From (i), $\quad \Rightarrow \quad a = 6$

$\therefore$ A.P. is 6, 11, 16,....

7. $S_n = 2n - 3n^2$

Hint: $a_1 = 5 - 6 = -1$

$a_2 = 5 - 12 = -7$

$a_3 = 5 - 18 = -13$

... $\quad d = a_2 - a_1 = -6$

... $\quad S_n = \frac{n}{2}\{-2 + (n-1)(-6)\}$

$= \frac{n}{2}\{4 - 6n\}$

$= n(2 - 3n) = 2n - 3n^2.$

8. $a = 18$; $d = 15\frac{1}{2} - 18 = -2\frac{1}{2} = -\frac{1}{2}$

$\therefore \quad a_n = -49\frac{1}{2} \quad \therefore\ a_n = a + (n-1)d$

$\Rightarrow -49\frac{1}{2} = 18 + (n-1).\left(-\frac{5}{2}\right)$

$= 18 - \frac{5}{2}n + \frac{5}{2}$

$\Rightarrow \quad -70 = -\frac{5}{2}n \quad \Rightarrow \quad n = 28.$

Also

Sum $= \frac{n}{2}\{a + a_n\}$

$= \frac{28}{2}\left\{18 - 49\frac{1}{2}\right\} = 14\left\{18 - \frac{99}{2}\right\}$

$= 14 \times \left(-\frac{63}{2}\right) = -441.$

9. $60°, 80°, 100°, 120°$

Hint: Let the angles be:

$$a - 3d, a - d, a + d, a + 3d.$$

OR

$\because \quad S_n = 5n^2 - 3n$

$\therefore \quad S_{n-1} = 5(n-1)^2 - 3(n-1)$

$= 5n^2 + 5 - 10n - 3n + 3$

$= 5n^2 - 13n + 8$

n^{th} term $(a_n) = S_n - S_{n-1}$

$= 5n^2 - 3n - (5n^2 - 13n + 8)$

$= 10n - 8$

$a_1 = 10 \times 1 - 8 = 2$

$a_2 = 10 \times 2 - 8 = 12$

$a_3 = 10 \times 3 - 8 = 22$

Therefore, the A.P. is 2, 12, 22,.......

Substituting $n = 10$ in $a_n = 10n - 8$, we get 10^{th} term $= 10 \times 10 - 8 = 92$.

WORKSHEET – 44

1. $5a_5 = 10a_{10}$

$\Rightarrow \quad 5(a + 4d) = 10(a + 9d)$

$\Rightarrow \quad 5a = -70d \quad \Rightarrow \quad a = -14d$

Now $\quad a_{15} = -14d + 14d = 0.$

2. $a_n = 505 \quad \Rightarrow \quad a + (n-1) \times d = 505$

$\Rightarrow \quad 1 + 7n - 7 = 505$

$\Rightarrow \quad n = \frac{511}{7} = 73$

$\therefore$ Middle term is $\left(\frac{n+1}{2}\right)^{th} = 37th$

$\therefore \; a_{37} = 1 + 36 \times 7 = 253.$

3. $2(p + 10) = 2p + 3p + 2$

$\Rightarrow \quad 2p + 20 = 5p + 2$

$\Rightarrow \quad 18 = 3p$

$\Rightarrow \quad p = 6.$

4. Let if possible:

$a_n = 0$

$\Rightarrow \quad 0 = 31 + (n-1) \times d = 31 + (n-1) \times (-3)$

$\{\because \; d = 28 - 31 = -3\}$

$= 31 - 3n + 3$

$0 = 34 - 3n \quad \Rightarrow \quad 3n = 34$

$n = \frac{34}{3}$ which is not a natural number.

$\Rightarrow$ 0 can't be any term of given sequence.

5. 15^{th} term from end of $-10, -20, -30, ..., -980, -990, -1000$

$= 15^{th}$ term of $-1000, -990, -980, ..., -20, -10$

$= -1000 + (15 - 1) \times (-990 + 1000)$

$= -1000 + 140 = -860.$

6. $6n - 1$

Hint: Use

$a_n = S_n - S_{n-1}$

7. Hint: Use $S_{20} = S_{30}$ and show that $S_{50} = 0$.

8. A.P.: 63, 65, 67,...........

$a = 63, d = 65 - 63 = 2$

$\therefore \; n^{th}$ term of 63, 65, 67,.....

$= a + (n-1) \times d = 63 + (n-1) \times 2 \quad ...(i)$

A.P.: 3, 10, 17,.........

$a' = 3, d' = 10 - 3 = 7$

$\therefore \; n^{th}$ term of 3, 10, 17,.......

$= a' + (n-1)d' = 3 + (n-1) \times 7 \quad ...(ii)$

According to the question,

$63 + (n-1) \times 2 = 3 + (n-1) \times 7$

[Using (*i*) and (*ii*)]

$\Rightarrow \quad 2n - 2 - 7n + 7 = 3 - 63$

$\Rightarrow \quad -5n + 5 = -60$

$\Rightarrow \quad -5n = -65 \quad \Rightarrow \quad n = 13.$

9. The sequence of savings (in rupees) is 4, 5.75, 7.5, ..., 19.75

Here, $a = 4$

$d = 1.75$

$a_n = 19.75$

$\Rightarrow \quad a + (n-1) \times d = 19.75$

$\Rightarrow \quad 4 + (n-1) \times (1.75) = 19.75$

$\Rightarrow \quad 4 + 1.75n - 1.75 = 19.75$

$\Rightarrow \quad 1.75n = 19.75 - 2.25$

$\Rightarrow \quad 1.75n = 17.50$

$\Rightarrow \quad n = 10$

In 10th week her saving will be ` 19.75.

OR

Let first term be a and common difference be d.

According to question,

$$a_4 + a_8 = 24$$

$$\Rightarrow \quad a + 3d + a + 7d = 24$$

$$a + 5d = 12 \quad \text{...}(i)$$

and $$a_6 + a_{10} = 44$$

$$\Rightarrow \quad a + 5d + a + 9d = 44$$

$$a + 7d = 22 \quad \text{...}(ii)$$

Subtracting (*i*) from (*ii*), we get

$$2d = 10$$

$$\therefore \quad d = 5$$

Putting $d = 5$ in (*i*), we get

$$a = -13$$

$\therefore$ First three terms of this A.P. will be –13, –8, –3.

WORKSHEET – 45

1. Hint: 12, 16, 20,...., 248

$$\therefore \quad 248 = 12 + (n-1)4$$

$$\Rightarrow \quad \frac{236}{4} = n - 1$$

$$\Rightarrow \quad n = 60.$$

2. $S_n = 90 \quad \Rightarrow \quad 90 = \frac{n}{2}\left\{4 + (n-1) \times 8\right\}$

$$\Rightarrow \quad 180 = n\,(8n - 4)$$

$$\Rightarrow \quad 2n^2 - n - 45 = 0$$

$$\Rightarrow (2n + 9)(n - 5) = 0$$

$$\Rightarrow \quad n = 5$$

$\therefore a_n = a_5 = a + 4 \times d = 2 + 4 \times 8 = 34.$

3. $$a_n = a'_n$$

$$\Rightarrow \quad 63 + (n-1) \times 2 = 3 + (n-1) \times 7$$

$$\Rightarrow \quad 2n + 61 = 7n - 4$$

$$\Rightarrow \quad 5n = 65$$

$$\Rightarrow \quad n = 13.$$

4. Common difference $= 2p - 1 - p$

$$= 7 - (2p - 1)$$

$$\therefore \quad 2p - p + 2p = 7 + 1 + 1$$

$$\Rightarrow \quad 3p = 9 \quad \Rightarrow \quad p = 3.$$

5. $a = 103;\ d = 101 - 103 = -2$

... $a_n = 49$

$$\Rightarrow \quad 103 + (n-1) \times (-2) = 49$$

$$\Rightarrow \quad -2n = 49 - 105$$

$$\Rightarrow \quad n = 28$$

$$S = \frac{28}{2}\{103 + 49\}$$

$$= 14 \times 152 = 2128.$$

6. –1, 4, 740

Hint: $a_3 = 7$

$a_7 = 3 \times a_3 + 2.$

7. Let the first term be *a* and the common difference be *d*.

$$\because \ S_n = \frac{n}{2}[2a + (n-1)d] \ \therefore \ 42 = \frac{6}{2}[2a + 5d]$$

$$\Rightarrow \ 2a + 5d = 14 \quad \text{...}(i)$$

$$\because \ a_n = a + (n-1)d \quad \therefore \ \frac{a + 9d}{a + 29d} = \frac{1}{3}$$

$$\Rightarrow 3a + 27d = a + 29d \Rightarrow 2a - 2d = 0 \Rightarrow a = d$$

Substituting $a = d$ in equation (*i*), we have

$$d = 2 \quad \text{and so} \quad a = 2$$

Now, $a_{18} = 2 + 17d = 2 + 17 \times 2 = 36.$

Hence the first term is 2 and 18th term is 36.

8. Hint: $ma_m = na_n$

$$\Rightarrow \quad m\{a + (m-1)d\} = n\{a + (n-1)d\}$$

$$\Rightarrow \quad m\{a + (m-1)d\} - n\{a + (n-1)d\} = 0$$

$$\Rightarrow \quad a(m-n) + \{m(m-1) - n(n-1)\}d = 0$$

$$\Rightarrow \quad a(m-n) + \{(m^2 - n^2) - (m-n\}d = 0$$

$$\Rightarrow \quad a(m-n) + \{(m-n)(m+n-1)d = 0$$

$$\Rightarrow \quad a + (m+n-1)d = 0 \qquad \{\because m \neq n\}$$

$$\Rightarrow \quad a_{m+n} = 0.$$

9. (*i*) Here, penalty for delay on

1st day = ` 200

2nd day = ` 250

3rd day = ` 300

....

....

Now, 200, 250, 300, ... are in A.P. such that

$$a = 200, \quad d = 250 - 200 = 50$$

$\therefore S_{30}$ is given by

$$S_{30} = \frac{30}{2}\,[2\,(200) + (30 - 1) \times 50]$$

$$\left[\text{Using } S_n = \frac{n}{2}\left[2a + (n-1)d\right]\right]$$

$$= 15\,[400 + 29 \times 50]$$

$$= 15\,[400 + 1450]$$

$$= 15 \times 1850 = 27{,}750$$

Thus, penalty for the delay for 30 days is ` 27,750.

(*ii*) One should be punctual and show dedication to his work, failing of which may result loss.

WORKSHEET – 46

1. $a_1 = x; a_2 = y; l = 2x$

$\therefore \quad d = y - x$

$\therefore \quad 2x = x + (n - 1).(y - x)$

$\Rightarrow \quad \frac{x}{y-x} = n - 1$

$n = \frac{x+y-x}{y-x} = \frac{y}{y-x}$

$\therefore \quad S_n = \frac{y}{y-x} \times \frac{1}{2}(x+2x) = \frac{3xy}{2(y-x)}.$

2. 10^{th} term from end of 4, 9,....., 244, 249, 254
= 10^{th} term from begining of 254, 249, 244, ..., 9, 4.

$= 254+9\times(-5)=254 - 45 = 209.$

3. 105, 112, 119, ..., 994

$a_n = a + (n - 1)d$

$\Rightarrow \quad 994 = 105 + (n - 1) \times 7$

$\Rightarrow \quad 994 = 105 + 7n - 7$

$\Rightarrow \quad 994 = 98 + 7n$

$\therefore \quad n = \frac{994-98}{7} = 128.$

4. $S_n = \frac{n}{2}(a + l) \Rightarrow 144 = \frac{9}{2}(a + 28)$

$\Rightarrow a + 28 = 32 \Rightarrow a = 4.$

5. Yes,

Hint: $a_{30} - a_{20} = a + 29d - a - 19d$
$= 10d = -40.$

6. Let the first term and common difference of first A.P. be A and D respectively and that of the second A.P. be a and d respectively.

$$\frac{\frac{n}{2}[2A + (n-1)D]}{\frac{n}{2}[2a + (n-1)d]} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{2A + (n-1)D}{2a + (n-1)d} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{A + \left(\frac{n-1}{2}\right)D}{a + \left(\frac{n-1}{2}\right)d} = \frac{7n+1}{4n+27}$$

To prepare the 5th term in numerator and denominator of LHS of this last equation, we should put $\frac{n-1}{2} = 4$, *i.e.*, $n = 9$.

Therefore,

$$\frac{A + 4D}{a + 4d} = \frac{7 \times 9 + 1}{4 \times 9 + 27} \Rightarrow \frac{A_5}{a_5} = \frac{64}{63}$$

Hence, the required ratio is 64 : 63.

7. Hint: Use $a' + (p - 1)d = a$
$a' + (q - 1)d = b$
$a' + (r - 1)d = c.$

OR

Let the first term be a and the common difference be d.

Now, $a_{19} = 3 \times a_6 \quad \Rightarrow \quad a + 18d = 3(a+5d)$

$2a = 3d$ ...(*i*)

Also, $a_9 = 19 \quad \Rightarrow \quad a + 8d = 19$ ...(*ii*)

From equations (*i*) and (*ii*),

$\frac{3}{2}d + 8d = 19 \quad \Rightarrow \quad 19d = 38$

$\Rightarrow \quad d = 2$ and so $a = 3$.

[From equation (*i*)]

Hence, the A.P. is $a, a + d, a + 2d$, ...
i.e., 3, 5, 7, ...

8. Let a = first term
d = common difference

$\therefore \quad a_n = a + (n - 1)d$

$\therefore \quad a_{17} = a + 16d$

$a_8 = a + 7d$

$a_{11} = a + 10d$

Now $a_{17} = 2.a_8 + 5$

$\Rightarrow \quad a + 16d = 2(a + 7d) + 5$
$= 2a + 14d + 5$

$\Rightarrow \quad 2d = a + 5$

$\Rightarrow \quad a = 2d - 5$ ...(*i*)

Also as $a_{11} = 43$

$\Rightarrow \quad a + 10d = 43$

Using (*i*), $2d - 5 + 10d = 43$

$\Rightarrow \quad 12d = 48$

$d = 4$

$\therefore$ from (i) $\Rightarrow$ $a = 2 \times 4 - 5 = 3$

$\therefore$ $a_n = a + (n-1) . d$

$\Rightarrow$ $a_n = 3 + (n-1) . 4$

$= 3 + 4n - 4$

$a_n = 4n - 1$

9. Hint: Saving is sum of 17 terms of the AP 5000, 5500, 6000, ….

(i) S = 5000 + 5500 + 6000 + 6500 + …

$$= \frac{17}{2}\{2 \times 5000 + 16 \times 500\}$$

$$= \frac{17}{2}\{10000 + 8000\} = 18000 \times \frac{17}{2}$$

$= 153000$

(ii) Sum of n-terms of an Arithmetic Progression

(iii) Caring, responsible and proactive.

WORKSHEET – 47

1. $S_n = 3n^2 - n$

Put $n = 1$ $\therefore$ $S_1 = a_1 = 2$

Put $n = 2$ $\therefore$ $S_2 = a_1 + a_2 = 10$

$\Rightarrow$ $a_2 = 8$

$\therefore$ $d = a_2 - a_1 = 6.$

2. Hint: $a + 2d = 4$

$a + 8d = -8$

$-\quad -\quad +$

$-6d = 12$

$d = -2$

$a = 8$

$a_{10} = a + 9d$

$= 8 - 18 = -10.$

3. Hint: $a_n = S_n - S_{n-1}.$

4. Let the n^{th} term be – 44.

$\therefore$ $a_n = -44$

$\Rightarrow$ $a + (n-1)d = -44$

$\Rightarrow$ $40 + (n-1)(-4) = -44$

$\Rightarrow$ $(n-1) = 21$ $\Rightarrow$ $n = 22.$

5. –8930

Hint: $a = -5$; $d = -8 - (-5) = -3.$

$a_n = -230$. Find n.

Then use $S_n = \frac{n}{2}\{a + l\}$.

6. n^{th} term is $a_n = 5n - 3$

Substituting $n = n - 1$, we have

$(n-1)^{th}$ term is $a_{n-1} = 5(n-1) - 3 = 5n - 8$

$\therefore$ Common difference is $d = a_n - a_{n-1}$

$= 5n - 3 - 5n + 8$

$= 5$

Substitute $n = 1$ in $a_n = 5n - 3$ to get first term

$a_1 = 5 \times 1 - 3$ $\Rightarrow$ $a_1 = 2$

Now, using

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

The sum of first 20 terms is

$$S_{20} = \frac{20}{2}[2 \times 2 + (20-1) \times 5] = 10 \times 99$$

$= 990.$

7. Let the same common difference be d.

30^{th} term of one A.P. $= 3 + (30-1) \times d$

$= 3 + 29d$ ...(i)

30^{th} term of other A.P. $= 8 + (30-1)d$

$= 8 + 29d$ ...(ii)

Now, the required difference

$= (8 + 29d) - (3 + 29d)$

[Using equations (i) and (ii)]

$= 8 + 29d - 3 - 29d = 5.$

8. Hint:

$$\frac{p}{2}\{2a' + (p-1)d\} = a$$

$$\frac{q}{2}\{2a' + (q-1)d\} = b$$

$$\frac{r}{2}\{2a' + (r-1)d\} = c$$

OR

Let the first term and the common difference of the given A.P. be a and d respectively.

5^{th} term $= 0$ $\Rightarrow a + 4d = 0$

$\Rightarrow$ $a = -4d$ ...(i)

23rd term: $a_{23} = a + 22d$

$\Rightarrow$ $a_{23} = -4d + 22d$

[From equation (i)]

$\Rightarrow$ $a_{23} = 18d$ ...(ii)

11th term: $a_{11} = a + 10d$

$\Rightarrow$ $a_{11} = -4d + 10d$

[From equation (i)]

$$\Rightarrow \quad a_{11} = 6d \quad \Rightarrow \quad a_{11} = 6 \times \frac{a_{23}}{18}$$

[From equation (*ii*)]

$$\Rightarrow \quad a_{23} = 3a_{11}$$

$\Rightarrow$ 23rd term = 3 × 11th term

Hence proved.

9. Let the digits of the number be $a-d$, a and $a+d$ such the required number is $100(a-d)+10a+a+d$ as the digits are in A.P.

So, the required number $= 111a - 99d$ ...(*i*)

Sum of the digits = 15

$$\Rightarrow \quad a - d + a + a + d = 15$$

$$\Rightarrow \quad a = 5 \quad ...(ii)$$

The number obtained by reversing the digits

$$= 100(a + d) + 10a + a - d$$

$$= 111a + 99d \quad ...(iii)$$

According the given condition, we have

$$111a - 99d = 594 + 111a + 99d$$

[Using equations (*i*) and (*iii*)]

$$\Rightarrow \quad -2 \times 99d = 594$$

$$\Rightarrow \quad d = -3 \quad ...(iv)$$

Using equations (*i*), (*ii*) and (*iv*), we arrive that the original number is

111 × 5 – 99 × (–3), that is 852.

OR

16 rows, 5 logs are placed in top row.

Hint: Put $S_n = 200$, $a = 20$, $d = -1$

in formula $S_n = \frac{n}{2}[2a + (n-1)d]$

So $41n - n^2 = 400$

$$\Rightarrow \quad n = 16, 25$$

$\therefore$ $n = 25$ not possible

because if $n = 25$ then the number of logs in top row

$$= -4$$

$$\therefore \quad n = 16 \text{ and } a_{16} = 5.$$

WORKSHEET – 48

1. $a = 10$, $d = 7 - 10 = -3$

$$a_{30} = a + 29d = 10 + 29(-3) = -77.$$

2. $a_n = 2n + 1 \quad \therefore \quad a_1 = 2 \times 1 + 1 = 3$

$$\text{Now, } S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(3 + 2n + 1)$$

$$= n(n+2)$$

3. No.

Let $a_n = 68$

$$\Rightarrow \quad a + (n-1) \times d = 68$$

$$\Rightarrow \quad 7 + (n-1) \times 3 = 68$$

$$\Rightarrow \quad 3n = 64 \Rightarrow n = \frac{64}{3}$$

which is not a whole number so $a_n = 68$ not possible.

4. General term is $a_n = (-1)^n \, 3^{n+1}$

Substituting $n = 1, 2, 3, 4$ successively we get

$$a_1 = (-1)\, 3^2 = -9,\; a_2 = (-1)^2\, 3^3 = 27$$

$$a_3 = (-1)^3\, 3^4 = -81,\; a_4 = (-1)^4\, 3^5 = 243.$$

Therefore, first four terms are –9, 27, –81, 243.

5. The sequence is 23, 21, 19, ...,5

$$\therefore \quad a = 23$$

$$d = 21 - (23) = -2$$

$$\therefore \quad a_n = 5$$

$$\Rightarrow \quad a + (n-1) \times d = 5$$

$$\Rightarrow \quad 23 + (n-1) \times (-2) = 5$$

$$\Rightarrow \quad 23 - 2n + 2 = 5$$

$$\Rightarrow \quad -2n = -20$$

$$\Rightarrow \quad n = 10.$$

Hence, number of rows is 10.

6. In the series $(-5) + (-8) + (-11) + ... + (-230)$, $a = -5$, $d = -8 - (-5) = -3$.

Let the number of terms be n, then

$$-230 = -5 + (n-1)(-3)$$

$[\because a_n = a + (n-1)d]$

$$\Rightarrow \quad n - 1 = \frac{-225}{-3} \quad \Rightarrow \quad n = 76$$

Now, sum of first n terms is given by

$$S_n = \frac{n}{2}(a + a_n)$$

$$= \frac{76}{2}(-5 - 230) \qquad (\because a_n = -230)$$

$$= -38 \times 235 = -8930.$$

7. $n = 50$; a = first term; d = common difference

$S_{10} = 210$ = sum of first 10 terms

Sum of last 15 terms = 2565

Now, $S_{10} = 210$

$$\Rightarrow \quad a + (a + d) + ... + (a + 9d) = 210$$

$$\Rightarrow \quad 10a + d(1 + 2 + ... + 9) = 210$$

$\Rightarrow \quad 10a + 45d = 210$

$\Rightarrow \quad 2a + 9d = 42 \quad ......(i)$

$\Rightarrow$ Also sum of last 15 terms $= 2565$

$\Rightarrow \quad a_{50} + a_{49} + ... + a_{36} = 2565$

$\Rightarrow (a + 49d) + (a + 48d) + ... + (a + 35d) = 2565$

$\Rightarrow \quad 15a + d\,(35 + 36 + ... + 49) = 2565$

(Sum of AP 35, 36, ..., 49)

$$= \frac{15}{2}(35 + 49) = \frac{84 \times 15}{2} = 630$$

$\Rightarrow 15a + d\,(630) = 2565$

$\Rightarrow \quad a + 42d = 171 \quad ...(ii)$

$\Rightarrow \quad 2a + 84d = 342 \quad ...(iii)$

Subtract equation (i) from equation (iii),

$75d = 300$

$$d = \frac{300}{75} = 4$$

From equation (ii),

$\Rightarrow a = 171 - 42 \times 4 = 171 - 168 = 3$

$\therefore \quad a = 3; d = 4 \quad \therefore$ AP is: 3, 7, 11, ...

8. To pick up the first potato, distance run

= 2(5)m

To pick up the second potato, distance run

= 2(5 + 3) = 2 (8)m

To pick up the third potato, distance run

= 2(8 + 3) = 2 (11)m

....

....

$\therefore$ Sequence of the distance run is:

2 (5), 2 (8), 2 (11), ..., till 10 terms.

$\therefore$ Total distance covered

= 2 [5 + 8 + 11 + ... + 10 terms]

$$= 2\left[\frac{10}{2}\left\{2(5) + (10 - 1)(3)\right\}\right]$$

$$= 2\left[5(37)\right] = 10 \times 37 = 370\text{m.}$$

WORKSHEET – 49

1. $\because$ 21 is an odd number $\quad \therefore \; a_{21} = 1$

$\therefore$ 40 is an even number $\quad \therefore \; a_{40} = -1$

2. Hint: $d = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$.

$a = \sqrt{2}$

Use $\quad S_n = \frac{n}{2}[2a + (n - 1)d]$.

3. Hint: $\quad a + 6d = 34$

and $a + 12d = 64$

$\Rightarrow \quad a = 4, d = 5.$

4. All three digit numbers which are multiple of 11 are: 110, 121, 132, 143, …, 990

$\therefore$ This is an AP

as $\quad a_n = a + (n - 1)d$

$\Rightarrow \quad 990 = 110 + (n - 1)11$

$\Rightarrow \quad n - 1 = 80$

$\Rightarrow \quad n = 81$

$\therefore$ Sum of all terms of above AP

$$= \frac{n}{2}\{a + a_n\} = \frac{81}{2}\{110 + 990\}$$

$$= \frac{81}{2} \times 1100 = 44955.$$

5. 28th term

Hint: Let $\quad a_n < 0$

$\therefore \quad \frac{83}{4} - \frac{3n}{4} < 0 \quad \Rightarrow 83 - 3n < 0$

$\Rightarrow \quad 3n > 83 \quad \Rightarrow \quad n > 27\frac{2}{3}$

$\Rightarrow \quad n = 28.$

6. Numbers are: 504, 511, 518, …, 896

Which forms an A.P.

$a = 504; d = 7$

Let $\quad a_n = 896$

$\Rightarrow \quad a + (n - 1)\,.\,d = 896$

$\Rightarrow \quad (n - 1)\,.\,7 = 896 - 504 = 392$

$\Rightarrow \quad n - 1 = \frac{392}{7} = 56 \quad \Rightarrow \quad n = 57$

$\therefore \quad \text{Sum} = \frac{n}{2}\{a + l\} = \frac{57}{2}\{504 + 896\}$

$$= \frac{57}{2} \times 1400 = 39900.$$

7. 6 or 12

Hint: Let $\quad S_n = 72$

$\Rightarrow \quad \frac{n}{2}[2a + (n - 1)d] = 72.$

8. $n = 9$; angle = 32°.

Hint: Sum of all angles = 360°.

9. Volume of concrete required to build the

1st step $= \frac{1}{4} \times \frac{1}{2} \times 50 \text{ m}^3$

Volume of concrete required for

2nd step $= \frac{2}{4} \times \left(\frac{1}{2}\right) \times 50 \text{ m}^3$

Volume of concrete required for

3rd step $= \frac{3}{4} \times \left(\frac{1}{2}\right) \times 50 \text{ m}^3$

....

....

Volume of concrete required for 15th step.

$$= \frac{15}{4} \times \frac{1}{2} \times 50 \text{ m}^3$$

$\therefore$ Total volume of concrete required:

$$S_{15} = \frac{50}{8} [1 + 2 + 3 + \ldots\ldots\ldots + 15]$$

$$= \frac{25}{4} \times \frac{15}{2} \times (1 + 15) \quad \left[\because S_n = \frac{n}{2}\{a + l\}\right]$$

$$= \frac{25 \times 15 \times 16}{8} = 750 \text{ m}^3.$$

WORKSHEET – 50

1. $a_n = 2n + 1$

$\therefore$ $a_1 = 3$

$a_2 = 5$

$a_3 = 7$

$\therefore$ $a_1 + a_2 + a_3 = 15.$

2. Hint: Use: $a_n = a + (n - 1)d$.

3. $a_{18} - a_{13} = a + 17d - a - 12d = 5d$

$= 5 \times 5 = 25. \quad [\because d = 5]$

4. $10n - 2$

Hint: $a_n = S_n - S_{n-1}$

$= 5n^2 + 3n - 5(n-1)^2 - 3(n-1)$

$= 5n^2 + 3n - 5(n^2 + 1 - 2n) - 3n + 3$

$= 5n^2 + 3n - 5n^2 - 5 + 10n - 3n + 3$

$a_n = 10n - 2.$

5. Here $a = -11$

$d = -7 - (-11) = 4$

$a_n = 49$

$\therefore$ $a_n = a + (n - 1) . d$

$\Rightarrow$ $49 = -11 + (n - 1) . 4$

$\Rightarrow$ $n = 16$

as n is even $\Rightarrow$ there will be two middle terms which are $\frac{16}{2}$, *i.e.*, 8th and 9th term

$\therefore$ $a_8 = a + 7d = -11 + 7 \times 4 = 17$

and $a_9 = a + 8d = -11 + 8 \times 4 = 21.$

6. Hint: $S_1 = \frac{n}{2}\{2a + (n-1)d\}$

$S_2 = \frac{2n}{2}\{2a + (2n-1)d\}$

$S_3 = \frac{3n}{2}\{2a + (3n-1)d\}$

Calculate $3(S_2 - S_1)$.

7. 900

Hint: $S_{24} = 12(a_1 + a_{24})$

Also note:

$a_5 + a_{20} = a_1 + a_{24}$

$a_{10} + a_{15} = a_1 + a_{24}$

Hence, given relation gives:

$3(a_1 + a_{24}) = 225$

$a_1 + a_{24} = 75$

$\therefore$ $S_{24} = 900.$

8. 7, 8, 9

Hint: Let the three numbers be:

$a - d, a, a + d.$

9. We have $\frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$

$\Rightarrow$ $2a^n + 2b^n = a^n + ab^{n-1} + ba^{n-1} + b^n$

$\Rightarrow$ $a^n + b^n - ab^{n-1} - ba^{n-1} = 0$

$\Rightarrow$ $a^{n-1}(a - b) - b^{n-1}(a - b) = 0$

$\Rightarrow$ $(a - b)(a^{n-1} - b^{n-1}) = 0$

$\Rightarrow$ $a = b$ or $\left(\frac{a}{b}\right)^{n-1} = 1$

Taking $\left(\frac{a}{b}\right)^{n-1} = 1 \Rightarrow \left(\frac{a}{b}\right)^{n-1} = \left(\frac{a}{b}\right)^0$

$\Rightarrow$ $n - 1 = 0 \quad \therefore \quad n = 1.$

OR

$a_n = x$

$\Rightarrow$ $a + (n - 1) \times d = x$

$\Rightarrow \quad 1 + (n-1) \times 3 = x$

$\Rightarrow \quad 3n - 2 = x \quad \text{...}(i)$

$\therefore \quad S = \frac{n}{2}(a + l)$

$590 = \frac{n}{2}(1 + x)$

$1180 = n(1 + 3n - 2)$

$1180 = 3n^2 - n$

$\Rightarrow \quad 3n^2 - n - 1180 = 0$

$\Rightarrow \quad 3n^2 - 60n + 59n - 1180 = 0$

$\Rightarrow \quad 3n(n-20) + 59(n-20) = 0$

$\Rightarrow \quad n = \frac{-59}{3}$ (Reject) or $n = 20$

$\therefore$ From $(i) \Rightarrow x = 3 \times 20 - 2$

$x = 60 - 2 \Rightarrow x = 58.$

WORKSHEET – 51

1. Let $x = n^{\text{th}}$ term

$\therefore \quad x = 2 + (n-1)\,3 \Rightarrow x = 3n - 1$

$\therefore \quad S_n = \frac{n}{2}\{2 + x\} \Rightarrow 155 = \frac{n}{2}\{2 + 3n - 1\}$

$\Rightarrow 310 = 3n^2 + n \quad \Rightarrow 3n^2 + n - 310 = 0$

$\Rightarrow n(3n+31) - 10(3n + 31) = 0 \Rightarrow n = 10$

$\therefore \quad x = 29.$

2. Hint: Use $a_n = a + (n-1).d.$

3. 19668

Hint: The sequence is:

103, 119, 135, ..., 791.

4. $n = 6$

Hint: Let $a = 3$, $b = 17$

$\therefore \quad a, x_1, x_2, \ldots\ldots\ldots, x_n, b$ are in A.P.

$\therefore \quad d = \frac{b-a}{n+1} = \frac{14}{n+1}$

$\therefore \quad x_1 = a + d = 3 + \frac{14}{n+1} = \frac{3n+17}{n+1} \quad \text{...}(i)$

$x_n = a + nd = 3 + \frac{14n}{n+1} = \frac{17n+3}{n+1} \quad \text{...}(ii)$

$\frac{x_1}{x_n} = \frac{1}{3} \Rightarrow 3x_1 = x_n$

$\therefore$ Using (i) and (ii), we get

$n = 6.$

5. Hint: $S_1 = \frac{n}{2}(n+1);\ a = 1,\ d = 1$

$S_2 = n^2;\ a = 1,\ d = 2$

$S_3 = \frac{n}{2}(3n-1);\ a = 1,\ d = 3$

$\therefore S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}(3n-1)$

$S_1 + S_3 = 2n^2 \quad \therefore \quad S_1 + S_3 = 2S_2.$

6. Let $a_1 = a$ and common difference $= d$.

Now, $a_1 + a_7 + a_{10} + a_{21} + a_{24} + a_{30} = 540$

$a + a + 6d + a + 9d + a + 20d + a + 23d + a + 29d = 540$

$\Rightarrow \quad 6a + 87d = 540$

$\Rightarrow \quad 2a + 29d = 180 \quad \text{...}(i)$

Further, the required sum

$S_{30} = \frac{30}{2}(a_1 + a_{30}) = 15(a + a + 29d)$

$= 15(2a + 29d)$

$= 15 \times 180$ [Using equation (i)]

$= 2700.$

7. Hint: Let the first term and the common difference be a and d respectively.

$a_9 = 0 \Rightarrow a + 8d = 0 \Rightarrow a = -8d$

$a_{29} = a + 28d = -8d + 28d = 20d$

$a_{19} = a + 18d = -8d + 18d = 10d.$

8. 2, 6, 10, 14.

Hint: Let the four parts be:

$a - 3d,\ a - d,\ a + d,\ a + 3d.$

OR

The sequence is: 150, 146, 142, ...

$\therefore$ Total number of workers who worked all the n days

$= 150 + 146 + \ldots\ldots + n$ terms.

$\therefore \quad = n(152 - 2n)$

Now had the workers not dropped then the work would have finished in $(n-8)$ days with 150 workers working on each day.

$\therefore$ Total number of workers who would have worked all the n days is $150(n-8)$.

$\therefore \quad n(152 - 2n) = 150(n-8)$

$\Rightarrow \quad n^2 - n - 600 = 0$

$\Rightarrow \quad (n-25)(n+24) = 0$

$\Rightarrow \quad n = 25$ or $n = -24$ (Reject)

$\therefore \quad n = 25.$

WORKSHEET – 52

1. As sum of first n odd natural number is $= n^2$

$\therefore$ replacing n by 20 we get

the sum of first 20 odd natural number $= 400$.

2. Given, AP $= -11, -8, -5, ..., 49$

where $a = -11$, $d = -8 + 11 = 3$

$\therefore$ From the end $t_4 = l - (n-1)d$

$= 49 - (4-1)\cdot 3$

$= 49 - 9 = 40.$

3. $S_n = 2n^2 + 5n$

$\Rightarrow \quad S_{n-1} = 2(n-1)^2 + 5(n-1)$

$= 2(n^2 + 1 - 2n) + 5n - 5$

$= 2n^2 + n - 3$

n^{th} term $= S_n - S_{n-1}$

$= 2n^2 + 5n - (2n^2 + n - 3)$

$= 4n + 3.$

4. The numbers are: 12, 15, 18, …, 99 which is an A.P.

Let $a_n = a + (n-1)\,.\,d$

$\Rightarrow \quad 99 = 12 + (n-1)\,.\,(3)$

$\Rightarrow \quad = 9 + 3n$

$3n = 90$

$\Rightarrow \quad n = 30$

5. First term of the A.P. $= a = -\dfrac{4}{3}$

Common difference of the A.P. $= d$

$$= -1 - \left(-\frac{4}{3}\right) = \frac{1}{3}$$

Let the A.P. consists n terms.

$$\therefore \quad n^{th} \text{ term} = 4\frac{1}{3} = \frac{13}{3} \qquad ...(i)$$

But n^{th} term is given by

$$a_n = a + (n-1)d$$

$$= -\frac{4}{3} + (n-1)\frac{1}{3}$$

$$= \frac{n}{3} - \frac{5}{3} \qquad ...(ii)$$

From equations (i) and (ii), we get

$$\frac{n}{3} - \frac{5}{3} = \frac{13}{3} \Rightarrow n = 18$$

Since $n = 18$ is even number so, the middle most terms will be $\left(\frac{18}{2}\right)^{th}$ and $\left(\frac{18}{2}+1\right)^{th}$ terms, *i.e.*, 9^{th} and 10^{th} terms.

Now, $$a_9 = -\frac{4}{3} + (9-1)\times\frac{1}{3}$$

$$= -\frac{4}{3} + \frac{8}{3} = \frac{4}{3}$$

And $$a_{10} = -\frac{4}{3} + (10-1)\frac{1}{3}$$

$$= -\frac{4}{3} + \frac{9}{3} = \frac{5}{3}$$

Therefore, the required sum

$$= a_9 + a_{10} = \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3.$$

6. According to question, we have

$$A = \frac{a-b}{a+b}$$

$$D = \frac{3a-2}{a+b} - \frac{a-b}{a+b}$$

$$= \frac{3a-2b-a+b}{a+b} = \frac{2a-b}{a+b}$$

Sum of n terms of an AP is

$$S_n = \frac{n}{2}[2A + (n-1)D]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{a-b}{a+b}\right) + (11-1)\left(\frac{2a-b}{a+b}\right)\right]$$

$$= \frac{11}{2}\left[\frac{2a-2b+20a-10b}{a+b}\right]$$

$$= \frac{11}{2}\left[\frac{22a-12b}{a+b}\right] = \frac{11(11a-6b)}{a+b}.$$

7. Let the first term and the common difference of the given A.P. be a and d respectively.

According to the given condition,

$$\frac{a_{11}}{a_{18}} = \frac{2}{3} \Rightarrow \frac{a+10d}{a+17d} = \frac{2}{3}$$

[Using $a_n = a+(n-1)d$]

$\Rightarrow$ $3a + 30d = 2a + 34d$

$\Rightarrow$ $a = 4d$

Now, $$\frac{a_5}{a_{21}} = \frac{a+4d}{a+20d}$$

$$= \frac{4d+4d}{4d+20d} \text{ (Substituting } a = 4d)$$

$$= \frac{8d}{24d} = \frac{1}{3}$$

i.e., $a_5 : a_{21} = 1 : 3$.

Now, $$\frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a+4d]}{\frac{21}{2}[2a+20d]}$$

$$\left[\text{Using } S_n = \frac{n}{2}\{2a+(n-1)d\}\right]$$

$$= \frac{5(8d+4d)}{21(8d+20d)}$$

[Substituting $a = 4d$]

$$= \frac{60d}{588d} = \frac{5}{49}$$

i.e., $S_5 : S_{21} = 5 : 49$.

8. Distances covered by a girl during 1st minute, 2nd minute, 3rd minute,..... are respectively 20 m, 18 m, 16 m,.......which form an A.P. with first term (a) = 20 m and common difference (d) = –2 m.

(*i*) Distance covered in 10th minute

= 10th term of the A.P.

$= a+(10-1)d = 20+9\times(-2) = 20-18$

= 2 m.

(*ii*) Distance covered in 10 minutes

= sum of first 10 terms

$= \frac{10}{2}[2a + (10-1)d] = 5[2 \times 20 + 9(-2)]$

$= 5 \times 22 = 110$ m.

CHAPTER TEST

1. $S_n = \frac{n}{2}(a + a_n) \Rightarrow 399 = \frac{n}{2}(1 + 20)$

$\Rightarrow 21n = 2 \times 399 \quad \Rightarrow \quad n = 38.$

2. $\because \quad a_p = \frac{3p-1}{6}$

$\therefore \quad a_n = \frac{3n-1}{6}$ and $a_1 = \frac{1}{3}$

Now, $S_n = \frac{n}{2}\left(\frac{1}{3} + \frac{3n-1}{6}\right) = \frac{n}{12}(3n + 1).$

3. $a + d = 13$ and $a + 4d = 25$

$\Rightarrow \quad a = 9, d = 4$

Now, $a_7 = a + 6d = 9 + 24 = 33.$

4. $a = 7$; $a_n = 49$

Now, $S_n = 420$

$\Rightarrow \quad 420 = \frac{n}{2}(7 + 49)$

$\Rightarrow \quad n = 15$

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad 49 = 7 + 14d$

$\Rightarrow \quad d = 3$

5. True, the reason is:

$d = 14 - 8 = 6, a = 8$

$a_{53} = a + 52d = 8 + 52 \times 6 = 320$

$a_{41} = a + 40d = 8 + 40 \times 6 = 248$

Now, $a_{53} - a_{41} = 72.$

6. $a_4 = 11$

$\Rightarrow \quad a + 3d = 11 \quad \ldots(i)$

$a_5 + a_7 = 34$

$\Rightarrow \quad a + 4d + a + 6d = 34$

$\Rightarrow \quad 2a + 10d = 34$

$\Rightarrow \quad a + 5d = 17 \quad \ldots(ii)$

Subtracting (*ii*) from (*i*), we get

$\Rightarrow \quad -2d = -6$

$d = 3.$

7. Let the profit to be ceased at nth day.

Sale on first day = ` 8100

Sale on second day = ` (8100 – 150)

= ` 7950

So, the sale (in ₹) will be day by day as follows:

8100, 7950, 7800,..........n terms

Here, $a = 8100$, $d = -150$

The profit will be ceased when it is equal to or less than ₹ 1500.

Therefore, $8100 + (n-1) \times (-150) \leq 1500$

$[\because a_n = a + (n-1)d]$

$\Rightarrow 8100 - 150n + 150 \leq 1500$

$\Rightarrow 150n \geq 6750 \Rightarrow n \geq 45$

Hence, the profit to be ceased at 45th day.

8. If x, y and z are in A.P., then

$$y = \frac{x+z}{2} \quad ...(i)$$

Now, in the given equation,

LHS $= (x + 2y - z)(2y + z - x)(z + x - y)$

$= (x + x + z - z)(x + z + z - x)(2y - y)$

[Using equation (i)]

$= 2x \times 2z \times y = 4xyz$

= RHS. **Hence proved.**

9. (i) Number of classes = 12

$\because$ Each class has 3 sections.

$\therefore$ Number of plants planted by class I

$= 1 \times 3 = 3$

Number of plants planted by class II

$= 2 \times 3 = 6$

Number of plants planted by class III

$= 3 \times 3 = 9$

Number of plants planted by class IV

$= 4 \times 3 = 12$

....

Number of plants planted by class XII

$= 12 \times 3 = 36$

The numbers 3, 6, 9, 12, ..., 36 are in A.P.

Here, $a = 3$ and $d = 6 - 3 = 3$

$\because$ Number of classes = 12

i.e., $n = 12$

$\therefore$ Sum the n terms of the above A.P., is given by

$$S_{12} = \frac{12}{2}[2(3) + (12-1)3]$$

$$\left[\text{Using } S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$= 6[6 + 11 \times 3]$

$= 6[6 + 33]$

$= 6 \times 39 = 234$

Thus, the total number of trees planted = 234.

(ii) Sum of an arithmetic progression upto n terms

(iii) Love for environment

❑❑

Chapter 6 INTRODUCTION TO TRIGONOMETRY

WORKSHEET – 54

1. As $\sin A = \cos B = \sin (90 - B)$

$\Rightarrow A = 90 - B$

$A + B = 90°$.

2. As $\cos (90 - \theta) = \sin \theta$

$\sin (90 - \theta) = \cos \theta$.

$\sin \theta \cos (90 - \theta) + \cos \theta \sin (90 - \theta)$

$= \sin^2 \theta + \cos^2 \theta$

$= 1$.

3. $\sin \theta = \frac{24}{25} \Rightarrow \sin^2 \theta = \left(\frac{24}{25}\right)^2$

$\Rightarrow 1 - \sin^2 \theta = 1 - \frac{24^2}{25^2}$

$\Rightarrow \cos^2 \theta = \frac{7^2}{25^2}$

$\Rightarrow \cos \theta = \frac{7}{25}$

Now, $\tan\theta + \sec\theta = \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos\theta}$

$= \frac{\frac{24}{25}}{\frac{7}{25}} + \frac{1}{\frac{7}{25}} = \frac{24}{7} + \frac{25}{7} = \frac{49}{7} = 7.$

4. $\tan \theta = \frac{\sin \theta}{\sqrt{1-\sin^2 \theta}}$.

5. $\cot 25° + \tan 41°$.

6. LHS $= \sqrt{\frac{\cot^2 \theta}{1 + \cot^2 \theta}}$

$= \sqrt{\frac{\cot^2 \theta}{\text{cosec}^2 \theta}}$

$\{\because \text{cosec}^2 \theta - \cot^2 \theta = 1\}$

$= \sqrt{\frac{\cos^2 \theta}{\sin^2 \theta \times \frac{1}{\sin^2 \theta}}} \quad \left\{\because \cot \theta = \frac{\cos \theta}{\sin \theta}\right\}$

$= \sqrt{\cos^2 \theta} = \cos \theta = \text{RHS}$

7. $\frac{4}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cos^2 45°$

$= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2} = \frac{8+8-3}{6}$

$= \frac{13}{6}$.

8. $\sin (x + y) = 1$ and $\cos (x - y) = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin (x + y) = \sin 90°$ and $\cos (x - y) = \cos 30°$

$\Rightarrow x + y = 90°$ and $x - y = 30°$

Adding and subtracting, we get respectively

$2x = 120°$ and $2y = 60°$

i.e., $x = 60°$ and $y = 30°$.

9. $\text{cosec } A = \sqrt{10}$

$\sin A = \frac{1}{\text{cosec } A} = \frac{1}{\sqrt{10}}$

$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{1}{10}} = \frac{3}{\sqrt{10}}$

$\tan A = \frac{\sin A}{\cos A} = \frac{1}{3}$

$\cot A = \frac{1}{\tan A} = 3$

$\sec A = \frac{1}{\cos A} = \frac{\sqrt{10}}{3}$.

10. Hint: RHS = $\dfrac{\sin^2 A}{1-\cos A} = \dfrac{1-\cos^2 A}{1-\cos A}$

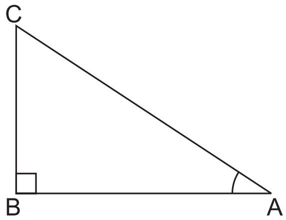
$= \dfrac{(1-\cos A)(1+\cos A)}{1-\cos A}$

OR

Hint: LHS = $\dfrac{\sin\theta\,(1-2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta-1)}$.

WORKSHEET – 55

1.



As AC = 25 cm; BC = 7 cm

$\Rightarrow$ Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow (25)^2 = (AB)^2 + (7)^2$

$\Rightarrow 625 - 49 = AB^2$

$\Rightarrow AB^2 = 576 = (24)^2$

$\Rightarrow AB = 24$

$\therefore \tan A = \dfrac{BC}{AB} = \dfrac{7}{24}$.

2. 0

Hint: Divide numerator and denominator by cos θ.

3. As $\cos 52° = \cos(90 - 38°)$

$= \sin 38°$

$[\because \cos(90-\theta) = \sin\theta]$

$\therefore \sin 38° - \cos 52° = \sin 38° - \sin 38° = 0$.

4. Hint: $\angle A = 30°, \angle B = 90°, \angle C = 60°$.

5. $\sec\theta + \tan\theta = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{(1+\sin\theta)}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\sqrt{1-\sin^2\theta}} = \dfrac{1+\frac{a}{b}}{\sqrt{1-\frac{a^2}{b^2}}} = \dfrac{b+a}{\sqrt{b^2-a^2}}$

$= \dfrac{b+a}{\sqrt{b+a}\,\sqrt{b-a}} = \sqrt{\dfrac{b+a}{b-a}}$.

6. $\sin A = \dfrac{7}{25}, \cos A = \dfrac{24}{25}$,

$\sin C = \dfrac{24}{25}$ and $\cos C = \dfrac{7}{25}$.

7. $\dfrac{\cos 60° + \sin 30° - \cot 30°}{\tan 60° + \sec 45° - \text{cosec } 45°}$

$= \dfrac{\frac{1}{2}+\frac{1}{2}-\sqrt{3}}{\sqrt{3}+\sqrt{2}-\sqrt{2}} = \dfrac{1-\sqrt{3}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{3}-3}{3}$.

8. Given expression

$= \dfrac{\cot\theta\tan(90°-\theta) - \sec(90°-\theta)\,\text{cosec}\,\theta}{\sin\theta\cos(90°-\theta) + \cos\theta\sin(90°-\theta)}$

$= \dfrac{\cot\theta\cot\theta - \text{cosec}\,\theta\,\text{cosec}\,\theta}{\sin\theta\sin\theta + \cos\theta\cos\theta}$

$= \dfrac{\cot^2\theta - \text{cosec}^2\theta}{\sin^2\theta + \cos^2\theta}$

$= \text{cosec}^2\theta - 1 - \text{cosec}^2\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= -1$.

9. Draw Δ ABC with

AB = BC = AC = a (say)

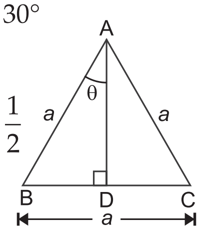
Draw AD ⊥ BC

$\therefore \angle BAD = \angle DAC = \theta = 30°$

and BD = DC = $a/2$

$\therefore \sin\theta = \dfrac{BD}{AB} = \dfrac{a/2}{a} = \dfrac{1}{2}$

$\Rightarrow \sin 30° = \dfrac{1}{2}$.



10. As $a^2 + b^2 = (a+b)^2 - 2ab$

$\therefore$ Taking $a = \sin^2\theta$; $b = \cos^2\theta$, we get

$(\sin^2\theta)^2 + (\cos^2\theta)^2 = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$\Rightarrow \sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$ ...(*i*)

also as $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

Put $a = \sin^2\theta$; $b = \cos^2\theta$

$\Rightarrow (\sin^2\theta)^3 + (\cos^2\theta)^3$
$= [\sin^2\theta + \cos^2\theta]\,[\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta]$

$\Rightarrow \sin^6\theta + \cos^6\theta = 1\,.\,[1 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$ [$\because$ Using (*i*)]
$= 1 - 3\sin^2\theta\cos^2\theta$ ...(*ii*)

$\therefore$ Consider

LHS $= 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta)$
$= 2(1 - 3\sin^2\theta\cos^2\theta) - 3(1 - 2\sin^2\theta\cos^2\theta)$ [$\because$ Using (*i*) and (*ii*)]
$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta$
$= -1 =$ RHS **Hence proved.**

OR

$$\text{LHS} = \frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

$$= \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)}$$

$$= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \text{RHS}$$

WORKSHEET – 56

1. Hint: $\tan 5^\circ = \cot 85^\circ$; $\tan 25^\circ = \cot 65^\circ$.

2. Hint: $(1 + \sin\theta)(1 - \sin\theta) = \cos^2\theta$
$$= \frac{1}{\sec^2\theta}.$$

3. $8\tan x = 15 \Rightarrow \tan^2 x = \frac{225}{64}$

$\Rightarrow \sec^2 x - 1 = \frac{225}{64} \Rightarrow \sec^2 x = \frac{289}{64}$

$\Rightarrow \sec x = \frac{17}{8} \Rightarrow \cos x = \frac{8}{17}$

Now, $\sin x - \cos x = \sqrt{1 - \cos^2 x} - \cos x$

$$= \sqrt{1 - \frac{64}{289}} - \frac{8}{17} = \frac{15 - 8}{17}$$

$$= \frac{7}{17}.$$

4. If $\theta = 0 \Rightarrow \cos 0^\circ = 1$

$$\therefore \frac{1 - \cos\theta}{1 + \cos\theta} = \frac{1-1}{1+1} = \frac{0}{2} = 0.$$

5. Hint: $\sec 4A = \operatorname{cosec}(90^\circ - 4A)$.

6. Hint: $\cos(90^\circ - \theta) = \sin\theta$,
$\sin(90^\circ - \theta) = \cos\theta$.

7. $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

OR

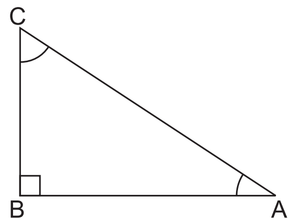
$$\frac{\operatorname{cosec}^2\theta + \cot^2\theta}{\operatorname{cosec}^2\theta - \sec^2\theta}$$

$$= \frac{1+\cot^2\theta+\cot^2\theta}{1+\cot^2\theta-(1+\tan^2\theta)}$$

$$= \frac{1+2\cot^2\theta}{\cot^2\theta-\tan^2\theta}$$

$$= \frac{1+2\times 3}{3-\frac{1}{3}} = \frac{7}{\frac{8}{3}} = \frac{21}{8}.$$

8.



$\tan A = \sqrt{3} \Rightarrow A = 60^\circ$
($\because \tan 60^\circ = \sqrt{3}$)

$\tan B = \frac{1}{\sqrt{3}} \Rightarrow B = 30^\circ$
$\left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$

$\therefore \sin A\cos B - \cos A\sin B$
$= \sin 60^\circ\,.\,\cos 30^\circ - \cos 60^\circ\,.\,\sin 30^\circ$

$$= \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} - \frac{1}{2}\times\frac{1}{2}$$

$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

9. Hint:

$$\text{LHS} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{(\sec\theta - \tan\theta)^2}$$

OR

Hint:

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

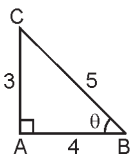
$= \cos A + \sin A$.

10. Hint: $1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$= \cos^2\theta - \sin^2\theta$.

WORKSHEET – 57

1. $\tan\theta = \dfrac{3}{4} = \dfrac{\text{Perpendicular}}{\text{Base}}$



$BC = \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$

$\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{4}{5}$

$$\therefore \quad \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{1}{9}.$$

2. Hint:

$1 + \tan\theta + \sec\theta$

$$= 1 + \frac{1}{\cot\theta} + \sec\theta = \frac{1+\cot\theta+\operatorname{cosec}\theta}{\cot\theta}$$

3. Hint: $A + B = 90^\circ$; $A - B = 30^\circ$.

4. $\tan 2\theta = \cot(\theta + 9^\circ)$

$\Rightarrow \tan 2\theta = \tan[90^\circ - (\theta + 9^\circ)]$

$\Rightarrow 2\theta = 90^\circ - \theta - 9^\circ \Rightarrow 3\theta = 81^\circ$

$\Rightarrow \theta = 27^\circ$.

5. $\cot\theta = \dfrac{\cos\theta}{\sqrt{1-\cos^2\theta}}$.

6. True

Hint: $A^6 + B^6$

$= (A^2 + B^2)\,[(A^2 + B^2)^2 - 3A^2B^2]$.

7. Hint:

$$\text{LHS} = \frac{1 - 2\sin\theta\cos\theta + 1 + 2\sin\theta\cos\theta}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}.$$

8. $\text{LHS} = \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing numerator and denominator by $\sin A$, we get

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\,[1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1}$$

$= \operatorname{cosec} A + \cot A = \text{RHS}$.

9. Given expression

$$= \frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$$

$$= \frac{2\sin(90^\circ - 22^\circ)}{\cos 22^\circ} - \frac{2\cot(90^\circ - 75^\circ)}{5\tan 75^\circ} - \frac{3\times 1\times \tan(90^\circ - 70^\circ)\tan(90^\circ - 50^\circ)\tan 50^\circ \tan 70^\circ}{5}$$

$$= \frac{2\cos 22^\circ}{\cos 22^\circ} - \frac{2\tan 75^\circ}{5\tan 75^\circ} - \frac{3\cot 70^\circ \cot 50^\circ \tan 50^\circ \tan 70^\circ}{5}$$

$$= 2 - \frac{2}{5} - \frac{3\times\frac{1}{\tan 70^\circ}\times\frac{1}{\tan 50^\circ}\tan 50^\circ \tan 70^\circ}{5}$$

$$= 2 - \frac{2}{5} - \frac{3}{5} = \frac{10-2-3}{5} = \frac{5}{5} = 1.$$

10. Given expression

$= 8\sqrt{3}\ \text{cosec}^2 30°.\ \sin 60°.\ \cos 60°.\ \cos^2 45°.\ \sin 45°.\ \tan 30°.\ \text{cosec}^3 45°.$

$= 8\sqrt{3} \times \dfrac{1}{\sin^2 30°}\ .\ \sin(90° - 30°).\ \cos(90° - 30°)\ \cos^2(90° - 45°).\ \sin 45°.\ \dfrac{\sin 30°}{\cos 30°}\ \dfrac{1}{\sin^3 45°}$

$= 8\sqrt{3} \times \dfrac{1}{\sin^2 30°} \times \cos 30°.\ \sin 30°.\ \sin^2 45°.\ \sin 45°.\ \dfrac{\sin 30°}{\cos 30°} \times \dfrac{1}{\sin^3 45°}$

$= 8\sqrt{3} \times \left(\dfrac{\sin 30°.\ \sin 30°}{\sin^2 30°}\right) \times \dfrac{\cos 30°}{\cos 30°} \times \dfrac{\sin^2 45°\ \sin 45°}{\sin^3 45°}$

$= 8\sqrt{3} \times 1 \times 1 \times 1 = 8\sqrt{3}.$

OR

Hint: $\sec^2\theta = x^2 + \dfrac{1}{16x^2} + \dfrac{1}{2}$

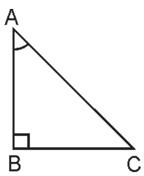
$\therefore \sec^2\theta - 1 = x^2 + \dfrac{1}{16x^2} - \dfrac{1}{2}$

$\therefore \quad \tan^2\theta = \left(x - \dfrac{1}{4x}\right)^2$

$\Rightarrow \quad \tan\theta = \pm\left(x - \dfrac{1}{4x}\right).$

WORKSHEET – 58

1. As $\sin A = \dfrac{3}{4}$,



let BC = $3x$ and CA = $4x$

$\therefore$ AB $= \sqrt{(4x)^2 - (3x)^2} = \sqrt{7}\,x$

Now, $\tan A = \dfrac{BC}{AB} = \dfrac{3x}{\sqrt{7}x} = \dfrac{3}{\sqrt{7}}.$

2. $\dfrac{2\tan 30°}{1 + \tan^2 30°} = \dfrac{2 \times \dfrac{1}{\sqrt{3}}}{1 + \left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{\dfrac{2}{\sqrt{3}}}{\dfrac{4}{3}} = \dfrac{\sqrt{3}}{2}.$

3. Hint: $\tan x = \dfrac{15}{8}$

$\Rightarrow \sin x = \dfrac{8}{17}, \cos x = \dfrac{8}{17}$

$\therefore \sin^2 x - \cos^2 x = \dfrac{225}{289} - \dfrac{64}{289} = \dfrac{161}{289}.$

4. $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$

$= \dfrac{\dfrac{1}{2} + 1 - \dfrac{2}{\sqrt{3}}}{\dfrac{2}{\sqrt{3}} + \dfrac{1}{2} + 1} = \dfrac{\sqrt{3} + 2\sqrt{3} - 4}{4 + \sqrt{3} + 2\sqrt{3}}$

$= \dfrac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \dfrac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$

$= \dfrac{27 + 16 - 24\sqrt{3}}{27 - 16} = \dfrac{43 - 24\sqrt{3}}{11}.$

5. $\because$ A + B + C = 180°

$\therefore \quad \text{LHS} = \cot\dfrac{C + A}{2} = \cot\dfrac{180° - B}{2}$

$= \cot\left(90° - \dfrac{B}{2}\right) = \tan\dfrac{B}{2}$

= RHS.

6. Yes.

Hint: Both sides $= \dfrac{7}{25}$.

7. LHS = (cosec A – sin A) (sec A – cos A)

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$= \dfrac{1 - \sin^2 A}{\sin A} \times \dfrac{1 - \cos^2 A}{\cos A}$

$= \dfrac{\cos^2 A \times \sin^2 A}{\sin A \cos A}$

$= \sin A \cos A$ ... (*i*)

$\text{RHS} = \dfrac{1}{\tan A + \cot A} = \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

$= \sin A \cos A$ ...(*ii*)

($\because$ $\sin^2 A + \cos^2 A = 1$)

From equations (*i*) and (*ii*), we obtain LHS = RHS.

8. $7 \sin^2 \theta + 3(1 - \sin^2 \theta) = 4$

Let $\sin \theta = x$

$\therefore \quad 7x^2 + 3 - 3x^2 = 4$

$$\Rightarrow 4x^2 = 1 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}$$

$$\therefore \sin \theta = \frac{1}{2} \text{ or } \sin \theta = \frac{-1}{2}$$

$\sin \theta = -\frac{1}{2}$ is not possible as θ is acute.

$$\Rightarrow \operatorname{cosec} \theta = 2 \qquad \therefore \cos \theta = \frac{\sqrt{3}}{2}$$

$\therefore \sec \theta + \operatorname{cosec} \theta = \frac{2}{\sqrt{3}} + 2$. **Hence proved**

9. LHS $= (\sec \theta + \tan \theta)^2$

$= (\sec \theta + \tan \theta)(\sec \theta + \tan \theta)$

$= (\sec \theta + \tan \theta)(\sec \theta + \tan \theta) \times \frac{(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta)}$

$$= \frac{(\sec \theta + \tan \theta)(\sec^2 \theta - \tan^2 \theta)}{(\sec \theta - \tan \theta)}$$

$$= \frac{(\sec \theta + \tan \theta) . 1}{\sec \theta - \tan \theta} = \frac{\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}}$$

$$= \frac{1 + \sin \theta}{1 - \sin \theta} = \frac{\sin \theta \left(\frac{1}{\sin \theta} + 1\right)}{\sin \theta \left(\frac{1}{\sin \theta} - 1\right)}$$

$$= \frac{\operatorname{cosec} \theta - 1}{\operatorname{cosec} \theta + 1} = \text{RHS}$$

10. LHS $= m^2 - n^2 = (\tan \theta + \sin \theta)^2 - (\tan \theta - \sin \theta)^2 = 4 \sin \theta \tan \theta$ ...(*i*)

$$\text{RHS} = 4\sqrt{mn} = 4\sqrt{\frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta}$$

$$= 4 \sin \theta \sqrt{\sec^2 \theta - 1}$$

$= 4 \sin \theta \tan \theta$ ...(*ii*)

From (*i*) and (*ii*), LHS = RHS.

WORKSHEET – 59

1. Required value $= 25 \left(\frac{64}{100} + 2 \times \frac{36}{100} - \frac{8}{6}\right)$

$$= 25 \times \frac{1}{300} (192 + 216 - 400)$$

$$= \frac{1}{12} \times 8 = \frac{2}{3}.$$

2. $\sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - 0.36} = 0.8$

And $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{0.8}{0.6} = \frac{4}{3}$

Now, $5 \sin \theta - 3 \tan \theta = 5 \times 0.8 - 3 \times \frac{4}{3} = 0$

3. Hint: Divide numerator and denominator by sin A.

$$\frac{1 + \cot A}{1 - \cot A} = \frac{1 + \frac{3}{4}}{1 - \frac{3}{4}}.$$

4. $\sec A = \frac{2}{\sqrt{3}} \Rightarrow \sec A = \sec 30° \Rightarrow A = 30°$

$\Rightarrow A + B = 90° \Rightarrow B = 90° - 30° = 60°$

Now, $\operatorname{cosec} B = \operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$

5. Given expression

$$= \frac{\left(\sqrt{3}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 5 \times 0}{2 + 2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{3 + 2 + 4}{4 - 3} = 9.$$

6. False

Hint: $\angle A = 30°$, $\angle B = 60°$.

7. $\sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}}$

$$= \sqrt{\frac{\frac{1}{\cos \theta} - \frac{1}{\sin \theta}}{\frac{1}{\cos \theta} + \frac{1}{\sin \theta}}} = \sqrt{\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta}}$$

$$= \sqrt{\frac{(\sin\theta - \cos\theta)\times\frac{1}{\sin\theta}}{(\sin\theta + \cos\theta)\times\frac{1}{\sin\theta}}} = \sqrt{\frac{1-\cot\theta}{1+\cot\theta}}$$

$$= \sqrt{\frac{1-\frac{3}{4}}{1+\frac{3}{4}}} = \sqrt{\frac{1}{7}}.$$

Hence proved.

8. $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\cot\theta - 1}{\cot\theta + 1}.$

(Dividing numerator and denominator by $\sin\theta$)

$$= \frac{\frac{p}{q}-1}{\frac{p}{q}+1} = \frac{p-q}{p+q}.$$

9. Given expression

$$= \left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 55°}{\sin 35°}\right)^2 - 2\cos 60°$$

$$= \left\{\frac{\sin 35°}{\cos(90°-35°)}\right\}^2 + \left\{\frac{\cos 55°}{\sin(90°-55°)}\right\}^2 - 2\cos 60°$$

$$= \left(\frac{\sin 35°}{\sin 35°}\right)^2 + \left(\frac{\cos 55°}{\cos 55°}\right)^2 - 2\cos 60°$$

$= 1 + 1 - 2\cos 60°$

$= 2 - 2\times\frac{1}{2} = 2 - 1 = 1$

OR

Given expression

$$= \frac{\cos 58°}{\sin 32°} + \frac{\sin 22°}{\cos 68°} - \frac{\cos 38° \operatorname{cosec} 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°}$$

$$= \frac{\cos 58°}{\sin(90°-58°)} + \frac{\sin 22°}{\cos(90°-22°)} - \frac{\cos 38° \operatorname{cosec}(90°-38°)}{\tan 18° \tan 35° \tan 60° \tan(90°-18°)\tan(90°-35°)}$$

$$= \frac{\cos 58°}{\cos 58°} + \frac{\sin 22°}{\sin 22°} - \frac{\cos 38° \sec 38°}{\tan 18° \tan 35° \tan 60° \cot 18° \cot 35°}$$

$$= 2 - \frac{\cos 38° \times \frac{1}{\cos 38°} \times \tan 18° \times \tan 35°}{\tan 18° \tan 35° \tan 60°}$$

$$= 2 - \frac{1}{\tan 60°}$$

$$= 2 - \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6-\sqrt{3}}{3}.$$

10. $\tan A = n \tan B$

$\Rightarrow \quad \cot B = \frac{n}{\tan A}$ and $\sin A = m \sin B$

$\Rightarrow \quad \sin B = \frac{1}{m}\sin A$

$\Rightarrow \quad \operatorname{cosec} B = \frac{m}{\sin A}$

$\therefore \quad \operatorname{cosec}^2 B - \cot^2 B = 1$

$\Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$

$\Rightarrow \quad \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1$

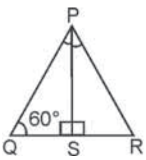
$\Rightarrow \quad m^2 - 1 = (n^2 - 1)\cos^2 A$

$\Rightarrow \quad \frac{m^2-1}{n^2-1} = \cos^2 A.$

Hence proved

OR

Consider an equilateral triangle PQR in which PS $\perp$ QR. Since PS $\perp$ QR so PS bisects $\angle$P as well as base QR. We observe that ΔPQS is a right triangle, right-angled at S with $\angle$QPS = 30° and $\angle$PQS = 60°.



For finding the trigonometric ratios, we need to know the length of the sides of the triangle. So, let us suppose PQ = x

Then , QS = $\frac{1}{2}$ QR = $\frac{x}{2}$

and $(PS)^2 = (PQ)^2 - (QS)^2$

$$= x^2 - \frac{x^2}{4} = \frac{3x^2}{4}$$

$$\therefore \quad PS = \frac{\sqrt{3}x}{2}$$

(*i*) $\cos 60° = \frac{QS}{PQ} = \frac{\frac{x}{2}}{x} = \frac{1}{2}$

(*ii*) $\sin 60° = \frac{PS}{PQ} = \frac{\frac{\sqrt{3}x}{2}}{x} = \frac{\sqrt{3}}{2}$

(*iii*) $\tan 30° = \frac{QS}{PS} = \frac{\frac{x}{2}}{\frac{\sqrt{3}x}{2}} = \frac{1}{\sqrt{3}}$.

WORKSHEET – 60

1. $b^2x^2 + a^2y^2 = b^2a^2\cos^2\theta + a^2b^2\sin^2\theta$
$= a^2b^2$.

2. $A = 90° - 60° = 30°$
$\therefore$ cosec A = cosec 30° = 2.

3. $\tan\theta = \frac{12}{5}$

$\Rightarrow \quad 1 + \tan^2\theta = 1 + \frac{12^2}{5^2}$

$\Rightarrow \quad \sec\theta = \frac{13}{5}$

Now, $\frac{1+\sin\theta}{1-\sin\theta} = \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}} = \frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

$$= \frac{\frac{13}{5}+\frac{12}{5}}{\frac{13}{5}-\frac{12}{5}} = \frac{\frac{25}{5}}{\frac{1}{5}} = 25.$$

4. $\left(\frac{\sin 29°}{\cos 61°}\right) + \left(\frac{\cos 27°}{\sin 63°}\right)^2 - 4\cos^2 45°$

$$= \frac{\sin 29°}{\cos(90° - 29°)} + \left\{\frac{\cos 27°}{\sin(90° - 27°)}\right\}^2 - 4 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1 + 1^2 - \frac{4}{2} = 0.$$

5. Given expression

$$= 4\left\{\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right\} - 3\left\{\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right\} - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 4\left(\frac{1}{16} + \frac{1}{4}\right) - 3\left(\frac{1}{2} - 1\right) - \frac{3}{4}$$

$$= \frac{1}{4} + 1 - \frac{3}{2} + 3 - \frac{3}{4}$$

$$= \frac{17}{4} - \frac{9}{4} = \frac{8}{4} = 2.$$

6. $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$

$$= \frac{\cos A + \cos^2 A + \sin^2 A}{\sin A(1+\cos A)}$$

$$= \frac{1+\cos A}{\sin A(1+\cos A)} = \text{cosec } A$$

$= 2.$

7. $\because \quad \sin\theta = \frac{3}{4} \quad \therefore \text{cosec }\theta = \frac{4}{3}$

$\because \quad \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\therefore \quad \sec\theta = \frac{4}{\sqrt{7}}$ and $\cot\theta = \frac{\sqrt{7}}{3}$

Now, LHS

$$= \sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{\frac{16}{9}-\frac{7}{9}}{\frac{16}{7}-1}} = \sqrt{\frac{\frac{9}{9}}{\frac{9}{7}}}$$

$= \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$ = RHS. **Hence proved.**

8. Hint: LHS $= \frac{1}{\frac{1}{\sin A} - \frac{\cos A}{\sin A}} - \frac{1}{\sin A}$

$$= \frac{\sin A}{1-\cos A} - \frac{1}{\sin A}$$

$$= \frac{\sin^2 A - 1 + \cos A}{\sin A(1-\cos A)}$$

$$= \frac{1 - \cos^2 A - 1 + \cos A}{\sin A\,(1 - \cos A)}$$

$$= \frac{\cos A\,(1 - \cos A)}{\sin A\,(1 - \cos A)} = \cot A.$$

OR

Using $a^3 + b^3 = (a^2 + b^2 - ab)(a + b)$, we get

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cdot\cos\theta$$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cdot\cos\theta)}{\sin\theta + \cos\theta} + \sin\theta\cdot\cos\theta$$

$$= 1 - \sin\theta\cdot\cos\theta + \sin\theta\cdot\cos\theta = 1.$$

9. $\dfrac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$

$$= \frac{2\,(1 + \sin\theta)(1 - \sin\theta)}{2\,(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \frac{1 - \sin^2\theta}{1 - \cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$

$$= \left(\frac{15}{8}\right)^2 = \frac{225}{64}.$$

10. Hint: $p^2 - 1 = \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1$

$= 2\tan\theta\,(\tan\theta + \sec\theta)$

Similarly $p^2 + 1 = 2\sec\theta\,(\tan\theta + \sec\theta)$.

WORKSHEET – 61

1. Hint: $x + y = 2\cot A$

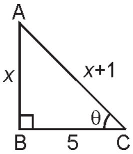
$x - y = 2\cos A$

$\therefore \quad \left(\dfrac{x - y}{x + y}\right)^2 = \sin^2 A$

and $\quad \left(\dfrac{x - y}{2}\right)^2 = \cos^2 A$

$\therefore \quad \sin^2 A + \cos^2 A = 1.$

2. 5



Hint: $(x + 1)^2 = x^2 + 5^2$

3. $\tan A = \dfrac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow \therefore \quad \angle A = 30°$

$\therefore \quad \angle C = 180° - \angle A - \angle B = 180° - 120°$

$= 60°$

Now, $\sin A\cos C + \cos A\sin C$

$= \sin 30°\cos 60° + \cos 30°\sin 60°$

$= \dfrac{1}{2} \times \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} = 1.$

4. $\cos\alpha = \dfrac{1}{2} \Rightarrow \cos\alpha = \cos 60°$

$\Rightarrow \quad \alpha = 60°$

$\tan\beta = \dfrac{1}{\sqrt{3}} \Rightarrow \tan\beta = \tan 30°$

$\Rightarrow \quad \beta = 30°.$

Now, $\sin(\alpha + \beta) = \sin(60° + 30°) = \sin 90° = 1.$

5. $\tan 1°\ \tan 2° \ldots \tan 43°\ \tan 44°\ \tan 45°\ \tan 46°\ \tan 47° \ldots \tan 88°\ \tan 89°$

$= (\tan 1°\ \tan 89°)(\tan 2°\ \tan 88°) \ldots (\tan 43°\ \tan 47°)(\tan 44°\ \tan 46°)\ \tan 45°$

$= (\tan 1°\ \cot 1°)(\tan 2°\ \cot 2°) \ldots (\tan 43°\ \cot 43°)(\tan 44°\ \cot 44°)\ \tan 45°$

$= (1) \times (1) \times \ldots \times (1) \times (1) \times \tan 45°$

$= (1 \times 1 \times \ldots \times 1 \times 1) \times \tan 45°$

$= 1 \times 1 = 1.$

6. Given expression

$$= \frac{\tan 50° + \sec 50°}{\cot 40° + \operatorname{cosec} 40°} + \cos 40°\operatorname{cosec} 50°$$

$$= \frac{\tan 50° + \sec 50°}{\cot(90° - 50°) + \operatorname{cosec}(90° - 50°)} + \cos 40°\operatorname{cosec}(90° - 40°)$$

$$= \frac{\tan 50° + \sec 50°}{\tan 50° + \sec 50°} + \cos 40°.\ \frac{1}{\cos 40°}$$

$= 1 + 1 = 2.$

7. LHS $= \tan(A - B) = \tan(60° - 30°) = \tan 30°$

$= \dfrac{1}{\sqrt{3}}.$

$$\text{RHS} = \frac{\tan A - \tan B}{1 + \tan A\cdot\tan B} = \frac{\tan 60° - \tan 30°}{1 + \tan 60°\cdot\tan 30°}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2}$$

$= \frac{1}{\sqrt{3}}$ = LHS. **Hence verified.**

8. RHS $= \frac{\sin^6 \theta}{\cos^6 \theta} + \frac{3 \sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\cos^2} + 1$

$$= \frac{\sin^6 \theta + \cos^6 \theta + 3 \sin^2 \theta \cos^2 \theta}{\cos^6 \theta}$$

$$= \frac{\left(\sin^2 \theta + \cos^2 \theta\right)^3}{\cos^6 \theta}$$

$= \sec^6 \theta$ = LHS. $[\because \sin^2 \theta + \cos^2 \theta = 1]$

OR

Hint: Numerator of

LHS = $\tan \theta + \sec \theta - (\sec^2 \theta - \tan^2 \theta)$

$= (\tan \theta + \sec \theta) - (\tan \theta + \sec \theta)(\sec \theta - \tan \theta)$

$= (\tan \theta + \sec \theta)(1 - \sec \theta + \tan \theta)$.

9. $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$

Squaring both sides, we get

$\cos^2 \theta + 2 \cos \theta \sin \theta + \sin^2 \theta = 2 \cos^2 \theta$

$\Rightarrow 2 \cos^2 \theta - \cos^2 \theta - 2 \cos \theta \sin \theta = \sin^2 \theta$

$\Rightarrow \cos^2 \theta - 2 \cos \theta \sin \theta = \sin^2 \theta$

Adding $\sin^2 \theta$ to both sides, we have

$\sin^2 \theta + \cos^2 \theta - 2 \cos \theta \sin \theta = \sin^2 \theta + \sin^2 \theta$

$\Rightarrow (\cos \theta - \sin \theta)^2 = 2 \sin^2 \theta$

$\Rightarrow \cos \theta - \sin \theta = \sqrt{2} \sin \theta$ **Hence proved.**

10. Hint: $l \tan \theta + m \sec \theta = n$ ...(i) $\times l'$

$l' \tan \theta - m' \sec \theta = n'$ ...(ii) $\times l$

$\Rightarrow ll' \tan \theta + ml' \sec \theta = nl'$

$l'l \tan \theta - m'l \sec \theta = n'l$

$-\quad\quad + \quad\quad -$

$(m'l + ml') \sec \theta = nl' - n'l$

$$\Rightarrow \sec \theta = \frac{nl' - n'l}{m'l + ml'}$$

Similarly, $\tan \theta = \frac{nm' + mn'}{lm' + ml'}$.

WORKSHEET – 62

1. Given expression

$$= \frac{\cos^2 (90° - 70°) + \cos^2 70°}{\sec^2 (90° - 40°) - \cot^2 40°} + 2 \{\text{cosec}^2 58° - \cot 58° \tan (90° - 58°)\}$$

$$= \frac{\sin^2 70° + \cos^2 70°}{\text{cosec}^2 40° - \cot^2 40°} + 2 (\text{cosec}^2 58° - \cot^2 58°)$$

$= \frac{1}{1} + 2(1) = 1 + 2 = 3.$

2. $\sec 5A = \text{cosec} (A - 36°)$

$\Rightarrow \sec 5A = \sec \{90° - (A - 36°)\}$

$\Rightarrow 5A = -A + 126° \Rightarrow A = 21°.$

3. Given expression

$= \sin^2 5° + \sin^2 10° \ldots + \sin^2 40° + \sin^2 45° + \sin^2 50° + \ldots + \sin^2 80° + \sin^2 85° + \sin^2 90°$

$= \cos^2 85° + \cos^2 80° + \ldots + \cos^2 50° + \left(\frac{1}{\sqrt{2}}\right)^2 + \sin^2 50° + \ldots + \sin^2 80° + \sin^2 85° + (1)^2$

$= (\cos^2 85° + \sin^2 85°) + (\cos^2 80° + \sin^2 80°) + \ldots + (\cos^2 50° + \sin^2 50°) + \frac{1}{2} + 1$

$= (1 + 1 + \ldots \text{ 8 terms}) + \frac{1}{2} + 1$

$= 8 + \frac{1}{2} + 1 = 9\frac{1}{2}.$

4. $\tan 3x = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$

$\Rightarrow \tan 3x = \tan 45° \Rightarrow x = \frac{45°}{3} = 15°.$

5. $\text{cosec } A = \sqrt{2} \Rightarrow \sin A = \frac{1}{\sqrt{2}}$

$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$

$\tan A = 1, \cot A = 1$

Now, $\frac{2 \sin^2 A + 3 \cot^2 A}{4 \left(\tan^2 A - \cos^2 A\right)} = \frac{2 \times \frac{1}{2} + 3 \times 1}{4\left(1 - \frac{1}{2}\right)}$

$= \frac{4}{2} = 2.$

6. True

Hint:

$$a\cos\theta + b\sin\theta = 4 \quad ...(i) \times \sin\theta$$

$$a\sin\theta - b\cos\theta = 3 \quad ...(ii) \times \cos\theta$$

$$\Rightarrow \quad a\cos\theta\sin\theta + b\sin^2\theta = 4\sin\theta$$

$$\underset{-}{} \quad a\sin\theta\cos\theta \underset{+}{-} b\cos^2\theta = \underset{-}{3}\cos\theta$$

$$b = 4\sin\theta - 3\cos\theta$$

Similarly, $a = 4\cos\theta + 3\sin\theta$

$$\therefore \ a^2+b^2 = 16\sin^2\theta + 9\cos^2\theta - 12\sin\theta\cos\theta + 16\cos^2\theta + 9\sin^2\theta + 12\sin\theta\cos\theta$$

$$= 16 + 9 = 25.$$

7. $(a^2 - b^2)\sin\theta + 2ab\cdot\cos\theta = a^2 + b^2$

Divide by $\cos\theta$

$$(a^2 - b^2)\tan\theta + 2ab = \frac{a^2+b^2}{\cos\theta}$$

$$\Rightarrow (a^2 - b^2)\tan\theta + 2ab = (a^2 + b^2)\cdot\sec\theta$$

$$= (a^2 + b^2)\cdot\sqrt{1+\tan^2\theta}$$

Squaring both sides:

$$(a^2 - b^2)^2\tan^2\theta + 4a^2b^2 + 4ab(a^2 - b^2)\tan\theta$$

$$= (a^2+b^2)^2(1 + \tan^2\theta)$$

$$= (a^2+b^2)^2 + (a^2+b^2)^2\tan^2\theta$$

$$[(a^2 - b^2)^2 - (a^2 + b^2)^2]\tan^2\theta + 4a^2b^2 + 4ab(a^2 - b^2)\tan\theta - (a^2+b^2)^2 = 0$$

$$\Rightarrow -4a^2b^2\tan^2\theta + 4ab(a^2 - b^2)\tan\theta - a^4 - b^4 + 2a^2b^2 = 0$$

$$\Rightarrow -4a^2b^2\tan^2\theta + 4ab(a^2 - b^2)\tan\theta - (a^2 - b^2)^2 = 0$$

$$\Rightarrow 4a^2b^2\tan^2\theta - 4ab(a^2 - b^2)\tan\theta + (a^2 - b^2)^2 = 0$$

$$\Rightarrow [2ab\tan\theta - (a^2 - b^2)]^2 = 0$$

$$\Rightarrow 2ab\tan\theta = a^2 - b^2$$

$$\Rightarrow \tan\theta = \frac{a^2-b^2}{2ab}.$$

8. Hint: Use $(a^2+b^2)^3 = a^6+b^6+3a^2b^2(a^2+b^2)$.

9. LHS

$$= \frac{(1+\cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \text{cosec}^3 A}$$

$$= \frac{\left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)}{\frac{1}{\cos^3 A} - \frac{1}{\sin^3 A}}$$

$$= \frac{\frac{(\sin A\cos A + \cos^2 A + \sin^2 A)(\sin A - \cos A)}{\sin A\cos A}}{\frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A\cos A)}{\sin^3 A\cos^3 A}}$$

$= \sin^2 A\cdot\cos^2 A = \text{RHS}.$ **Hence proved.**

10. $m = \text{cosec}\,\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta$

$$= \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

$$n = \sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

Now, LHS $= (m^2n)^{\frac{2}{3}} + (mn^2)^{\frac{2}{3}}$

$$= \left(\frac{\cos^4\theta}{\sin^2\theta}\times\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} + \left(\frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^4\theta}{\cos^2\theta}\right)^{\frac{2}{3}}$$

$$= (\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}}$$

$$= \cos^2\theta + \sin^2\theta = 1 = \text{RHS}.$$

OR

LHS

$$= (1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A)$$

$$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$= \frac{\sin A + \cos A - 1}{\sin A} \times \frac{\cos A + \sin A + 1}{\cos A}$$

$$= \frac{(\sin A + \cos A)^2 - 1^2}{\sin A\cos A}$$

$$= \frac{\sin^2 A + 2\sin A\cos A + \cos^2 A - 1}{\sin A\cos A}$$

$$= \frac{2\sin A\cos A}{\sin A\cos A} = 2$$

= RHS. **Hence proved.**

WORKSHEET – 63

1. Given expression

$= \sin 25° \cos (90° - 25°) + \cos 25° \sin (90° - 25°)$

$= \sin^2 25° + \cos^2 25° = 1.$

2. $$\frac{2\sin\theta - \cos\theta}{2\sin\theta + \cos\theta} = \frac{\frac{2\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\frac{2\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}$$

$$= \frac{2\tan\theta - 1}{2\tan\theta + 1} = \frac{2\times\frac{4}{3} - 1}{2\times\frac{4}{3} + 1} = \frac{5}{11}.$$

3. $$\frac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2(1+\sqrt{3})}{\sqrt{3}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}\times(\sqrt{3}-1)}{2\sqrt{2}(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-\sqrt{3}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3\sqrt{2} - \sqrt{6}}{8}.$$

4. False, because $\cos^2 23° - \sin^2 67° = 0$, 0 is not a positive value.

5. LHS $= \dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A}$

$$= \frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A}$$

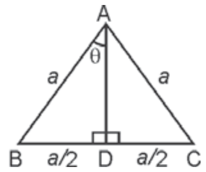
$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1 + \sin A)\cos A}$$

$$= \frac{2 + 2\sin A}{(1 + \sin A)\cos A} = \frac{2(1 + \sin A)}{(1 + \sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2\sec A$$

= RHS. **Hence proved.**

6. Let us construct a triangle ABC in which AB = BC = AC = a (say). Draw AD ⊥ BC.



AD bisects BC

$\Rightarrow \quad BD = DC = \dfrac{a}{2}$

AD bisects ∠BAC

$\Rightarrow \quad \theta = 30°$

In right-angled ΔABD.

$$AD^2 = AB^2 - BD^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$= a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$\Rightarrow \quad AD = \dfrac{\sqrt{3}}{2} a$

Now, in Δ ABD,

$$\tan\theta = \frac{BD}{AD} \Rightarrow \tan 30° = \frac{\frac{a}{2}}{\sqrt{3}\frac{a}{2}}$$

$\Rightarrow \quad \tan 30° = \dfrac{1}{\sqrt{3}}.$

7. $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$ (Given)

Divide both sides by $\cos\theta$ to get

$(a^2 - b^2)\tan\theta + 2ab = (a^2 + b^2)\sec\theta$

Squaring both sides, we get

$(a^2 - b^2)^2\tan^2\theta + 4a^2b^2 + 4ab(a^2 - b^2)\tan\theta = (a^2 + b^2)^2\sec^2\theta$

$\Rightarrow (a^2 - b^2)^2\tan^2\theta - (a^2 + b^2)^2\tan^2\theta + 4ab(a^2 - b^2)\tan\theta - (a^2 + b^2)^2 + 4a^2b^2 = 0$

$(\because \sec^2\theta = 1 + \tan^2\theta)$

$\Rightarrow -4a^2b^2\tan^2\theta + 4ab(a^2 - b^2)\tan\theta - (a^2 - b^2)^2 = 0$

$\Rightarrow -4a^2b^2x^2 + 4ab(a^2 - b^2)x - (a^2 - b^2)^2 = 0$

where $x = \tan\theta$

This is a quadratic equation in x.

Here, discriminant,

$$D = \sqrt{16a^2b^2(a^2 - b^2)^2 - 4\times 4a^2b^2(a^2 - b^2)^2}$$

$= 0$

$$\therefore x = \frac{-4ab(a^2 - b^2) - \sqrt{0}}{2\times(-4a^2b^2)} = \frac{a^2 - b^2}{2ab}$$

$\Rightarrow \tan\theta = \dfrac{a^2 - b^2}{2ab}.$ **Hence proved.**

8. Since ABC is a acute angled triangle so, $\angle A < 90°$, $\angle B < 90°$ and $\angle C < 90°$.

Also $\angle A + \angle B + \angle C = 180°$ ...(i)

$\sin(A + B - C) = \frac{1}{2}$ (Given)

$\Rightarrow \sin(A + B - C) = \sin 30°$

$\Rightarrow \angle A + \angle B - \angle C = 30°$ ...(ii)

Similarly, $\angle B + \angle C - \angle A = 45°$ ...(iii)

Add equations (ii) and (iii) to get

$$2\angle B = 75° \Rightarrow \angle B = 37\frac{1}{2}^{\circ}$$

Subtract equation (ii) from equation (i) to get

$$2\angle C = 150° \Rightarrow \angle C = 75°$$

Subtract equation (iii) from equation (i) to get

$$2\angle A = 135° \Rightarrow \angle A = 67\frac{1}{2}^{\circ}$$

Thus, $\angle A = 67\frac{1}{2}^{\circ}$, $\angle B = 37\frac{1}{2}^{\circ}$ and $\angle C = 75°$.

CHAPTER TEST

1. $x = \frac{\sec\theta}{2}$ and $\frac{1}{x} = \frac{\tan\theta}{2}$

$$\therefore \quad 2\left(x^2 - \frac{1}{x^2}\right) = 2\left(\frac{\sec^2\theta}{4} - \frac{\tan^2\theta}{4}\right)$$

$$= 2\left(\frac{\sec^2\theta - \tan^2\theta}{4}\right) = \frac{1}{2}.$$

2. $$\frac{\cos^2 20° + \cos^2 70°}{2(\sin^2 59° + \sin^2 31°)} = \frac{2}{k}$$

$$\Rightarrow \frac{\sin^2 70° + \cos^2 70°}{2(\sin^2 59° + \cos^2 59°)} = \frac{2}{k}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{k} \Rightarrow k = 4.$$

3. $\sin^4\theta + \cos^4\theta = 1 + 4k\sin^2\theta\cos^2\theta$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 + 4k\sin^2\theta\cos^2\theta$

$\Rightarrow 2\sin^2\theta\cos^2\theta(-1 - 2k) = 0$

$\Rightarrow -1 - 2k = 0 \Rightarrow k = -\frac{1}{2}$.

4. $$\tan\theta = 4$$

$$\Rightarrow \tan^2\theta + 1 = 4^2 + 1$$

$$\Rightarrow \sec^2\theta = 17$$

$$\therefore \frac{1}{10}(\tan^2\theta + 2\sec^2\theta) = \frac{1}{10}(16 + 2 \times 17) = 5.$$

5. **False.**

Suppose $A = 30°$ and $B = 60°$

Then, LHS $= \tan(A + B) = \tan(30° + 60°) = \tan 90°$

$\Rightarrow$ LHS = undefined ...(i)

and RHS $= \tan A + \tan B = \tan 30° + \tan 60°$

$$= \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{1+3}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$\Rightarrow$ RHS = a real number ...(ii)

From results (i) and (ii), it is clear that the given identity is false.

6. $\frac{-1}{7}$

Hint: $\cos 55° = \cos(90° - 35°) = \sin 35°$

$\cos 70° = \sin 20°$

and $\tan 5° = \cot 85°$.

7. $\frac{13}{4}$.

Hint: $\sin 30° = \frac{1}{2} = \cos 60°$, $\sin 60° = \frac{\sqrt{3}}{2}$,

$\cos 45° = \frac{1}{\sqrt{2}} = \sin 45°$, $\sin 90° = 1$.

8. $\sin\theta + \cos\theta = a$

Squaring both sides.

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = a^2$

$\Rightarrow 2\sin\theta\cos\theta = a^2 - 1$

$$\Rightarrow \sin\theta\cos\theta = \frac{a^2 - 1}{2} \quad ...(i)$$

Now, $\sin^6\theta + \cos^6\theta = (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$$= 1^3 - 3\left(\frac{a^2-1}{2}\right)^2(1)$$

[Using equation (i)]

$$= 1 - \frac{3}{4}\left(a^2 - 1\right)^2 = \frac{4 - 3(a^2 - 1)^2}{4}.$$

Hence proved.

9. LHS $= \dfrac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1}$$

$$= \frac{\left(\sec^2\theta - 1\right) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \left(\tan^2\theta + 1\right) + 2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)}$$

$$= \frac{\tan\theta}{\sec\theta} = \tan\theta\cos\theta$$

$$= \frac{\sin\theta}{\cos\theta}.\cos\theta = \sin\theta = \text{RHS}.$$

Hence proved.

OR

$$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$

$$= \frac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A - \cos B)(\cos A + \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)}$$

$= 0$ which is an integer.

❑❑

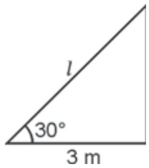
Chapter 7

SOME APPLICATIONS OF TRIGONOMETRY

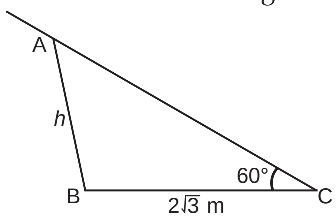
WORKSHEET – 65

1. $\cos 30^\circ = \dfrac{3}{l}$

$\Rightarrow \quad l = 2\sqrt{3}$ m.



2. From figure, let h = AB = height of pole; BC = $2\sqrt{3}$ m = shadow length



∴ In right-angled ΔABC;

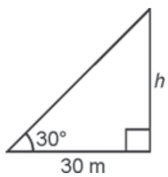
$$\tan 60^\circ = \frac{AB}{BC} = \frac{h}{2\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{2\sqrt{3}}$$

$$\Rightarrow \quad h = 2\sqrt{3} \times \sqrt{3} = 6 \text{ m}.$$

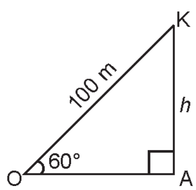
3. Let the height of the tower be h.

$$\tan 30^\circ = \frac{\text{Perpendicular}}{\text{Base}}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30} \Rightarrow h = 10\sqrt{3} \text{ m}.$$

4. Let OA be the horizontal ground and K be the position of the kite at a height h m above the ground, then AK = h m. It is given that OK = 100 m, ∠AOK = 60°.



In ΔAOK, right angled at A, we have

$$\sin 60^\circ = \frac{h}{100} \Rightarrow h = 100 \sin 60^\circ$$

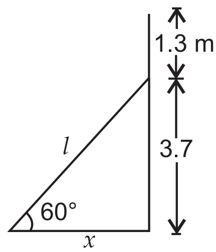
$$\Rightarrow \quad h = 100 \times \frac{\sqrt{3}}{2} = 50\sqrt{3} = 50 \times 1.732$$

$\therefore \quad h = 86.60$ m.

5. 4.28 m, 2.14 m

Hint: $\sin 60^\circ = \dfrac{3.7}{l}$

$\tan 60^\circ = \dfrac{3.7}{x}$.



6. Height = 94.64 m, Distance = 109.3 m

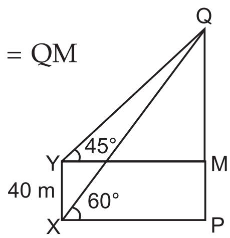
Hint:

$\tan 45^\circ = \dfrac{QM}{YM} \Rightarrow YM = QM$

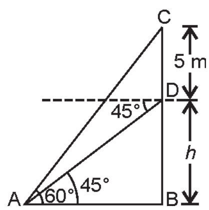
But XP = YM

∴ XP = QM

$\tan 60^\circ = \dfrac{40 + QM}{QM}$.



7. Let BD be the tower of height h m and CD be the pole. In right-angled triangle ABD,



$$\tan 45^\circ = \frac{BD}{AB}$$

$$\Rightarrow \quad 1 = \frac{h}{AB} \Rightarrow AB = h$$

In right-angled triangle ABC,

$$\tan 60^\circ = \frac{BC}{AB} \Rightarrow \sqrt{3} = \frac{BD + CD}{AB}$$

$$\Rightarrow \quad \frac{h+5}{h} = \sqrt{3}$$

$$\Rightarrow \quad h = \frac{5}{\sqrt{3} - 1} \Rightarrow h = \frac{5}{1.732 - 1}$$

$$\Rightarrow \quad h = 6.83 \text{ m}.$$

8. Let AB = height of building

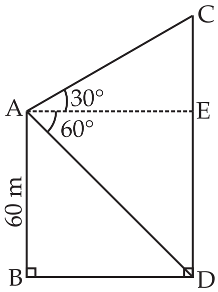
and CD = height of tower

∴ To find: (*i*) Difference between heights

= CD – DE [∵ AB = DE]

(*ii*) BD = Distance between bottoms

In right-angled ΔABD,
∠ADB = ∠EAD = 60°



$\therefore \quad \tan 60° = \frac{AB}{BD}$

$\Rightarrow \quad \sqrt{3} = \frac{60}{BD}$

$\Rightarrow \quad BD = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$ m

$\therefore \quad BD = 20\sqrt{3}$ m

Also as ABDE is a rectangle

∴ AB = DE = 60 m and BD = AE = $20\sqrt{3}$ m

∴ In right-angled ΔAEC,

$\tan 30° = \frac{CE}{AE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CE}{20\sqrt{3}}$

$\Rightarrow \quad CE = \frac{20\sqrt{3}}{\sqrt{3}} = 20$ m

∴ Difference between heights = CE = 20 m

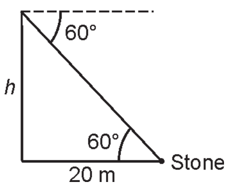
WORKSHEET – 66

1. ∠ACB = ∠XAC = 45°

$\sin(\angle ACB) = \frac{20}{x}$ and $\tan(\angle ACB) = \frac{20}{y}$

$\Rightarrow \quad x = 20\sqrt{2}$ m and $y = 20$ m.

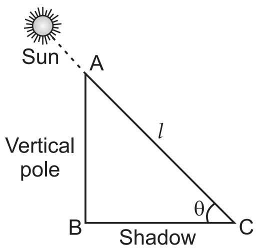
2. $\tan 60° = \frac{h}{20}$.



$\Rightarrow \quad \sqrt{3} = \frac{h}{20}$

$\Rightarrow \quad h = 20\sqrt{3}$ m.

3. Let the length of shadow of pole AB be BC = x, then AB = x.

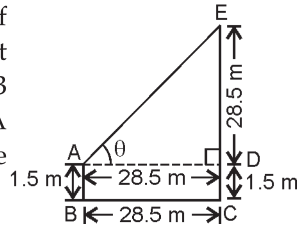


Also let θ be the angle of elevation of Sun's altitude. In right-angled triangle ABC,

$\tan\theta = \frac{x}{x} \Rightarrow \theta = 45°$

Hence, the angle of elevation of the Sun's altitude is 45°.

4. Let the angle of elevation be θ. Let the observer be AB with his eye at A and the tower be EC.



∴ CD = AB = 1.5 m

ED = 30 – 1.5 = 28.5 m

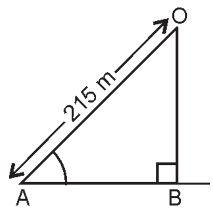
And AD = BC = 28.5 m

In right-angled ΔADE,

$\tan\theta = \frac{DE}{AD} = \frac{28.5}{28.5} = 1 \quad \Rightarrow \quad \theta = 45°.$

5. Let the balloon be at the point O, the thread be OA and the required height be OB.

Case I: The cable is inclined at 60°.



$\Rightarrow \quad \sin 60° = \frac{OB}{OA}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{OB}{215}$

$\Rightarrow \quad OB = \frac{215\sqrt{3}}{2}$

$= \frac{215 \times 1.732}{2} = 186.19$ m.

Case II: The cable is inclined at 60° – 15° = 45°

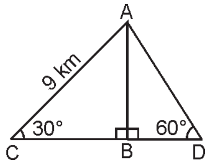
$\sin 45° = \frac{OB}{OA} \Rightarrow \frac{1}{\sqrt{2}} = \frac{OB}{215}$

$\Rightarrow \quad OB = \frac{215}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{215\sqrt{2}}{2}$

$= \frac{215 \times 1.414}{2} = 152$ m (approx.)

So, reduced height = 186.19 m – 152 m = 34.19 m.

6. Let AB is a hill and C and D be two city centres subject to the angles of elevation of the top A of hill AB at C and D are 30° and 60° respectively, then ∠ACB = 30°, ∠ADB = 60°, AC = 9 km.



In right-angled ΔABC,

$$\sin 30° = \frac{AB}{9}$$

$$\Rightarrow \quad AB = 9 \times \sin 30° = 9 \times \frac{1}{2} = 4.5$$

In right-angled ΔABD, we have

$$\sin 60° = \frac{AB}{AD}$$

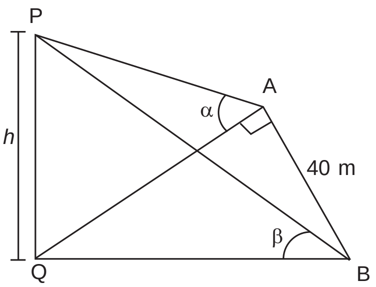
$$\Rightarrow \quad AD = AB \text{ cosec } 60°$$

$$\Rightarrow \quad AD = 4.5 \times \frac{2}{\sqrt{3}} = \frac{9 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= 3\sqrt{3} = 3 \times 1.732$$

$$= 5.196 \approx 5.20 \text{ km.}$$

7. Let $\angle PAQ = \alpha$ and $\angle PBQ = \beta$



$\therefore$ It is given that $\cot \alpha = \frac{3}{10}$ and $\cot \beta = \frac{1}{2}$.

Clearly, since Q, A and B are in same plane

$$\therefore \quad \angle PQA = \angle PQB = 90°.$$

and it is given that

$$\angle QAB = 90°.$$

$\therefore$ In right-angled ΔQAB, let $QA = x$.

$$QB^2 = QA^2 + AB^2$$

$$\Rightarrow \quad QB^2 = x^2 + 40^2$$

$$\Rightarrow \quad QB = \sqrt{x^2 + 1600}$$

Now, in ΔPQA;

$$\cot \alpha = \frac{AQ}{PQ}$$

$$\Rightarrow \quad \frac{x}{h} = \frac{3}{10}$$

$$\Rightarrow \quad x = \frac{3h}{10} \quad ...(i)$$

Also, in ΔPQB; $\frac{QB}{PQ} = \cot \beta = \frac{1}{2}$

$$\Rightarrow \quad \frac{\sqrt{x^2 + 1600}}{h} = \frac{1}{2} \quad \{\because \text{ Using } (i)\}$$

$$\Rightarrow \quad x^2 + 1600 = \frac{h^2}{4} \quad ...(ii)$$

Using (*i*) in (*ii*),

$$\left(\frac{3h}{10}\right)^2 + 1600 = \frac{h^2}{4}$$

$$\Rightarrow \quad \frac{9h^2}{100} + 1600 = \frac{h^2}{4}$$

$$\Rightarrow \quad \frac{h^2}{4} - \frac{9h^2}{100} = 1600$$

$$\Rightarrow \quad 16h^2 = 1600 \times 100$$

$$\Rightarrow \quad h = 100$$

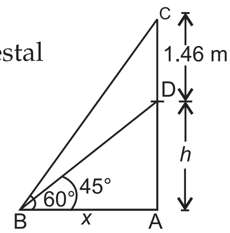
$\therefore$ Height of tower is 100 m.

8. 2 m

Hint: h = height of pedestal

$$\tan 45° = \frac{h}{x} \Rightarrow x = h$$

$$\tan 60° = \frac{h + 1.46}{h}.$$

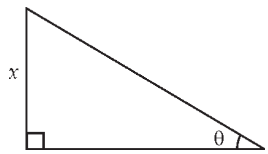


WORKSHEET – 67

1. $y = \sqrt{3}x$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{y} = \tan \theta$$

$$\Rightarrow \quad \theta = 30°$$

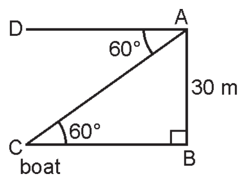


y = length of shadow

2. In ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

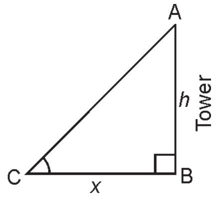
$$\Rightarrow \quad \sqrt{3} = \frac{30}{BC}$$



$$BC = \frac{30}{\sqrt{3}}, \quad \therefore BC = 10\sqrt{3} \text{ m.}$$

3. True.

$$\tan C = \frac{AB}{BC} = \frac{h}{x}$$

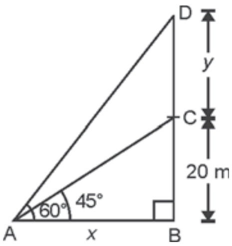


If $\quad AB = \frac{11h}{10}$

and $\quad BC = \frac{11x}{10}$

Then, $\tan C = \frac{AB}{BC} = \frac{h}{x}$.

4. Let the height of the tower CD be y metres and the horizontal distance of point A from the building BC is AB = x metres.



In right-angled triangle ABC,

$$\tan 45° = \frac{20}{x} \Rightarrow x = 20 \text{ m}$$

Also, in right-angled triangle ABD,

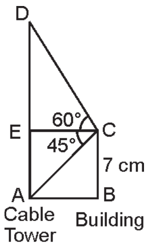
$$\tan 60° = \frac{20+y}{x}$$

$$\Rightarrow \quad 20\sqrt{3} = 20 + y$$

$$\Rightarrow \quad y = 20\left(\sqrt{3}-1\right) \text{m}$$

Thus, the height of the tower is $20\left(\sqrt{3}-1\right)$ m.

5. $7\left(\sqrt{3}+1\right)$ m

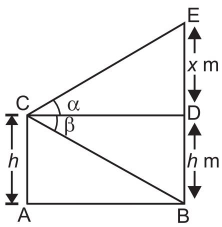


Hint: $\tan 45° = \frac{AE}{EC}$

$\Rightarrow \quad EC = 7\text{m}$

$\tan 60° = \frac{DE}{EC}$.

6. Let C be the position of a window of house AC which is h metres above the ground, *i.e.*, AC = h m. BE be the house on the opposite side of the street. The angle of elevation and depression of the top and foot of the opposite house from the window C be α and β, respectively.



Then according to question, we have

$\angle DCE = \alpha$

and $\angle BCD = \beta$

Let $\quad DE = x$ m

In right triangle CDE, we have

$$\tan \alpha = \frac{DE}{CD}$$

$$\Rightarrow \quad \tan \alpha = \frac{x}{CD} \Rightarrow CD = \frac{x}{\tan \alpha}$$

$$\Rightarrow \quad CD = x \cot \alpha \qquad ...(i)$$

In right triangle BCD, we have

$$\frac{BD}{CD} = \tan \beta$$

$$\Rightarrow \quad CD = \frac{BD}{\tan \beta} \Rightarrow CD = \frac{h}{\tan \beta}$$

$$\Rightarrow \quad CD = h \cot \beta \qquad ...(ii)$$

Comparing (*i*) and (*ii*), we get

$$x \cot \alpha = h \cot \beta$$

$$\Rightarrow \quad x = \frac{h \cot \beta}{\cot \alpha}$$

$$\Rightarrow \quad x = h \cot \beta . \tan \alpha$$

Hence, height of the opposite house (BE)

$= BD + DE$

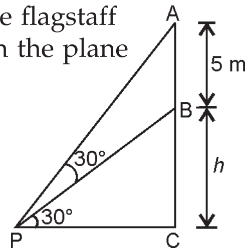
$= h + x$

$= h + h \cot \beta \,.\, \tan \alpha$

$= h\,(1 + \cot \beta \,.\, \tan \alpha)$

Hence proved.

7. Let the tower be BC the flagstaff be AB and the point on the plane be P.



Let BC = h

In right-angled ΔBCP,

$$\tan 30° = \frac{h}{PC}$$

$$\Rightarrow \quad PC = h \cot 30° \qquad ...(i)$$

In right-angled ΔACP,

$$\tan 60° = \frac{5+h}{PC}$$

$$\Rightarrow \quad PC = (5 + h) \cot 60° \qquad ...(ii)$$

Comparing equations (*i*) and (*ii*), we have

$$h \cot 30° = (5 + h) \cot 60°$$

$$\Rightarrow \quad h\sqrt{3} = (5+h)\frac{1}{\sqrt{3}}$$

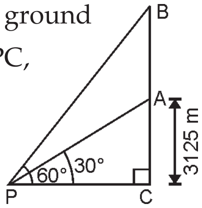
$$\Rightarrow \quad 3h = 5 + h$$

$\Rightarrow \quad h = 2.5$

Hence, the height of the tower is 2.5 m.

OR

Let the two planes be at A and B respectively. Also P be the point on the ground



In right-angled triangle APC,

$$\tan 30° = \frac{3125}{PC}$$

$$\Rightarrow \quad PC = 3125\sqrt{3} \text{ m}$$

Also in right-angled triangle BPC,

$$\tan 60° = \frac{BC}{PC}$$

$$\Rightarrow \quad BC = 3125\sqrt{3} \times \sqrt{3} = 3 \times 3125$$

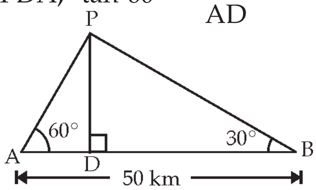
$$\therefore \quad AB = BC - AC = 3 \times 3125 - 3125$$
$$= 2 \times 3125 = 6250 \text{ m}.$$

Hence, distance between the two planes is 6250 m.

8. Let $PD \perp AB$

Let $AD = x \;\; \therefore \; DB = 50 - x$

(*i*) In right-angled ΔPDA; $\tan 60° = \frac{PD}{AD}$



$$\Rightarrow \sqrt{3} = \frac{PD}{x}$$

$$\Rightarrow PD = x\sqrt{3}.$$

Also in right-angled ΔPDB; $\tan 30° = \frac{PD}{DB}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{50-x} \Rightarrow 50 - x = 3x$$

$$\Rightarrow 50 = 4x \Rightarrow x = \frac{50}{4} = \frac{25}{2} = 12.5 \text{ m}$$

$\therefore$ In right-angled ΔADP; $\sin 60° = \frac{PD}{AP}$

$$\Rightarrow AP = \frac{PD}{\sin 60°} = \frac{x\sqrt{3}}{\frac{\sqrt{3}}{2}} = 12.5 \times 2 = 25 \text{ m}$$

$\therefore$ AP = 25 m

(*ii*) In right-angled ΔPDB; $\sin 30° = \frac{PD}{PB}$

$$\Rightarrow \frac{1}{2} = \frac{PD}{PB}$$

$$\Rightarrow PB = 2PD = 2 \times x\sqrt{3} = 2 \times 12.5 \times \sqrt{3}$$

$$PB = 25\sqrt{3} \text{ m}.$$

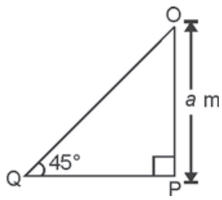
(*iii*) Clearly as PA < PB

$\Rightarrow$ Team of A should send its team.

(*iv*) Cooperation and responsibility.

WORKSHEET – 68

1. $\tan 45° = \frac{OP}{PQ} = \frac{a}{QP}$



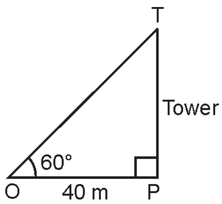
$$\Rightarrow \quad 1 = \frac{a}{QP}$$

$$\Rightarrow \quad QP = a \text{ m}$$

$$\therefore \; ar(\Delta OPQ) = \frac{1}{2} \times QP \times OP$$

$$= \frac{1}{2} \times a \times a = \frac{1}{2}a^2.$$

2. $\tan 60° = \frac{TP}{PO}$

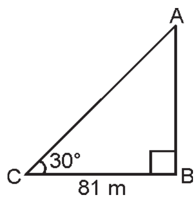


$$\Rightarrow \quad \sqrt{3} = \frac{TP}{40}$$

$$\Rightarrow \quad TP = 40\sqrt{3} \text{ m}.$$

3. True

$$\tan 30° = \frac{AB}{BC}$$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{81}$$

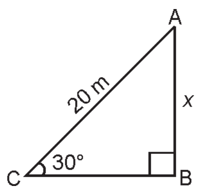
$$\Rightarrow \quad AB = \frac{81}{\sqrt{3}} = \frac{81\sqrt{3}}{3}$$

$$\Rightarrow \quad AB = \frac{81 \times 1.732}{3} = 46.76 \text{ m}.$$

4. Let the height of the pole AB = x m

Length of the rope AC = 20 m

In ΔABC,



$$\angle ACB = 30°$$

$$\therefore \quad \sin 30° = \frac{AB}{AC}$$

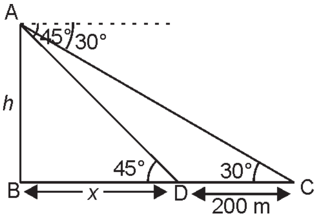
$$\Rightarrow \quad \frac{1}{2} = \frac{x}{20}$$

$$\Rightarrow \quad x = 10 \text{ m}$$

$\therefore$ Height of the pole = 10 m.

5. Let (AB = h) is the height of the light house. Point D and C are position of the ships from the root of the light house. Distance between D and C = 200 m, *i.e.*, (DC = 200). Again let BD = x

In right triangle ABD, we have



$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{h}{x}$$

$$x = h \qquad \ldots(i)$$

In right triangle ABC

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+200}$$

$$x + 200 = \sqrt{3}h \qquad \ldots(ii)$$

From (i) and (ii), we have

$$h + 200 = \sqrt{3}h$$

$$200 = \sqrt{3}h - h$$

$$200 = h\left(\sqrt{3}-1\right)$$

$$\frac{200}{\sqrt{3}-1} = h$$

$$\therefore \quad h = \frac{200}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{200(\sqrt{3}+1)}{3-1}$$

$$= \frac{200(\sqrt{3}+1)}{2} = 100 \times \sqrt{3} + 100$$

$$= 100 \times 1.732 + 100$$

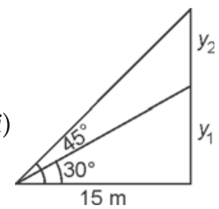
$$= \frac{100 \times 1732}{1000} + 100$$

$$= 173.2 + 100 = 273.2 \text{ m.}$$

6. 6.34 m

Hint: $\tan 45° = \dfrac{y_1 + y_2}{15}$

$$y_1 + y_2 = 15 \quad \ldots(i)$$



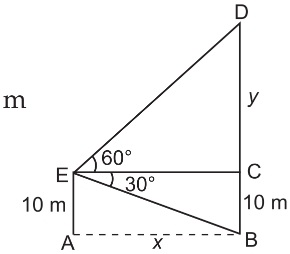
$$\tan 30° = \frac{y_1}{15}$$

$$\Rightarrow \quad y_1 = \frac{15}{\sqrt{3}} \qquad \ldots(ii)$$

$$\therefore \quad y_2 = 15 - y_1 = 6.34 \text{ m.}$$

7. Distance = 17.32 m, Height = 40 m

Hint: $\tan 30° = \dfrac{10}{x}$



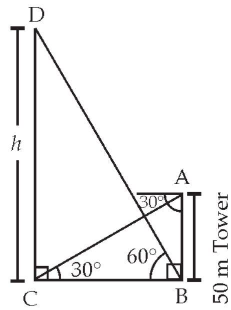
$$\Rightarrow \quad x = 10\sqrt{3} \text{ m}$$

$$\tan 60° = \frac{y}{10\sqrt{3}}$$

$$\Rightarrow \quad y = 30 \text{ m.}$$

8. Let height of hill = h

In right ΔABC, $\tan 30° = \dfrac{AB}{BC}$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{BC}$$

$$\Rightarrow \quad BC = 50\sqrt{3} \text{ m}$$

In right ΔDCB;

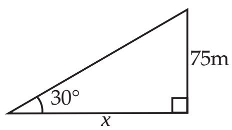
$$\tan 60° = \frac{DC}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{50\sqrt{3}} \Rightarrow h = 50 \times 3 = 150 \text{ m.}$$

WORKSHEET – 69

1. Let distance = x

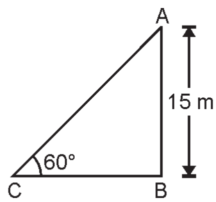
$$\tan 30° = \frac{75}{x}$$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{x}$$

$$\Rightarrow \quad x = 75\sqrt{3} \text{ m.}$$

2. $\tan C = \dfrac{AB}{BC}$



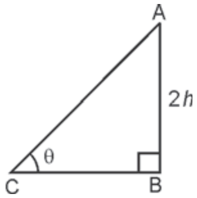
$$\Rightarrow \quad \tan 60° = \frac{15}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{15}{BC}$$

$$\Rightarrow \quad BC = \frac{15}{\sqrt{3}} \quad \Rightarrow \quad BC = 5\sqrt{3} \text{ m.}$$

3. False.

Let height of the tower is h metres so the angle of elevation is 30°.



$$\tan 30° = \frac{h}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BC}$$

$$\Rightarrow \quad BC = h\sqrt{3} \quad \ldots (i)$$

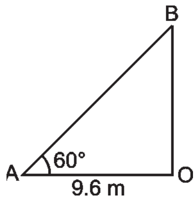
When height = $2h$,

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{2h}{h\sqrt{3}} \quad \text{[From } (i)\text{]}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{\sqrt{3}} \neq \tan 60°.$$

4. Let AB be the ladder leaning against a wall OB such that ∠OAB = 60° and OA = 9.6 m. In ΔOAB right angled at O, we have

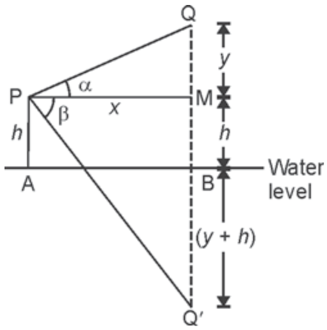


$$\cos 60° = \frac{OA}{AB}$$

$$\Rightarrow \quad AB = \frac{OA}{\cos 60°}$$

$$\Rightarrow \quad AB = \frac{9.6}{0.5} = 19.2 \text{ m.}$$

5. Let the point, cloud and reflection of the cloud be at P, Q and Q′ respectively.



Let PM = x, QM = y

We have to find QB, *i.e.*, $y + h$

In right-angled triangle QPM,

$$\tan\alpha = \frac{y}{x} \Rightarrow x = \frac{y}{\tan\alpha} \quad \ldots (i)$$

Also in right-angled triangle Q′PM,

$$\tan\beta = \frac{y + 2h}{x}$$

$$\Rightarrow \quad \frac{y\tan\beta}{\tan\alpha} = y + 2h$$

[From equation (i)]

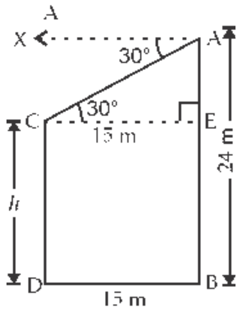
$$\Rightarrow \quad y\left(\frac{\tan\beta}{\tan\alpha} - 1\right) = 2h$$

$$\Rightarrow \quad y + h = h\left[1 + \frac{2\tan\alpha}{\tan\beta - \tan\alpha}\right]$$

$$= \frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}.$$

Hence proved.

6.



Let AB = 24 m; CD = h; CE = DB = 15 m

∴ In right ΔAEC:

$$\tan 30° = \frac{AE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{15}$$

$$\Rightarrow \quad AE = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3} \text{ m.}$$

∴ Height of first pole = CD = h = AB – AE

$$= 24 - 5\sqrt{3}$$
$$= 24 - 5 \times 1.732$$
$$= 24 - 8.660 = 15.340 \text{ m.}$$

7. Let CD = 60 m = height of building

AB = h = height of light house.

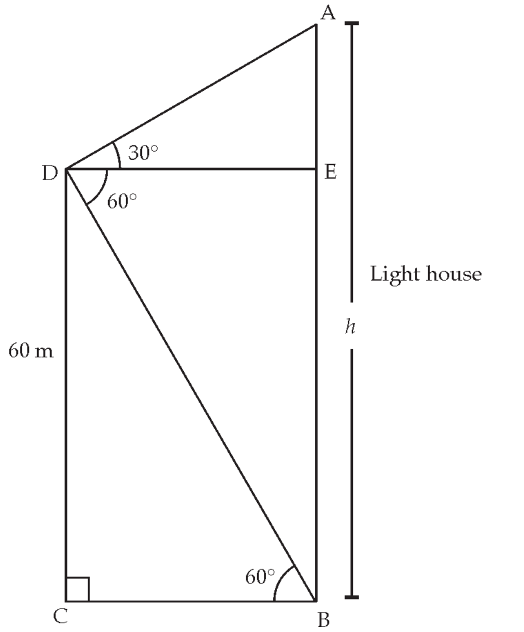
∴ AE = difference between height

and BC = distance between building and light house

∴ In right ΔDCB, $\tan 60° = \frac{DC}{BC}$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{BC} \Rightarrow BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$= 20\sqrt{3}$ m.



Also In right ΔAED,

$$\tan 30° = \frac{AE}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{BC} \qquad [\because DE = BC]$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}} \quad \Rightarrow AE = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m.}$$

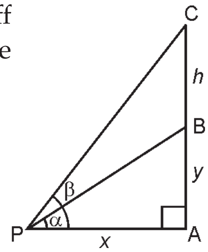
∴ (*i*) difference between heights = 20 m

(*ii*) distance between building = $20\sqrt{3}$ m.

8. Let the tower, the flagstaff and the point on the plane be AB, BC and P respectively.

Let AB = y and AP = x

In ΔABP,



$$\tan \alpha = \frac{y}{x}$$

$$\Rightarrow \frac{1}{x} = \frac{\tan\alpha}{y} \qquad ...(i)$$

In ΔACP, $\tan \beta = \dfrac{h+y}{x}$

$$\Rightarrow \frac{1}{x} = \frac{\tan\beta}{h+y} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{\tan\alpha}{y} = \frac{\tan\beta}{h+y}$$

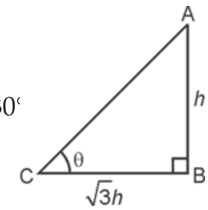
$\Rightarrow y \tan \beta = h \tan \alpha + y \tan \alpha$

$\Rightarrow y (\tan \beta - \tan \alpha) = h \tan \alpha$

$$\Rightarrow \quad y = \frac{h \tan\alpha}{\tan\beta - \tan\alpha}.$$ **Hence proved.**

WORKSHEET – 70

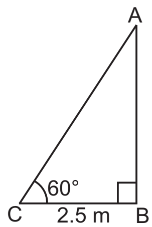
1. $\tan C = \dfrac{AB}{BC} = \dfrac{h}{\sqrt{3}h}$



$\Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow \quad \theta = 30°.$

2. Let AC = l = length of ladder



and $\angle ACB = 60°$

BC = 2.5 m

∠ In right-angled ∠ΔABC;

$$\cos\theta = \frac{BC}{AC}$$

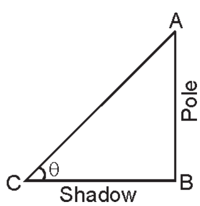
$$\Rightarrow \cos 60° = \frac{2.5}{l}$$

$$\Rightarrow \frac{1}{2} = \frac{2.5}{l}$$

$\Rightarrow \quad l = 5$ m.

3. True.

As $\tan\theta = \dfrac{AB}{BC}$



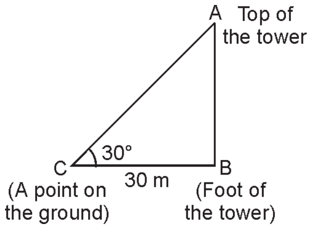
$\Rightarrow \quad \tan\theta = \dfrac{AB}{AB}$

($\because$ AB = BC)

$\Rightarrow \quad \tan\theta = 1 = \tan 45°$

$\therefore \quad \theta = 45°.$

4. Let AB be the tower and C be the point on the ground.



In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

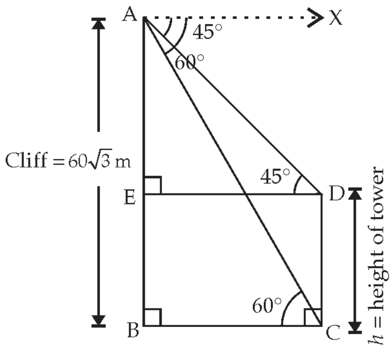
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$\Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m.}$$

Hence, the height of the tower is $10\sqrt{3}$ m.

5. Let AB = height of cliff = $60\sqrt{3}$ m

CD = h = height of tower



$\therefore$ In right ΔABC,

$$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{60\sqrt{3}}{BC}$$

$\Rightarrow$ BC = 60 m $\therefore$ ED = BC = 60 m

$\therefore$ In right ΔAED,

$$\tan 45° = \frac{AE}{ED}$$

$$1 = \frac{AE}{60} \Rightarrow AE = 60 \text{ m}$$

$\therefore$ h = DC = EB = AB – AE

$= 60\sqrt{3} - 60$

$= 60\left(\sqrt{3}-1\right)$ m.

6. Let the aeroplane's first situation be at A and second at B. Let the point of observation be at O.

From right-angled ΔAOD,

$$\tan 45° = \frac{AD}{OD} \Rightarrow OD = 3000 \text{ m}$$

Again from right-angled ΔBOC,

$$\tan 30° = \frac{3000}{3000 + DC}$$

$$\Rightarrow DC = 3000\left(\sqrt{3} - 1\right) \text{m}$$

Now speed of the plane

$$= \frac{\text{Distance}}{\text{Time}} = \frac{3000\left(\sqrt{3}-1\right)}{15}$$

$[\because DC = AB]$

$= 146.42$ m/sec

7. Let AB = 60 m = height of tower

CD = h = height of building

$$\therefore \text{ In right } \Delta ABD, \tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{60}{BD} \Rightarrow BD = \frac{60}{\sqrt{3}} \quad \ldots(i)$$

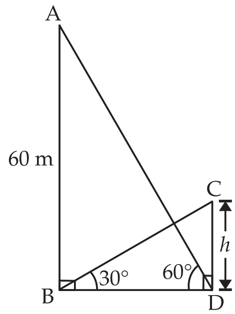
$$\text{Also in right } \Delta CDB; \tan 30° = \frac{CD}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BD}$$

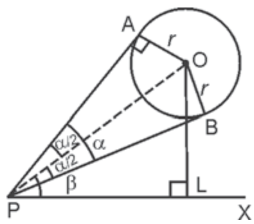
$$\Rightarrow h = \frac{BD}{\sqrt{3}}$$

$$= \frac{60}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{60}{3} = 20 \text{ m}$$



8. Let O be centre of the balloon of radius r and P the eye of the observer. Let PA, PB be tangents from P to the balloon. Then,



$$\angle APB = \alpha$$

$$\therefore \quad \angle APO = \angle BPO = \frac{\alpha}{2}$$

$\therefore$ OL $\perp$ PX, $\angle OPL = \beta$

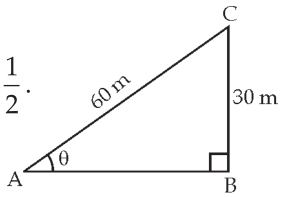
$$\therefore \quad \text{In } \Delta OAP, \sin\frac{\alpha}{2} = \frac{OA}{OP}$$

$$\Rightarrow \quad OP = r \operatorname{cosec}\frac{\alpha}{2}$$

In ΔOPL, $\sin\beta = \frac{OL}{OP}$

$\Rightarrow \quad OL = r \operatorname{cosec} \frac{\alpha}{2} \sin\beta.$

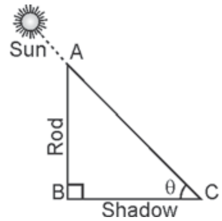
WORKSHEET – 71



1. $\sin\theta = \frac{30}{60} = \frac{1}{2}.$

$\Rightarrow \quad \theta = 30°.$

2. Given:



$AB : BC = 1 : \frac{1}{\sqrt{3}}$

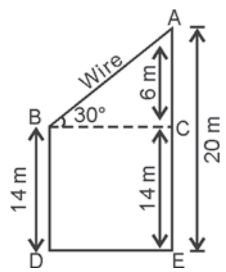
i.e., $AB : BC = \sqrt{3} : 1$

i.e., $\frac{AB}{BC} = \frac{\sqrt{3}}{1}$

$\therefore \quad \tan\theta = \frac{AB}{BC} = \sqrt{3} = \tan 60°$

$\Rightarrow \quad \theta = 60°.$

3. Wire is AB.



$CE = BD = 14$ m.

$AE = AC + CE$

$\Rightarrow \quad 20 = AC + 14$

$\Rightarrow \quad AC = 6$ m

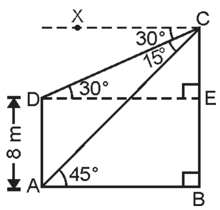
In ΔABC, $\angle C = 90°$,

$\sin 30° = \frac{6}{AB}$

$\Rightarrow \quad \frac{1}{2} = \frac{6}{AB} \Rightarrow AB = 12$ m.

4. False, because the tangent of the angle of elevation doubles not the angle of elevation.

5. BC is the multi-storeyed building with the foot B and the top C as the point of observation. AD is the building with bottom A and the top D. Draw DE || AB (see figure).



Given angles are $\angle XCD = 30°$ and $\angle XCA = 45°$. $\angle CDE$ and $\angle XCD$ are alternate interior angles.

$\therefore \quad \angle CDE = \angle XCD = 30°.$

Similarly, $\angle CAB = \angle XCA = 45°$

$BE = AD = 8$ m

In right triangle ABC,

$$\tan 45° = \frac{CE+BE}{AB} \Rightarrow 1 = \frac{CE+8}{AB}$$

$\Rightarrow \quad AB = CE + 8 \quad ...(i)$

Also, in right triangle DCE,

$$\tan 30° = \frac{CE}{DE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CE}{AB}$$

$(\because DE = AB)$

$\Rightarrow \quad AB = \sqrt{3}\, CE \quad ...(ii)$

From equations (*i*) and (*ii*), we get

$(\sqrt{3} - 1)\, CE = 8$

$$\Rightarrow \quad CE = \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{8 \times (1.73 + 1)}{3 - 1} = 10.92 \text{ m.}$$

Substituting $CE = 10.92$ in (*i*), we get

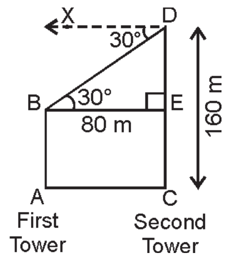
$AB = 10.92 + 8 = 18.92$ m

Further, $BC = BE + CE = 8 + 10.92$

$= 18.92$ m

Hence, the height of the multi-storyed building and the distance between the two buildings is 18.92 metres each.

6. Let AB be the first tower with bottom A and CD be the second tower with bottom C.



$BE = 80$ m

$CD = 160$ m

$AB = CE$

$\because$ XD || BE and BD is the transversal

$\therefore \angle DBE = \angle XDB = 30°$

In right triangle BDE,

$$\tan 30° = \frac{DE}{BE} = \frac{CD-CE}{BE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{160 - AB}{80}$$

$$\Rightarrow \quad 160 - AB = \frac{80}{\sqrt{3}}$$

$$\Rightarrow \quad AB = 160 - \frac{80}{\sqrt{3}} = 160 - \frac{80\sqrt{3}}{3}$$

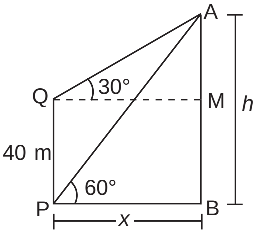
$$= \frac{480 - 80\sqrt{3}}{3}$$

$$= \frac{480 - 80 \times 1.732}{3} = 113.81$$

Hence, the height of the first tower is 113.81 metres.

7. Let h = AB = height of tower

x = PB = distance of P from B.



$\angle APB = 60°$; $\angle AQM = 30°$ are given as PBMQ is a rectangle

$$\Rightarrow \quad QP = MB = 40 \text{ m}$$

$$\therefore \quad AM = AB - MB = (h - 40) \text{ m}$$

$\therefore$ In right-angled ΔAMQ;

$$\tan 30° = \frac{AM}{QM}$$

$$\therefore \quad \frac{1}{\sqrt{3}} = \frac{h - 40}{x}.$$

$$\Rightarrow \quad x = \sqrt{3}(h - 40) \qquad \ldots(i)$$

Also in right-angled ΔABP

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

From equation (i),

$$\sqrt{3} = \frac{h}{\sqrt{3}(h - 40)}$$

$$\Rightarrow \quad 3(h - 40) = h$$

$$\Rightarrow \quad 3h - 120 = h$$

$$\Rightarrow \quad 2h = 120$$

$$\Rightarrow \quad h = 60 \text{ m}$$

$\therefore$ From equation (i),

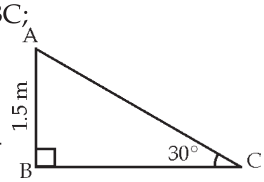
$$x = \sqrt{3}(60 - 40)$$

$$\Rightarrow \quad = 20\sqrt{3} \text{ m}.$$

8. (i) **Case I.** For children below the age of 5 years.

Length of slide = AC

$\therefore$ In right-angled ΔABC;



$$\frac{AB}{AC} = \sin 30°$$

$$\Rightarrow AC = \frac{AB}{\sin 30°} = \frac{1.5}{\frac{1}{2}}$$

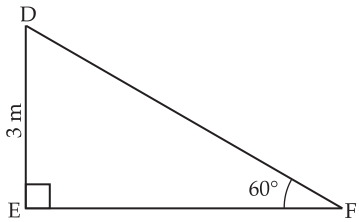
$$= 3 \text{ m}.$$

Case II. For older children:

Length of slide = DF

$\therefore$ In right-angled ΔDEF

$$\frac{DE}{DF} = \sin 60°$$

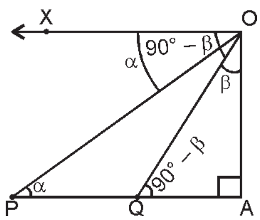


$$\Rightarrow DF = DE.\frac{2}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

(ii) Rationality.

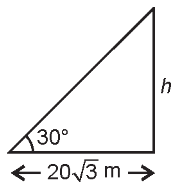
CHAPTER TEST

1. From the adjoining figure, angle of depression of P is $\angle XOP = \alpha$ and angle of depression of Q is $\angle XOQ = 90° - \beta$.



2. $$\tan 30° = \frac{h}{20\sqrt{3}}$$

$$\Rightarrow \quad h = 20 \text{ m}.$$



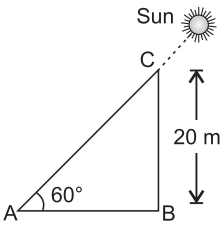
3. Let the tower be BC and the length of shadow be AB.

$$\tan 60° = \frac{BC}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{20}{AB}$$

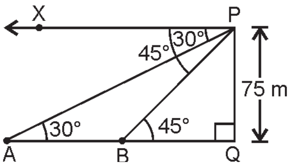
$$\Rightarrow \quad AB = \frac{20}{\sqrt{3}} \text{ m}$$

$$\Rightarrow \quad AB = \frac{20\sqrt{3}}{3} \text{ m.}$$



4. True, because the vertical tower, length of the shadow and the ray of the sun make a right angled isosceles triangle.

5. Let the ships be at A and B; and the tower be PQ.



$\angle PAQ = \angle XPA = 30°$

$\angle PBQ = \angle XPB = 45°$

In right ΔBPQ,

$\because \quad \angle PBQ = 45°, \qquad \therefore \quad \angle BPQ = 45°$

$\Rightarrow \quad BQ = PQ = 75 \qquad ...(i)$

In right ΔPAQ,

$$\tan 30° = \frac{PQ}{AB + BQ}$$

$\Rightarrow \quad AB + 75 = 75\sqrt{3} \qquad$ [Using (*i*)]

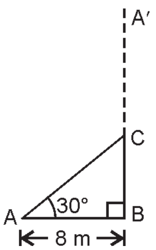
$\Rightarrow \quad AB = 75\left(\sqrt{3} - 1\right)$ m.

6. $8\sqrt{3}$ m

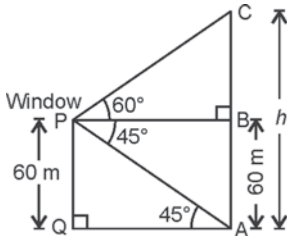
Hint: $A'C = AC$

$$\cos 30° = \frac{8}{AC}$$

$$\tan 30° = \frac{BC}{8}.$$



7. Let the window be at P and height of the opposite house be h.



In right ΔAPQ,

$$\tan 45° = \frac{60}{AQ}$$

$\Rightarrow \quad AQ = 60 \qquad \Rightarrow \quad BP = 60$

In right ΔBCP,

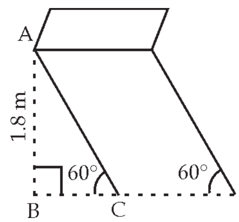
$$\tan 60° = \frac{h - 60}{60} \Rightarrow 60\sqrt{3} = h - 60$$

$$\Rightarrow \quad h = 60 + 60\sqrt{3} = 60$$

Thus, the required height is $60\left(1 + \sqrt{3}\right)$ m.

8. AC is the length of each leg of the stool.

In ΔABC, $\angle B = 60°$



$$\therefore \sin 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad AC = \frac{1.8}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{3.6}{\sqrt{3}} = \frac{3.6}{1.732}$$

$\Rightarrow \quad AC = 2.0785$ m.

(*ii*) Trigonometric ratios

(*iii*) Independent

Chapter 8

COORDINATE GEOMETRY

WORKSHEET – 73

1. PQ = QB

$\therefore \quad \frac{PQ}{QB} = 1 : 1$

$\therefore$ Q should be mid-point of PB

$\Rightarrow \quad y = \frac{-3-5}{2} = -4.$

2. Any point on *y*-axis be (0, *y*)

$\therefore \quad \sqrt{(6)^2 + (5-y)^2} = \sqrt{(0+4)^2 + (3-y)^2}$

$\Rightarrow \quad y = 9$

$\therefore$ Point is (0, 9).

3. Hint: Let the ratio is *k* : 1.

Now, use section formula.

4. Since diagonals of parallelogram bisect each other.

$\therefore$ Mid-point of AC = mid-point of BD

i.e., $\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$

$\Rightarrow \quad \frac{15}{2} = \frac{8+p}{2} \Rightarrow p = 7.$

OR

Given vertices are:

A(–3, 0), B(5, –2) and C(–8, 5). We know that

centroid G is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\therefore$ Centroid $= \left(\frac{-3+5-8}{3}, \frac{0-2+5}{3}\right) = (-2, 1).$

5. Since P is equidistant from A and B

$\therefore \quad AP = PB$

$\Rightarrow \quad AP^2 = PB^2$

$\Rightarrow (2-5)^2 + (4-k)^2 = (k-2)^2 + (7-4)^2$

$\Rightarrow \quad 9 + 16 + k^2 - 8k = k^2 + 4 - 4k + 9$

$4k = 12$

$\Rightarrow \quad k = 3.$

6. Let A(3, 0), B(6, 4) and C(–1, 3) are the vertices.

$\therefore$ Consider $AB = \sqrt{(6-3)^2 + (4-0)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$BC = \sqrt{(-1-6)^2 + (3-4)^2}$

$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

$AC = \sqrt{(-1-3)^2 + (3-0)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

Clearly AB = AC

$\Rightarrow$ Triangle is isosceles

also $AB^2 + AC^2 = 5^2 + 5^2 = 50$

and $BC^2 = \left(5\sqrt{2}\right)^2 = 50$

$\Rightarrow \quad AB^2 + AC^2 = BC^2$

$\therefore$ By converse of Pythagoras theorem $\angle A = 90°$.

$\Rightarrow$ ΔABC is right-angled isosceles triangle.

Hence proved.

7. Since A(*x*, *y*), B(3, 6) and C(–3, 4) are collinear

$\Rightarrow \quad ar\ \Delta ABC = 0$

$\Rightarrow x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow \quad x(6-4) + 3(4-y) + (-3)(y-6) = 0$

$\Rightarrow \quad 2x + 12 - 3y - 3y + 18 = 0$

$\Rightarrow \quad 2x - 6y + 30 = 0$

$x - 3y + 15 = 0.$

8. As the given points are collinear, the area of the triangle formed by these points must be zero.

Let (2, 1) ≡ (x_1, y_1); (*p*, –1) ≡ (x_2, y_2); and (–1, 3) ≡ (x_3, y_3)

Now, area of the triangle = 0

$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \quad \frac{1}{2}[2(-1-3) + p(3-1) - 1(1+1)] = 0$

$\Rightarrow \quad \frac{1}{2}(-8+2p-2) = 0$

$\Rightarrow \quad 2p = 10 \Rightarrow p = 5.$

9. Hint: ***Proceed as done in solved example 5.***

WORKSHEET – 74

1. Area $= \left|\frac{1}{2}\{4(-6+5) + 1(-5-5) - 4(5+6)\}\right|$

$= \left|\frac{1}{2}(-4-10-44)\right| = |-29|$

= 29 sq. units.

2. Condition of collinearity must be satisfied

$\therefore \quad -5(p+2) + 1(-2-1) + 4(1-p) = 0$

$\Rightarrow \quad -5p - 10 - 3 + 4 - 4p = 0$

$\Rightarrow \quad p = -1.$

3. 2 : 3

Hint: Use section formula.

4. True.

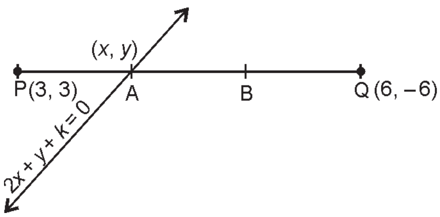
Hint: Any point P on x-axis will be of type P(x, 0)

$\therefore$ Let A(7, 6), B(–3, 4)

$\therefore$ Use PA = PB.

5. $k = -8$

Hint: Using section formula



Coordinates of A

$= \left(\frac{1 \times 6 + 2 \times 3}{3}, \frac{-6 \times 1 + 2 \times 3}{3}\right) = (4, 0)$

$\therefore$ As it lies on $2x + y + k = 0$

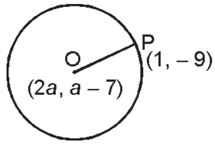
$\Rightarrow \quad 2 \times 4 + 0 + k = 0$

$\therefore \quad k = -8.$

6. Let O be the centre and P be the point on the circumference such that O ≡ (2a, a – 7) and P ≡ (1, – 9).

Radius = OP $= \frac{10\sqrt{2}}{2}$

$= 5\sqrt{2}$ units



i.e., $\sqrt{(2a-1)^2 + (a-7+9)^2} = 5\sqrt{2}$

Squaring both sides, we get

$(2a-1)^2 + (a+2)^2 = (5\sqrt{2})^2$

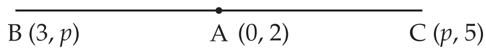
$\Rightarrow \quad 4a^2 - 4a + 1 + a^2 + 4a + 4 = 50$

$\Rightarrow \quad 5a^2 = 45 \Rightarrow a = \pm 3$

Thus, $a = \pm 3$.

7. As point A is equidistant from the points B and C.

$\therefore \quad AB = AC$



$\Rightarrow \sqrt{(3-0)^2 + (p-2)^2}$

$= \sqrt{(p-0)^2 + (5-2)^2}$

$\Rightarrow \quad 9 + (p-2)^2 = p^2 + 9$

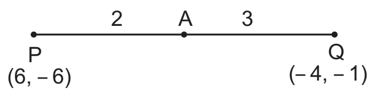
$\Rightarrow \quad (p-2)^2 = p^2$

$\Rightarrow \quad p^2 + 4 - 4p = p^2$

$\Rightarrow \quad 4p = 4 \Rightarrow p = 1$

$\therefore \quad AB = \sqrt{9+1} = \sqrt{10}$ units

8.



As $\frac{PA}{PQ} = \frac{2}{5} \Rightarrow \frac{PQ}{PA} = \frac{5}{2}$

$\Rightarrow \quad \frac{PQ}{PA} - 1 = \frac{5}{2} - 1$

$\Rightarrow \quad \frac{PQ - PA}{PA} = \frac{5-2}{2} \Rightarrow \frac{AQ}{PA} = \frac{3}{2}$

$\therefore \quad PA : AQ = 2 : 3$

Let A(x, y) then using section formula.

$x = \frac{2 \times (-4) + 3 \times 6}{2+3}$

$= \frac{-8+18}{5} = \frac{10}{5} = 2$

and $y = \dfrac{2\times(-1)+3\times(-6)}{2+3}$

$= \dfrac{-2-18}{5} = \dfrac{-20}{5} = -4$

$\therefore$ Coordinates of A are $(2, -4)$

Now, as P(6, – 6) lies on $3x + k(y + 1) = 0$

$\Rightarrow$ $3(6) + k(-6 + 1) = 0$

$\Rightarrow$ $18 - 5k = 0$

$\Rightarrow$ $k = \dfrac{18}{5}$.

9. Hint: ***Show that all sides are equal.***

WORKSHEET – 75

1. Centroid is $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

$= \left(\dfrac{-3+5-8}{3}, \dfrac{0-2+5}{3}\right) = (-2, 1).$

2. $ar(\Delta ABC)$

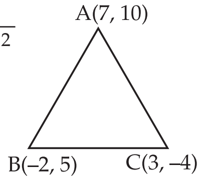
$= \left|\dfrac{1}{2}\{2(1+2)-2(-2-3)+3(3-1)\}\right|$

$= \left|\dfrac{1}{2}(6+10+6)\right| = 11$ sq. units.

3. $AB = \sqrt{(-2-2)^2 + (3-4)^2} = \sqrt{16+1}$

$= \sqrt{17}$.

4. Let $AB = \sqrt{(7+2)^2+(10-5)^2}$



$= \sqrt{81+25}$

$= \sqrt{106}$

$AC = \sqrt{(7-3)^2+(10+4)^2}$

$= \sqrt{16+196} = \sqrt{212}$

$BC = \sqrt{(3+2)^2+(-4-5)^2}$

$= \sqrt{25+81} = \sqrt{106}$

As AB = BC

$\Rightarrow$ ΔABC is isosceles

Also, $AC^2 = 212$

$AB^2 = 106$

$BC^2 = 106$

$\therefore$ $AC^2 = AB^2 + BC^2$

$\Rightarrow \Delta ABC$ is a right-angled triangle also.

5. True,

$\because$ Pythagoras Theorem is satisfied.

$AB^2 = (-2)^2 + (1-3)^2$

$= 4 + 4 = 8$

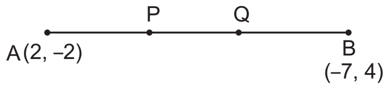
$AC^2 = (-1)^2 + (4-3)^2 = 2$

$BC^2 = (1)^2 + (3)^2 = 10$

$\therefore$ $AB^2 + AC^2 = BC^2$

$\Rightarrow$ $\angle A = 90°$.

6.



As AP = PQ = QB

$\Rightarrow$ AP : PB = 1 : 2; let P(x, y)

$\therefore$ Using section formula

$x = \dfrac{2\times 2+1\times(-7)}{1+2} = \dfrac{4-7}{3} = -1$

and $y = \dfrac{2\times(-2)+1\times(4)}{1+2} = \dfrac{-4+4}{3} = 0$

$\therefore$Coordinates of P are (–1, 0)

also then as Q is the mid-point of PB.

$\therefore$Coordinates of Q are given by:

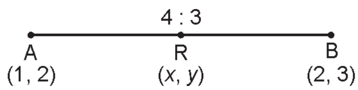
$Q\left(\dfrac{-1-7}{2}, \dfrac{0+4}{2}\right)$

{$\because$ Using mid–point formula}

$\Rightarrow$ Q(– 4, 2)

$\therefore$Coordinates of P and Q are (–1, 0) and (– 4, 2) respectively.

7. Let the coordinates of R be (x, y)



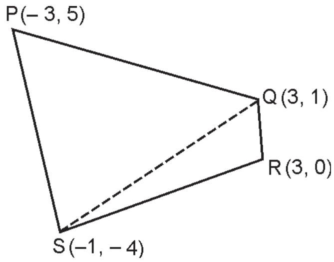
Then, $x = \dfrac{4\times2+3\times1}{4+3}$ and $y = \dfrac{4\times3+3\times2}{4+3}$

$\Rightarrow$ $x = \dfrac{11}{7}$ and $y = \dfrac{18}{7}$

Therefore, the coordinates of R are $\left(\dfrac{11}{7}, \dfrac{18}{7}\right)$.

8. 25 sq. units

Hint: Join SQ. Find *ar*(ΔPQS) and *ar*(ΔRQS)



∴ Required area = *ar*(ΔPQS) + *ar*(ΔRQS).

9. Each side of a square and rhombus are equal, but the diagonals of a square are equal and that of a rhombus may or may not equal.

$$\text{Side PQ} = \sqrt{(3-2)^2 + (4+1)^2} = \sqrt{1+25} = \sqrt{26}$$

$$\text{Side QR} = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

$$\text{Side RS} = \sqrt{(-3+2)^2 + (-2-3)^2} = \sqrt{1+25} = \sqrt{26}$$

$$\text{Side SP} = \sqrt{(2+3)^2 + (-1+2)^2} = \sqrt{25+1} = \sqrt{26}$$

$$\text{Diagonal PR} = \sqrt{(-2-2)^2 + (3+1)^2} = \sqrt{16+16} = 4\sqrt{2}$$

$$\text{Diagonal QS} = \sqrt{(-3-3)^2 + (-2-4)^2} = \sqrt{36+36} = 6\sqrt{2}$$

Clearly, the sides are equal but the diagonals are not equal. Hence, PQRS is a rhombus but not a square.

WORKSHEET – 76

1. As given points are collinear.

So, $a(b-1) + 0(1-0) + 1(0-b) = 0$

$\Rightarrow \quad ab - a - b = 0$

$\Rightarrow \quad a + b = ab$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} = 1.$

2. *ar* of ΔABC $= \frac{1}{2} \times 5 \times 3 = 7.5$. sq. unit.

3. Distance $= \sqrt{(\cos\theta - \sin\theta)^2 + (\sin\theta + \cos\theta)^2}$

$= \sqrt{2(\sin^2\theta + \cos^2\theta)} = \sqrt{2}$.

4. Let A ≡ $(x, -1)$ and B ≡ (3, 2).

AB = 5

$\Rightarrow \quad \sqrt{(3-x)^2 + (2+1)^2} = 5$

$\Rightarrow \quad 9 - 6x + x^2 + 9 = 25$ (On squaring)

$\Rightarrow \quad x^2 - 6x - 7 = 0$

$\Rightarrow \quad (x-7)(x+1) = 0 \Rightarrow x = 7$ or -1.

5. False.

As mid-point of AC $= \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$

and mid-point of BD $= \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$

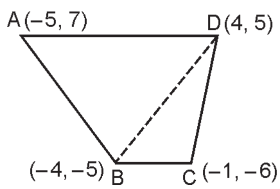
∴ Mid-point of AC = Mid-point of BD

$\Rightarrow \quad \frac{15}{2} = \frac{8+p}{2} \Rightarrow p = 7.$

6. *ar*(ΔABD)

$= \left|\frac{1}{2}\{-5(-5-5) - 4(5-7) + 4(7+5)\}\right|$

$= \left|\frac{1}{2}(50+8+48)\right| = 53$ sq. units.



ar(ΔBCD)

$= \left|\frac{1}{2}\{-4(-6-5) - 1(5+5) + 4(-5+6)\}\right|$

$= \left|\frac{1}{2}(44-10+4)\right| = 19$ sq. units.

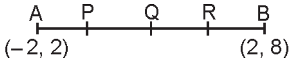
Now, *ar*(quadrilateral ABCD)

= *ar*(ΔABD) + *ar*(ΔBCD)

$= 53$ sq. units + 19 sq. units

$= 72$ sq. units.

7. Let the points P, Q and R divide AB into four equal parts AP = PQ = QR = RB as shown below in the adjoining figure.



Clearly, Q is the mid-point of AB

$\therefore \quad Q \equiv \left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$

i.e., $\quad Q \equiv (0, 5)$

P is the mid-point of AQ

$\therefore \quad P \equiv \left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$

i.e., $\quad P \equiv \left(-1, \frac{7}{2}\right)$

and R is the mid-point of QB.

$\therefore \quad R \equiv \left(\frac{0+2}{2}, \frac{5+8}{2}\right)$

i.e., $\quad R \equiv \left(1, \frac{13}{2}\right)$.

Hence, the required points are $P\left(-1, \frac{7}{2}\right)$, Q(0, 5) and $R\left(1, \frac{13}{2}\right)$.

8. We have A(4, 6), B(1, 5) and C(7, 2)

$\therefore$ Area of ΔABC is

$$= \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

$$= \left|\frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]\right|$$

$$= \left|\frac{1}{2}[12 - 4 + 7]\right|$$

$$= \frac{15}{2} \text{ sq. unit}$$

As $\quad \frac{AD}{AB} = \frac{1}{3} \Rightarrow \frac{AD}{DB} = \frac{1}{2}$

$\therefore$ If coordinates of D are (x, y)

Then using section formula:

$$x = \frac{2\times4+1\times1}{2+1} = \frac{9}{3} = 3$$

$$y = \frac{2\times6+1\times5}{2+1} = \frac{12+5}{3} = \frac{17}{3}$$

$\therefore \quad$ D is $\left(3, \frac{17}{3}\right)$

Similarly, if (a, b) be coordinates of E.

then $\quad a = \frac{2\times4+1\times7}{2+1} = \frac{15}{3} = 5$

$$b = \frac{2\times6+1\times2}{2+1} = \frac{14}{3}.$$

$\therefore \quad$ Coordinates of E are $\left(5, \frac{14}{3}\right)$.

$\therefore \quad$ area of ΔADE

$$= \left|\frac{1}{2}\left[4\left(\frac{17}{3}-\frac{14}{3}\right)+3\left(\frac{14}{3}-6\right)+5\left(6-\frac{17}{3}\right)\right]\right|$$

$$= \left|\frac{1}{2}\left[4-4+\frac{5}{3}\right]\right| = \frac{5}{6} \text{ sq. unit}$$

$$\therefore \quad \frac{ar \text{ of } \Delta ABC}{ar\ \Delta ADE} = \frac{\frac{15}{2}}{\frac{5}{6}} = \frac{15}{2}\times\frac{6}{5} = 9$$

$\Rightarrow \quad ar$ of ΔABC = 9. ($ar\ \Delta$ADE).

9. Let P(x, y), A(7, 1), B(3, 5)

As P is equidistant from A and B

$\therefore \quad PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow \quad (7-x)^2 + (1-y)^2 = (3-x)^2 + (5-y)^2$

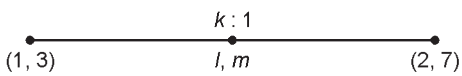
$\Rightarrow \quad 49 + x^2 - 14x + 1 + y^2 - 2y$

$\quad = 9 + x^2 - 6x + 25 + y^2 - 10y$

$\Rightarrow \quad -8x + 8y = -16 \Rightarrow x - y = 2.$

OR

Let the required ratio be $k : 1$ and the point of division be (l, m) using section formula, we have



$$l = \frac{2k+1}{k+1} \text{ and } m = \frac{7k+3}{k+1}$$

So, the point of division is $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$.

This point lies on the line $3x + y - 9 = 0$.

$\therefore \quad 3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0$

$\Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow 4k = 3 \quad \Rightarrow k = \frac{3}{4}$

Hence, the required ratio is 3 : 4.

WORKSHEET – 77

1. **Hint:** Use condition of collinearity.
2. **Hint:** Any point on x-axis be $(x, 0)$.
   Let ratio be $k : 1$
3. **Hint:** Origin is (0, 0).
4. Let A is (– 4, – 6) and B is (10, 12)
   Let P(0, y) be any point on y-axis which lies on AB.
   Let ratio be k : 1. Using section formula:

$$0 = \frac{k(10)-4}{k+1}$$

$$\Rightarrow \quad 10k = 4 \Rightarrow k = \frac{4}{10} = \frac{2}{5}$$

$\therefore$ Ratio is 2 : 5

Also then $y = \frac{12k-6}{k+1}$

$$\Rightarrow \quad y = \frac{12\left(\frac{2}{5}\right) - 6}{\frac{2}{5}+1} = \frac{24-30}{2+5} = \frac{-6}{7}$$

$\therefore$ The point P will be $\left(0, \frac{-6}{7}\right)$

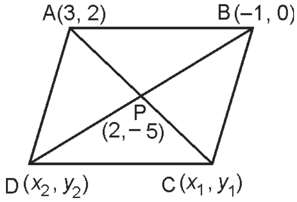
5. Diagonals AC and BD cut each other at the mid-point P.

$\therefore \quad \frac{3 + x_1}{2} = 2 \; ; \quad \frac{2 + y_1}{2} = -5$

$\Rightarrow \quad x_1 = 1 \; ; \quad y_1 = -12.$

Similarly, $\frac{x_2 - 1}{2} = 2 \; ; \quad \frac{y_2 + 0}{2} = -5$

$\Rightarrow \quad x_2 = 5 \; ; \quad y_2 = -10.$



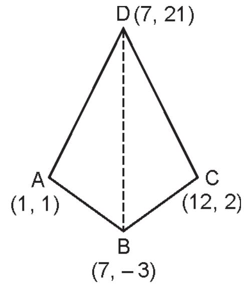
Hence, two other vertices of the parallelogram are (1,– 12) and (5, –10).

6. Area of quadrilateral ABCD

$= ar(\Delta ABD) + ar(\Delta BCD)$

$= \left|\frac{1}{2}\{1(-3-21)+7(21-1)+7(1+3)\}\right|$

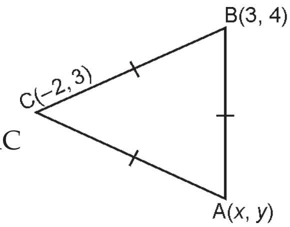
$+ \left|\frac{1}{2}\{7(2-21)+12(21+3)+7(-3-2)\}\right|$



$= \left|\frac{1}{2}(-24+140+28)\right| + \left|\frac{1}{2}(-133+288-35)\right|$

$= 72 + 60 = 132$ sq. units.

7. $\left(\frac{1 \pm \sqrt{3}}{2}, \frac{7 \pm 5\sqrt{3}}{2}\right)$



Hint:

As $\quad AB = BC = AC$

$\therefore \quad AB = BC$

$\Rightarrow \quad AB^2 = BC^2$

$\Rightarrow \; (x - 3)^2 + (y - 4)^2 = 26 \qquad ...(i)$

Similarly,

$AC = BC \Rightarrow (x + 2)^2 + (y - 3)^2 = 26 \; ...(ii)$

Solve (i) and (ii).

8. $\qquad PA = PB$

$\Rightarrow \qquad PA^2 = PB^2$

$\Rightarrow \quad (x-3)^2+(y-6)^2 = (x+3)^2+(y-4)^2$

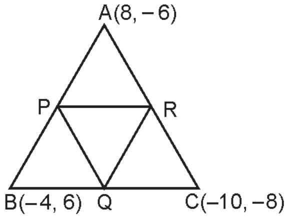
$\Rightarrow x^2+9-6x+y^2+36-12y$

$\qquad = x^2+9+6x+y^2+16-8y$

$\Rightarrow \qquad 3x+y = 5.$

9. Coordinates of P are $\left(\frac{8-4}{2}, \frac{-6+6}{2}\right)$,

i.e., (2, 0).



Coordinates of Q are $\left(\frac{-4-10}{2}, \frac{6-8}{2}\right)$

i.e., (–7, –1).

Coordinates of R are $\left(\frac{8-10}{2}, \frac{-6-8}{2}\right)$,

i.e., (– 1, – 7).

Now, $ar(\Delta ABC)$

$$= \left|\frac{1}{2}\{8(6+8)-4(-8+6)-10(-6-6)\}\right|$$

$$= \left|\frac{1}{2}(112+8+120)\right| = 120 \text{ sq. units.}$$

and $ar(\Delta PQR)$

$$= \left|\frac{1}{2}\{2(-1+7)-7(-7-0)-1(0+1)\}\right|$$

$$= \left|\frac{1}{2}(12+49-1)\right| = 30 \text{ sq. units.}$$

So, $$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{30}{120} = \frac{1}{4}$$

$$\Rightarrow \quad ar(\Delta PQR) = \frac{1}{4}ar(\Delta ABC).$$

Hence proved.

WORKSHEET – 78

1. Let (x, y) be required coordinates

$\therefore$ using mid-point formula:

$$-2 = \frac{x+2}{2} \text{ and } 5 = \frac{3+y}{2}$$

$$\Rightarrow \quad x = -6 \quad \text{and} \quad y = 7.$$

2. Using condition of collinearity,

$$k[3k-1]+3k[1-2k]+3[-k] = 0$$

$$\Rightarrow \quad 3k^2 - k + 3k - 6k^2 - 3k = 0$$

$$\Rightarrow \quad -3k^2 - k = 0$$

$$\Rightarrow \quad -k(3k+1) = 0$$

$$\Rightarrow \quad k = -\frac{1}{3} \text{ or } 0.$$

3. Using mid-point formula:

$$1 = \frac{2a-2}{2} \;;\; 2a + 1 = \frac{4+3b}{2}$$

$$\Rightarrow \quad a = 2 \quad ; \quad b = 2.$$

4. Let the ratio is $k : 1$

$\therefore$ Using section formula, the point is

$$p\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$$

As it lies on $x - y - 2 = 0$

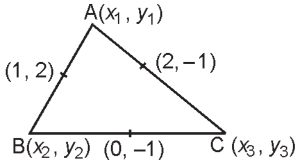
$$\Rightarrow \quad 8k + 3 - 9k + 1 - 2k - 2 = 0$$

$$\Rightarrow \quad -3k + 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{3}$$

$\therefore$ Ratio is 2 : 3.

5. Let the coordinates of vertices of the triangle ABC are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.



Since, the point (1, 2) is the mid-point of AB.

$$\therefore \quad \frac{x_1+x_2}{2} = 1, \; \frac{y_1+y_2}{2} = 2$$

i.e., $x_1 + x_2 = 2, y_1 + y_2 = 4$ ...(*i*)

Similarly, $x_2 + x_3 = 0, y_2 + y_3 = -2$ ...(*ii*)

and $x_3 + x_1 = 4, y_3 + y_1 = -2$ ...(*iii*)

$\therefore$ $x_1 + x_2 + x_3 = 3, y_1 + y_2 + y_3 = 0$...(*iv*)

Solving results (*i*), (*ii*), (*iii*) and (*iv*), we get $x_1 = 3, y_1 = 2, x_2 = -1, y_2 = 2, x_3 = 1, y_3 = -4$.

Hence the required vertices are A(3, 2), B(–1, 2) and C(1, – 4).

6. (7, 2) or (1, 0)

Hint: $ar(\Delta PAB) = 10$

$\Rightarrow ar(\Delta PAB) = +10$ or $ar(\Delta PAB) = -10$

Let $P(x, y)$ $\therefore$ Use area of triangle

$$= \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right].$$

7. The given points would be collinear, if the area of triangle formed by them as vertices is zero.

i.e., $\left|\frac{1}{2}\{a(c+a-a-b)+b(a+b-b-c)+c(b+c-c-a)\}\right| = 0$

Here, LHS

$= \left|\frac{1}{2}\{a(c-b)+b(a-c)+c(b-a)\}\right|$

$= \left|\frac{1}{2}(ac-ab+ab-bc+bc-ac)\right| = |0|$

= 0 = RHS. **Hence proved.**

8. 24 sq. units

Hint: Area of rhombus = $\frac{1}{2} d_1 \times d_2$

where d_1, d_2 = length of diagonals.

9. $ar(\Delta ABC) = 5$

$\Rightarrow \left|\frac{1}{2}\{k(6-1)-2(1-2k)+3(2k-6)\}\right| = 5$

$\Rightarrow \left|\frac{1}{2}(5k-2+4k+6k-18)\right| = 5$

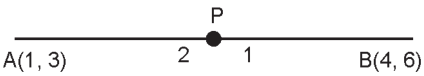
$\Rightarrow \left|\frac{1}{2}(15k-20)\right| = 5 \Rightarrow \left|\frac{15}{2}k-10\right| = 5$

$\Rightarrow \frac{15}{2}k-10 = \pm 5 \Rightarrow \frac{15}{2}k = 10 \pm 5$

$\Rightarrow \frac{15}{2}k = 15 \text{ or } 5 \Rightarrow k = 2 \text{ or } \frac{2}{3}.$

WORKSHEET – 79

1.



$\therefore \; P\left(\frac{2\times4+1}{3}, \frac{2\times6+3}{3}\right) \Rightarrow P(3, 5)$

2. Mid-point of (6,8) and (2,4) is P(4, 6).

$\therefore$ If A(1, 2), then

$AP = \sqrt{(4-1)^2+(6-2)^2}$

$= \sqrt{9+16} = 5$ units.

3. 2 or – 4

Hint: Use Pythagoras Theorem.

4.

$\therefore \; \frac{PA}{PQ} = \frac{2}{3} \Rightarrow PA : AQ = 2 : 1$

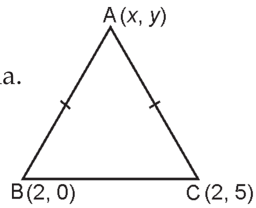
$\therefore$ Using section formula.

$x = \frac{2}{3} \;\; ; \;\; y = \frac{1}{3}.$

$\therefore$ Coordinates of A are $\left(\frac{2}{3}, \frac{1}{3}\right)$.

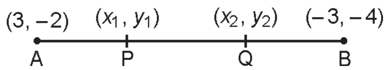
5. $\left(2-\frac{\sqrt{11}}{2}, \frac{5}{2}\right); \left(2+\frac{\sqrt{11}}{2}, \frac{5}{2}\right)$

Hint:
Let AB = AC = 3.
Use distance formula.



6. Let the points of trisection be $P(x_1, y_1)$ and $Q(x_2, y_2)$ such that P is the mid-point of A(3, –2), $Q(x_2, y_2)$ and Q is the mid-point of $P(x_1, y_1)$, B(– 3, –4).

i.e., AP = PQ = QB



$\Rightarrow$ AP : PB = 1 : 2

Using section formula,

$\therefore \; x_1 = \frac{-3+2\times3}{1+2}, \; y_1 = \frac{-4+2\times(-2)}{1+2}$

$\Rightarrow \; x_1 = 1, \; y_1 = -\frac{8}{3}$

Again AQ : QB = 2 : 1

$\therefore \; x_2 = \frac{2\times(-3)+3}{2+1}, \; y_2 = \frac{2\times(-4)-2}{2+1}$

$\Rightarrow \; x_2 = -1, \; y_2 = -\frac{10}{3}.$

Hence, the required points are $P\left(1, -\frac{8}{3}\right)$ and $Q\left(-1, -\frac{10}{3}\right)$.

7. A(1, 10), B(–7, –6), C(9, 2)

Hint: $x_1 + x_2 = 2 \times (-3) = -6$

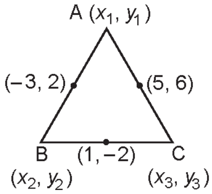
$x_2 + x_3 = 2$

Adding, $x_3 + x_1 = 10$

$2(x_1 + x_2 + x_3) = 6$

$x_1 + x_2 + x_3 = 3$

$\therefore \quad x_1 = 1, x_2 = -7, x_3 = 9$



Using same method,

$y_1 + y_2 + y_3 = 6$

and $y_1 = 10$, $y_2 = -6$, $y_3 = 2$.

8. Area of ΔDBC

$$= \left| \frac{1}{2}\left[x\{5-(-2)\} + (-3)(-2-3x) + 4(3x-5) \right] \right|$$

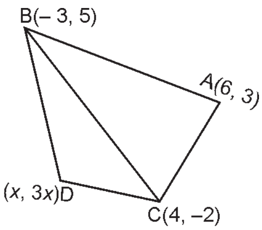
$$= \left| \frac{1}{2}(28x - 14) \right| = |(14x - 7)|$$

Area of ΔABC

$$= \left| \frac{1}{2}\left[6\{5-(-2)\} + (-3)(-2-3) + 4(3-5) \right] \right|$$

$$= \left| \frac{1}{2}(42 + 15 - 8) \right| = \frac{49}{2}$$

From question



$$\frac{ar(\Delta DBC)}{ar(\Delta ABC)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{|14x - 7|}{\frac{49}{2}} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{14x - 7}{\frac{49}{2}} = \frac{1}{2} \quad \text{or} \quad \frac{-14x + 7}{\frac{49}{2}} = \frac{1}{2}$$

$$\Rightarrow \quad 8x - 4 = 7 \quad \text{or} \quad -8x + 4 = 7$$

$$\Rightarrow \quad x = \frac{11}{8} \quad \text{or} \quad \frac{-3}{8}.$$

WORKSHEET – 80

1. A($a+b$, $a-b$), B($2a+b$, $2a-b$), C($a-b$, $a+b$), D(x, y)

Mid-point of AC = Mid-point of BD

$\Rightarrow \quad x = -b$

$\Rightarrow \quad y = b.$

2. As the distance of a point from x-axis is equal to its y-coordinate, *i.e.*, 3.

3. $\left(-\frac{1}{3}, 0\right); \left(\frac{-5}{3}, 2\right)$

Hint:

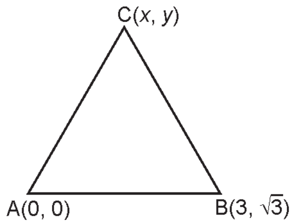
Use AP : PB = 1 : 2

and AQ : QB = 2 : 1.

4. Let the third vertex be C(x, y) of the given ΔABC. Using, AC = AB and BC = AB,

we have $x^2 + y^2 = 12$ and

$(x-3)^2 + (y-\sqrt{3})^2 = 12$

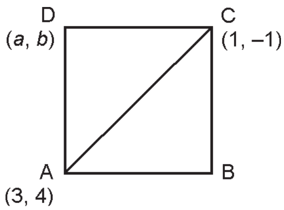


Solving these, we obtain

$x = 0, y = 2\sqrt{3}$ or $x = 3, y = -\sqrt{3}$.

Hence, the required vertex is $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$.

5. Let ABCD be the given square such that A(3, 4) and C(1, –1). Let D(a, b) be the unknown vertex.



Using $AD^2 = CD^2$, we have

$(a-3)^2 + (b-4)^2 = (a-1)^2 + (b+1)^2$

$\Rightarrow \quad 4a + 10b - 23 = 0$

$$\Rightarrow \quad b = \frac{23 - 4a}{10} \quad \text{...}(i)$$

Using $AD^2 + CD^2 = AC^2$, we have

$(a-3)^2 + (b-4)^2 + (a-1)^2 + (b+1)^2$

$= (3-1)^2 + (4+1)^2$

$\Rightarrow a^2+b^2-6a-8b+9+16+a^2-2a+b^2$
$+2b+1+1 = 4+25$

$\Rightarrow \quad a^2+b^2-4a-3b = 1 \quad ...(ii)$

Using equations (i) and (ii), we get

$$a^2+\left(\frac{23-4a}{10}\right)^2 - 4a-3\left(\frac{23-4a}{10}\right) = 1$$

$$\Rightarrow \quad 116a^2 - 464a - 261 = 0$$

$$\Rightarrow \quad 4a^2 - 16a - 9 = 0$$

(Dividing by 29)

$$\Rightarrow a = \frac{16\pm\sqrt{256+4\times4\times9}}{2\times4} = \frac{9}{2} \text{ or } -\frac{1}{2}$$

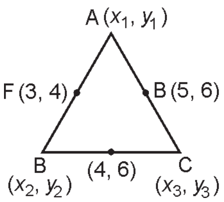
Substitute $a = \frac{9}{2}$ and $a = -\frac{1}{2}$ successively to get $b = \frac{1}{2}$ and $b = \frac{5}{2}$.

Hence, the required vertices are $\left(\frac{9}{2},\frac{1}{2}\right)$ and $\left(-\frac{1}{2},\frac{5}{2}\right)$.

OR

(4, 5), (2, 3), (6, 9)

Hint:



$x_1+x_2 = 6 \qquad y_1+y_2 = 8$

$\therefore \quad x_2+x_3 = 8 \qquad y_2+y_3 = 12$

$x_1+x_3 = 10 \qquad y_1+y_3 = 14$

Adding,

$2(x_1+x_2+x_3) = 24 \qquad 2(y_1+y_2+y_3) = 34$

$\Rightarrow \quad x_1+x_2+x_3 = 12 \quad \Rightarrow \quad y_1+y_2+y_3 = 17$

$\therefore \quad x_1 = 4 \quad \therefore \quad y_1 = 5$

$x_2 = 2 \qquad y_2 = 3$

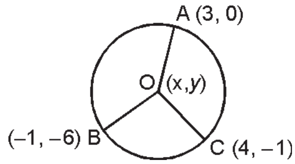
$x_3 = 6; \qquad y_3 = 9.$

6. Let O(x, y) be the circumcentre passing through A(3, 0), B(–1, – 6) and C(4, –1).

Then OA = OB = OC

Taking OA = OB

$\Rightarrow \quad OA^2 = OB^2$ (Squaring)



$\Rightarrow (x-3)^2 + (y-0)^2 = (x+1)^2 + (y+6)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 = x^2 + 2x + 1 + y^2 + 12y + 36$

$\Rightarrow \quad 2x + 3y + 7 = 0 \quad ...(i)$

and OA = OC $\Rightarrow$ $OA^2 = OC^2$ (Squaring)

$\Rightarrow \quad (x-3)^2 + y^2 = (x-4)^2 + (y+1)^2$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 = x^2 - 8x + 16 + y^2 + 2y + 1$

$\Rightarrow \quad x - y - 4 = 0 \quad ...(ii)$

Solving equations (i) and (ii), we get

$$x = 1, y = -3.$$

Thus, the coordinates of the centre are (1, –3)

Now, radius = OA = $\sqrt{(3-1)^2+(0+3)^2}$

$= \sqrt{4+9} = \sqrt{13}$ units.

OR

The area of the triangle formed by the given points must be zero.

i.e., $\left|\frac{1}{2}\left\{k(2k-6+2k)-(k-1)(6-2k-2+2k) -(4+k)(2-2k-2k)\right\}\right| = 0$

$\Rightarrow k(4k-6)-(k-1)\times4-(4+k)(2-4k) = 0$

$\Rightarrow \quad 4k^2-6k-4k+4-8+16k-2k+4k^2 = 0$

$\Rightarrow \quad 8k^2 + 4k - 4 = 0 \Rightarrow k^2 + \frac{1}{2}k - \frac{1}{2} = 0$

$\Rightarrow \quad k^2 + \frac{1}{2}k + \frac{1}{16} - \frac{1}{16} - \frac{1}{2} = 0$

$\Rightarrow \quad \left(k+\frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = 0$

$\Rightarrow \quad \left(k+\frac{1}{4}-\frac{3}{4}\right)\left(k+\frac{1}{4}+\frac{3}{4}\right) = 0$

$\Rightarrow \quad k = -1 \text{ or } \frac{1}{2}.$

7. $SP = \sqrt{(at^2 - a)^2 + (2at - 0)^2}$

$= \sqrt{a^2(t^2-1)^2 + 4a^2t^2}$

$= a\sqrt{t^4 - 2t^2 + 1 + 4t^2} \;= a\sqrt{t^4 + 2t^2 + 1}$

$= a(t^2 + 1)$

$SQ = \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{-2a}{t^2} - 0\right)^2}$

$= \sqrt{\frac{a^2}{t^4} - \frac{2a^2}{t^2} + a^2 + \frac{4a^2}{t^2}}$

$= a\sqrt{\frac{1}{t^4} + \frac{2}{t^2} + 1} \;= a\left(\frac{1}{t^2} + 1\right)$

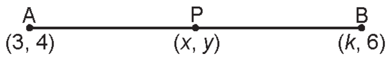
Now, $\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(t^2+1)} + \frac{1}{a\left(\frac{1}{t^2}+1\right)}$

$= \frac{1}{a}\left[\frac{1}{t^2+1} + \frac{t^2}{1+t^2}\right]$

$= \frac{1}{a}\left(\frac{1+t^2}{1+t^2}\right) = \frac{1}{a}$

which is independent of t.

8. As P(x, y) is mid-point of AB,



$x = \frac{3+k}{2}$ and $y = \frac{4+6}{2}$

i.e., $x = \frac{3}{2} + \frac{k}{2}$ and $y = 5$

The value of x and y will satisfy $x + y - 10 = 0$

$\therefore\ \frac{3}{2} + \frac{k}{2} + 5 - 10 = 0 \Rightarrow \frac{k}{2} = 5 - \frac{3}{2}$

$\Rightarrow k = 7.$

WORKSHEET – 81

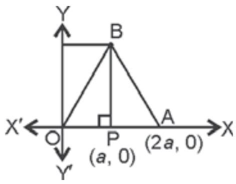
1. Let the coordinates of the third vertex C be (x, y).

$\therefore\ \frac{3-2+x}{3} = \frac{5}{3}$ and $\frac{2+1+y}{3} = \frac{1}{3}$

i.e. $x = 4$ and $y = -2$

$\therefore\ C \equiv (4, -2).$

2. As $BP = \frac{\sqrt{3}}{2} \times (2a)$



$= a\sqrt{3}$

and $OP = \frac{1}{2}\ OA = a$

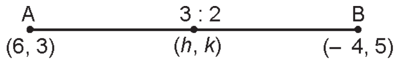
$\therefore$ Coordinates of B are (a, $\sqrt{3}\,a$).

3. Hint: Use AB = BC = AC

Take $AB^2 = BC^2$

and $BC^2 = AC^2$.

4. Let the required point be (h, k).



Then

$h = \frac{3\times(-4) + 2 \times 6}{3 + 2}$ and $k = \frac{3\times5+2\times3}{3+2}$

i.e., $h = 0$ and $k = \frac{21}{5}$.

So, the required point is $\left(0, \frac{21}{5}\right)$.

5. True.

Let O(0, 0), A(5,5) and B(–5,5) be the three points.

$\therefore\ OA = 5\sqrt{2} = OB$

and $AB^2 = 100 = OA^2 + OB^2$.

OR

True,

$AB = \sqrt{(-4+6)^2 + (6-10)^2} = \sqrt{20} = 2\sqrt{5}$

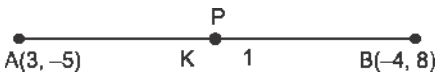
$BC = \sqrt{(3+4)^2 + (-8-6)^2} = \sqrt{245} = 7\sqrt{5}$

$AC = \sqrt{(3+6)^2 + (-8-10)^2} = \sqrt{405} = 9\sqrt{5}$

$\therefore\ AB + BC = AC.$

Also, $\frac{AB}{AC} = \frac{2}{9} \Rightarrow AB = \frac{2}{9}AC$.

6.



Let coordinates of P are (x, y)

$\therefore$ Using section formula:

$$x = \frac{-4K+3}{K+1}\ ;\ y = \frac{8K-5}{K+1}$$

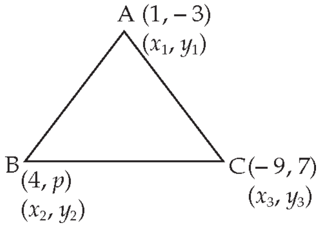
As P lies on $x + y = 0$

$$\Rightarrow \frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$$

$$\Rightarrow \quad 4k - 2 = 0$$

$$\Rightarrow \quad K = \frac{2}{4} = \frac{1}{2}.$$

7.



Area of ΔABC

$$= \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

$$\Rightarrow 15 = \frac{1}{2}[1(p - 7) + 4(7 + 3) + (-9)(-3 - p)]$$

$$30 = [p - 7 + 40 + 27 + 9p]$$

$$\Rightarrow 30 = [10p + 60] \quad \Rightarrow 10p = -30$$

$$\Rightarrow \quad p = -3.$$

8. Let the coordinates of P be (x, y).

$$PA = PB$$

$$\Rightarrow \quad PA^2 = PB^2 \qquad \text{(Squaring)}$$

$$\Rightarrow (x - 3)^2 + (y - 4)^2 = (x - 5)^2 + (y + 2)^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16$$
$$= x^2 - 10x + 25 + y^2 + 4y + 4$$

$$\Rightarrow \quad 4x - 12y - 4 = 0$$

$$\Rightarrow \quad x - 3y - 1 = 0 \qquad ...(i)$$

Area of ΔPAB = 10

$$\Rightarrow \left|\frac{1}{2}\{x(4+2) + 3(-2-y) + 5(y-4)\}\right| = 10$$

$$6x - 6 - 3y + 5y - 20 = \pm 20$$

$$6x + 2y - 26 = \pm 20$$

$$\Rightarrow \quad 3x + y - 3 = 0 \qquad ...(ii)$$

$$\text{or} \quad 3x + y - 23 = 0 \qquad ...(iii)$$

Now, we have to solve equations (*i*) and (*ii*) as well as equations (*i*) and (*iii*).

Solving equations (*i*) and (*ii*), we get

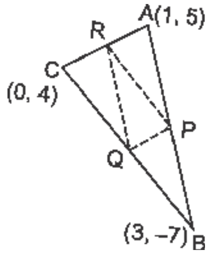
$x = 1, y = 0$

Solving equations (*i*) and (*iii*), we get

$x = 7, y = 2$

Hence, the coordinates of P are (1,0) or (7, 2).

OR



$\because$ P is mid-point of AB

$$\therefore P \equiv \left(\frac{1+3}{2}, \frac{5-7}{2}\right),$$

i.e., P $\equiv$ (2, –1)

$\because$ Q is mid-point of BC

$$\therefore Q \equiv \left(\frac{3+0}{2}, \frac{-7+4}{2}\right),\ i.e.,\ Q \equiv \left(\frac{3}{2}, -\frac{3}{2}\right)$$

$\because$ R is mid-point of CA

$$\therefore R \equiv \left(\frac{0+1}{2}, \frac{4+5}{2}\right),\ i.e.,\ R \equiv \left(\frac{1}{2}, \frac{9}{2}\right)$$

Now, $ar(\Delta PQR)$

$$= \left|\frac{1}{2}\left\{2\left(-\frac{3}{2} - \frac{9}{2}\right) + \frac{3}{2}\left(\frac{9}{2} + 1\right) + \frac{1}{2}\left(-1 + \frac{3}{2}\right)\right\}\right|$$

$$= \left|\frac{1}{2}\left(-12 + \frac{33}{4} + \frac{1}{4}\right)\right| = \left|\frac{-7}{4}\right| = \frac{7}{4} \qquad ...(i)$$

$ar(\Delta ABC)$

$$= \left|\frac{1}{2}\{1(-7-4) + 3(4-5) + 0(5+7)\}\right|$$

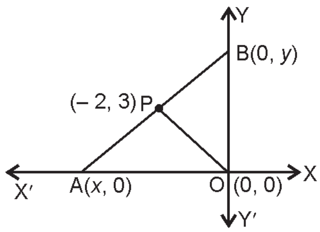
$$= \left|\frac{1}{2}(-11 - 3 + 0)\right| = |-7| = 7 \qquad ...(ii)$$

Dividing equation (*i*) by equation (*ii*), we have

$$\frac{ar(\Delta PQR)}{ar(\Delta ABC)} = \frac{\frac{7}{4}}{7}$$

$$\Rightarrow \quad ar(\Delta PQR) = \frac{1}{4}\ ar(\Delta ABC).$$

9. Since, A is on the x-axis, so its coordinates will be of the form $(x, 0)$. Similarly, the coordinates of B will be of the form $(0, y)$.



Since P is the mid-point of AB.

$\therefore \quad -2 = \dfrac{x+0}{2}$ and $3 = \dfrac{0+y}{2}$

$\therefore \quad x = -4$ and $y = 6$

$\therefore$ Coordinates of A are $(-4, 0)$ and coordinates of B are $(0, 6)$.

Now,

$$PO = \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$PA = \sqrt{(-4+2)^2 + (0-3)^2} = \sqrt{4+9} = \sqrt{13}$$

Clearly, PA = PB = PO

$\Rightarrow$ P is equidistant from A, B and the origin O.

WORKSHEET – 82

1. $(-5, 1)$, $(1, p)$ and $(4, -2)$ are collinear.

$\Rightarrow \; -5(p + 2) + 1\,(-2 -1) + 4(1 - p) = 0$

$\Rightarrow \; -5p - 10 - 3 + 4 - 4p = 0 \Rightarrow 9p = -9$

$\Rightarrow \; p = -1.$

2. $$\text{Area} = \left|\frac{1}{2}\{1(4-6) - 2(6-3) + 0\}\right|$$

$$= \left|\frac{1}{2}(-2-6)\right| = 4 \text{ sq. units.}$$

3. ***See Worksheet – 78, Sol. 4.***

4. **False,** because P does not lie on the line segment AB.

5. ***See Worksheet – 80, Sol. 5.***

6. As P is equidistant from A and B,

$PA = PB \Rightarrow PA^2 = PB^2$ (Squaring)

$\Rightarrow (a+b-x)^2 + (b-a-y)^2$

$= (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow (a+b-x)^2 - (a-b-x)^2$

$= (a+b-y)^2 - (b-a-y)^2$

$\Rightarrow (a+b-x+a-b-x)(a+b-x-a+b+x)$

$= (a + b - y + b - a - y)$

$(a + b - y - b + a + y)$

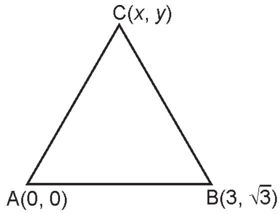
$\Rightarrow \; 2(a - x) \times 2b = 2(b - y) \times 2a$

$\Rightarrow \quad ab - bx = ab - ay \quad \Rightarrow \quad bx = ay$

Hence proved.

OR

Let the third vertex be $C(x, y)$ of the given ΔABC such that $A(0, 0)$ and $B(3, \sqrt{3})$.



Using $\quad AC = AB$

i.e. $\quad AC^2 = AB^2$ (Squaring)

$\Rightarrow \quad x^2 + y^2 = 9 + 3$

$\Rightarrow \quad x^2 + y^2 = 12$ ...(*i*)

Also, using $\quad BC = AC$

i.e., $\quad BC^2 = AC^2$ (Squaring)

$\Rightarrow \quad (x-3)^2 + (y - \sqrt{3})^2 = x^2 + y^2$

$\Rightarrow \; x^2 - 6x + 9 + y^2 - 2\sqrt{3}y + 3 = x^2 + y^2$

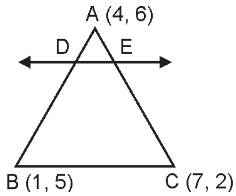
$\Rightarrow \quad 3x + \sqrt{3}\,y - 6 = 0$ ...(*ii*)

Solving equations (*i*) and (*ii*), we obtain

$x = 0, y = 2\sqrt{3}$ or $x = 3, y = -\sqrt{3}$

Hence, the third vertex is $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$.

7. $$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$



$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE} = \frac{4}{1}$$

$$\Rightarrow \frac{AB}{AD} - 1 = \frac{AC}{AE} - 1 = \frac{4}{1} - 1$$

(Subtracting 1 throughout)

$$\Rightarrow \frac{AB - AD}{AD} = \frac{AC - AE}{AE} = \frac{4-1}{1}$$

$$\Rightarrow \frac{BD}{AD} = \frac{CE}{AE} = \frac{3}{1}$$

$\Rightarrow \quad AD : BD = AE : EC = 1 : 3$

Let the coordinates of D and E be (x_1, y_1) and (x_2, y_2) respectively.

Let us use section formulae.

$$x_1 = \frac{1 \times 1 + 3 \times 4}{1 + 3}, y_1 = \frac{1 \times 5 + 3 \times 6}{1 + 3}$$

and $$x_2 = \frac{1 \times 7 + 3 \times 4}{1 + 3}, y_2 = \frac{1 \times 2 + 3 \times 6}{1 + 3}$$

i.e., $x_1 = \frac{13}{4}, y_1 = \frac{23}{4}$

and $x_2 = \frac{19}{4}, y_2 = 5$

So, the coordinates of D are $\left(\frac{13}{4}, \frac{23}{4}\right)$ and of E are $\left(\frac{19}{4}, 5\right)$.

ar(ΔADE)

$$= \left|\frac{1}{2}\left\{4\left(\frac{23}{4} - 5\right) + \frac{13}{4}(5-6) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right\}\right|$$

$$= \left|\frac{1}{2}\left\{4 \times \frac{3}{4} - \frac{13}{4} + \frac{19}{4} \times \frac{1}{4}\right\}\right|$$

$$= \frac{15}{32} \text{ sq. units.}$$

Again, *ar*(ΔABC)

$$= \left|\frac{1}{2}\{4(5-2)+1(2-6)+7(6-5)\}\right|$$

$$= \left|\frac{1}{2}(12-4+7)\right| = \frac{15}{2} \text{ sq. units}$$

$$\therefore\ ar(\Delta ADE) : ar(\Delta ABC) = \frac{\frac{15}{32}}{\frac{15}{2}} = 1 : 16.$$

8. (*i*) Distance covered by Ram = HT + TS

$$\therefore\ HT = \sqrt{(9-1)^2 + (-3-3)^2}$$

$$= \sqrt{64+36} = 10 \text{ units}$$

$$TS = \sqrt{(-3-9)^2 + (3-3)^2}$$

$$= \sqrt{144} = 12 \text{ units}$$

∴ Distance covered by Ram

= 10 + 12 = 22 units.

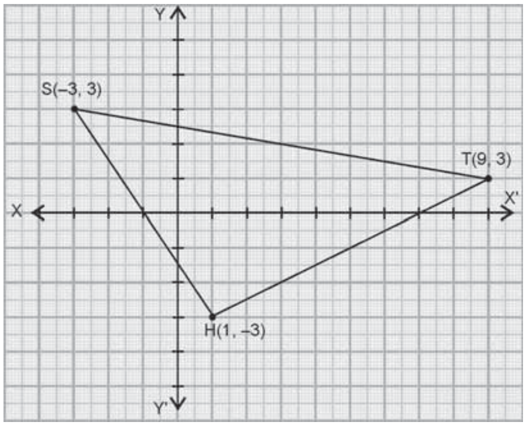
(*ii*) Distance covered by shyam = HS

$$= \sqrt{(-3-1)^2 + (3+3)^2}$$

$$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13} \text{ units}$$

(*iii*) Concept of distance formula in coordinate geometry

(*iv*) Mutual respect and Diligence.



WORKSHEET – 83

1. As points A, B and C are collinear.

$\therefore$ $ar(\Delta ABC) = 0$

$\Rightarrow \frac{1}{2}\,|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)| = 0$

$\Rightarrow |x(-4+5) + (-3)(-5-2) + 7(2+4)| = 0$

$\Rightarrow |x + 21 + 42| = 0$

$\Rightarrow x + 63 = 0$

$\Rightarrow x = -63.$

2. Let the coordinates of P and Q be $(x, 0)$ and $(0, y)$ respectively.

$\therefore \frac{x+0}{2} = 3$ and $\frac{0+y}{2} = -7$

i.e., $x = 6$ and $y = -14$

Here, P ≡ (6, 0) and Q ≡ (0, –14).

3. Let the required point be P(*h*, *k*).

Then PO = PA = PB

$\therefore h^2 + k^2 = (h-2x)^2 + k^2$

and $h^2 + k^2 = h^2 + (k-2y)^2$

$\Rightarrow h^2 = h^2 + 4x^2 - 4xh$

and $k^2 = k^2 - 4yk + 4y^2$

$\Rightarrow h = x$ and $k = y$

∴ P is (x, y).

4. False, because AB ≠ CD and BC ≠ AD as

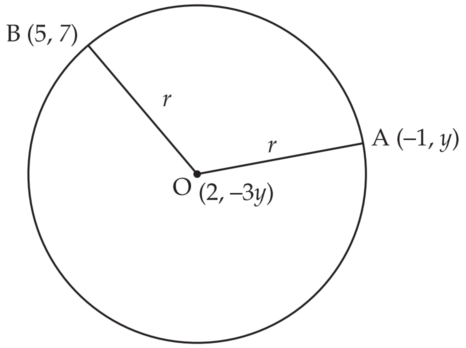
$AB = \sqrt{2^2+1^2} = \sqrt{5}$;

$BC = \sqrt{(-1)^2+(-10)^2} = \sqrt{101}$;

$$CD = \sqrt{(-8)^2 + 11^2} = \sqrt{185};$$

$$AD = \sqrt{(-7)^2 + 2^2} = \sqrt{53}.$$

5. Let radius = r



$\therefore$ OB = OA

$\Rightarrow$ $OB^2 = OA^2$

$\Rightarrow$ $(2-5)^2 + (-3y-7)^2 = (2+1)^2 + (-3y-y)^2$

$\Rightarrow$ $9 + 9y^2 + 49 + 42y = 9 + 16y^2$

$\Rightarrow$ $7y^2 - 42y - 49 = 0$

$\Rightarrow$ $y^2 - 6y - 7 = 0$

$\Rightarrow$ $y^2 - 7y + y - 7 = 0$

$\Rightarrow$ $y(y-7) + 1(y-7) = 0$

$\Rightarrow$ $(y-7)(y+1) = 0$

$\Rightarrow$ $y = 7$ or -1

$\therefore$ $r = OB = \sqrt{(2-5)^2 + (-3y-7)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$ (if $y = -1$)

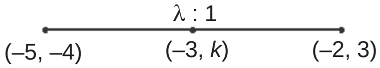
or

$r = OB = \sqrt{(2-5)^2 + (-28)^2} = \sqrt{9+784}$

$= \sqrt{793}$ (if $y = 7$)

6. Let the required ratio be $\lambda : 1$.
Here, we will use section formula as given below.

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$



In this question,

$$-3 = \frac{-2\lambda - 5}{\lambda + 1} \text{ and } k = \frac{3\lambda - 4}{\lambda + 1}$$

$\Rightarrow$ $-3\lambda - 3 = -2\lambda - 5$ and $k = \dfrac{3\lambda - 4}{\lambda + 1}$

$\Rightarrow$ $\lambda = 2$ and *i.e.*, $k = \dfrac{3 \times 2 - 4}{2+1}$

$\Rightarrow$ $\lambda : 1 = 2 : 1$ and $k = \dfrac{2}{3}$

Hence, the ratio is 2 : 1 and $k = \dfrac{2}{3}$.

7. First, we find the length of each side of quadrilateral ABCD.

$AB = \sqrt{(1-5)^2 + (5-6)^2} = \sqrt{(-4)^2 + (-1)^2}$

$= \sqrt{16 + 1} = \sqrt{17}$

$BC = \sqrt{(2-1)^2 + (1-5)^2} = \sqrt{1^2 + (-4)^2}$

$= \sqrt{1 + 16} = \sqrt{17}$

$CD = \sqrt{(6-2)^2 + (2-1)^2} = \sqrt{4^2 + 1^2}$

$= \sqrt{16+1} = \sqrt{17}$

$AD = \sqrt{(6-5)^2 + (2-6)^2} = \sqrt{1^2 + (-4)^2}$

$= \sqrt{1+16} = \sqrt{17}$

Clearly, AB = BC = CD = AD

All the sides of quadrilateral ABCD are equal.

Therefore, ABCD is a rhombus. It may be a square if diagonals are equal. To confirm it, we have to find out the lengths of diagonal AC and BD.

$AC = \sqrt{(2-5)^2 + (1-6)^2} = \sqrt{(-3)^2 + (-5)^2}$

$= \sqrt{9 + 25} = \sqrt{34}$

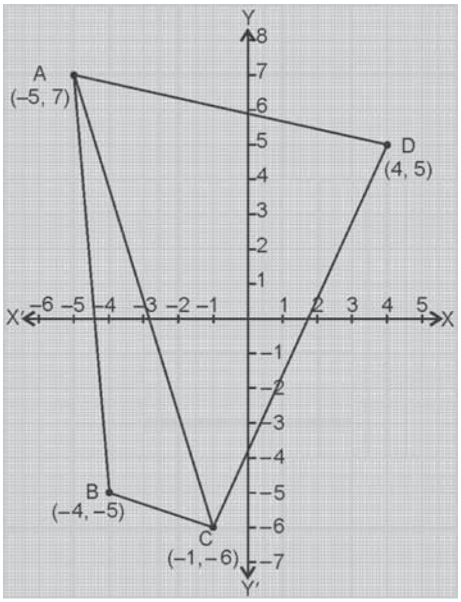
$BD = \sqrt{(6-1)^2 + (2-5)^2}$

$= \sqrt{5^2 + (-3)^2} = \sqrt{25+9} = \sqrt{34}$

Clearly, AC = BD.

Hence, quadrilateral ABCD is a square.

8. To find area of quadrilateral ABCD, we divide it into two parts by either diagonal (see graph).



Area of a triangle ABC

$= \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

$= \frac{1}{2}\left|-5\ (-5+6)-4(-6-7)-1(7+5)\right|$

$= \frac{1}{2}\left|-5+52-1\right| = \frac{1}{2} \times 35 = \frac{35}{2}$ sq. units

$ar(\Delta ACD) = \frac{1}{2}\left|-5(-6-5)-1\ (5-7)+4(7+6)\right|$

$= \frac{1}{2}\left|55+2+5\right| = \frac{1}{2} \times 109$

$= \frac{109}{2}$ sq. units

Now, ar(quadrilateral ABCD)

$= ar(\Delta ABC) + ar(\Delta ACD)$

$= \frac{35}{2} + \frac{109}{2} = \frac{144}{2}$

= 72 square units.

Alternative Method:

ar(quadrilateral ABCD)

$= \left|\frac{1}{2}\ [(x_1-x_3)\ (y_2-y_4)+(x_2-x_4)\ (y_3-y_1)\right|$

$= \left|\frac{1}{2}\ [(-5+1)\ (-5-5)+(-4-4)\ (-6-7)\right|$

$= \left|\frac{1}{2}\ [40+104]\right|$ = 72 sq. units.

CHAPTER TEST

1. Let the point of division be (x, y)

$$x = \frac{1\times 3+2\times 7}{1+2},\quad y = \frac{1\times 4+2(-6)}{1+2}$$

$$\Rightarrow\ x = \frac{17}{3},\ y = \frac{-8}{3}$$

$\Rightarrow \left(\frac{17}{3}, -\frac{8}{3}\right)$ lies in the IVth quadrant.

2. Mid-point of hypotenuse AB is equidistant from the vertices A, B and O.
Therefore, the required point is
$\left(\frac{0+2x}{2}, \frac{2y+0}{2}\right)$, *i.e.*, (x, y).

3. Let A(8, 1), B(3, –2k) and C(k, –5) are collinear.

$\Rightarrow$ Area ΔABC = 0

$\Rightarrow \left|x_1\ (y_2-y_3)+x_2(y_3-y_1) + x_3(y_1-y_2)\right| = 0$

$\Rightarrow \left|8(-2k+5)+3(-5-1) + k(1+2k)\right| = 0$

$\Rightarrow \left|-16k+40-18+k+2k^2\right| = 0$

$\Rightarrow \left|2k^2-15k+22\right| = 0$

$\Rightarrow 2k^2-15k+22 = 0$

$\Rightarrow 2k^2-11k-4k+22 = 0$

$\Rightarrow k(2k-11)-2(2k-11) = 0$

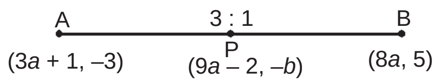
$\Rightarrow (k-2)\ (2k-11) = 0$

$\Rightarrow k-2 = 0$ or $2k-11 = 0$

$\Rightarrow k = 2$ or $k = \frac{11}{2}$.

4. **False,** because Q lies outside the circle as OQ > radius of circle.

5.



$$9a-2 = \frac{3\times 8a+1\times(3a+1)}{3+1}$$

and $$-b = \frac{3\times 5+1(-3)}{3+1}$$

$\Rightarrow\ 36a-8 = 24a+3a+1$

and $\ -3b-b = 15-3$

$\Rightarrow\ 9a = 9$ and $4b = -12$

Thus, $a = 1$ and $b = -3$.

6. As points A(–1, –4), B(*b*, *c*), C(5, –1) are collinear.

$\therefore\quad -1(c+1) + b(-1+4) + 5(-4-c) = 0$

$\Rightarrow\quad -c - 1 - b + 4b - 20 - 5c = 0$

$\Rightarrow\quad 3b - 6c = 21$

$\Rightarrow\quad b - 2c = 7$ ...(*i*)

Also $\quad 2b + c = 4$ (Given)

$\Rightarrow\quad c = 4 - 2b$

Using it in (*i*), $\quad b - 2(4 - 2b) = 7$

$\Rightarrow\quad b - 8 + 4b = 7$

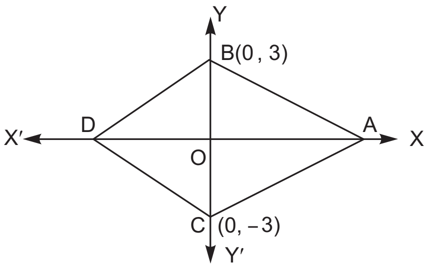
$\Rightarrow\quad 5b = 15$

$\Rightarrow\quad b = 3$

and $\quad c = 4 - 6$

$\Rightarrow\quad c = -2$

7. Since, BC lies on *y*-axis: Coordinates of B will be of type (0, *y*).



Coordinate of C is (0, –3) given

Since, mid-point BC is origin

$\therefore\quad \frac{y-3}{2} = 0 \quad\Rightarrow\quad y = 3$

$\Rightarrow$ Coordinates of B will be (0, 3)

$\therefore\quad BC = \sqrt{(0-0)^2 + (-3-3)^2} = \sqrt{36} = 6$

Clearly, Point A will lies on *x*-axis.

Let coordinates of A be $(x, 0)$.

$\therefore\quad AB = BC$

$\Rightarrow\quad \sqrt{x^2+9} = \sqrt{36}$

$\Rightarrow\quad x^2 + 9 = 36 \quad\Rightarrow\quad x^2 = 27$

$\Rightarrow\quad x = \pm 3\sqrt{3}$

$\therefore$ Two possible coordinates of A are $(3\sqrt{3}, 0)$ or $(-3\sqrt{3}, 0)$.

Moreover as ABCD is a rhombus.

$\therefore$ If A is $(3\sqrt{3}, 0)$, then D can be taken as $(-3\sqrt{3}, 0)$

and if A is $(-3\sqrt{3}, 0)$, then D can be taken as $(3\sqrt{3}, 0)$.

8. (*i*) Deepa is correct.

As A(3, 4), B(6, 7), C(9, 4), D(6, 1)

$\therefore\ AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$

$BC = \sqrt{(9-6)^2 + (4-7)^2} = 3\sqrt{2}$

$CD = \sqrt{(6-9)^2 + (1-4)^2} = 3\sqrt{2}$

and $\quad DA = \sqrt{(3-6)^2 + (4-1)^2} = 3\sqrt{2}$

Hence, AB = BC = CD = DA

Also, $\quad AC = \sqrt{(9-3)^2 + (4-4)^2}$

$= \sqrt{36+0} = 6$

$BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{36} = 6$

$\therefore\quad AC = BD$

$\Rightarrow$ Four sides and diagonals are respectively equal.

$\therefore$ ABCD is a square.

(*ii*) Distance formula in coordinate geometry is used.

(*iii*) Rationality being able to form judgement.

❑❑

Chapter 9 TRIANGLES

WORKSHEET – 84

1. Here, $\Delta ABC \sim \Delta RQP$

$\Rightarrow$ $\angle A \leftrightarrow \angle R$, $\angle B \leftrightarrow \angle Q$, $\angle C \leftrightarrow \angle P$

$\therefore$ $\angle P = \angle C = 180° - 80° - 60° = 40°$.

2. $$DC^2 = BC^2 + BD^2 = BC^2 + \left(\frac{AB}{2}\right)^2$$
$$= BC^2 + \frac{1}{4}(AC^2 - BC^2)$$
$$= 9 + \frac{1}{4}(25 - 9) = 9 + 4 = 13$$

$\Rightarrow$ $DC = \sqrt{13}$ cm.

3. $x = 8$cm

Hint: As DE ∥ BC

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$
$$\Rightarrow \quad \frac{2x-1}{x-3} = \frac{2x+5}{x-1}.$$

4. DE ∥ BC and DB is transversal

$\Rightarrow$ $\angle EDA = \angle ABC$ (Alternate interior angles)

Similarly, $\angle AED = \angle ACB$

Consequently,

$\Delta ADE \sim \angle ACB$ (AA similarity)

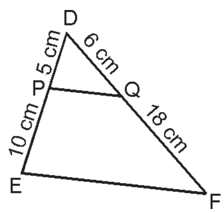
$$\therefore \quad \frac{AD^2}{AB^2} = \frac{ar(\Delta ADE)}{ar(\Delta ABC)}$$
$$\Rightarrow \quad \frac{AD^2}{9AD^2} = \frac{ar(\Delta ADE)}{153}$$
$$\Rightarrow \quad ar(\Delta ADE) = 17cm^2.$$

5. No.

Here, $\frac{DP}{PE} = \frac{5}{10} = \frac{1}{2}$

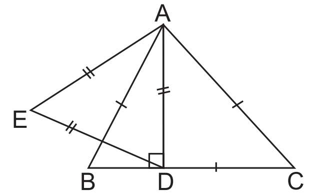
And $\frac{DQ}{QF} = \frac{6}{18} = \frac{1}{3}$

$\because$ $\frac{DP}{PE} \neq \frac{DQ}{QF}$



Therefore, PQ is not parallel to EF.

6.



Let each side of $\Delta ABC = x$

$$\therefore \quad AD^2 = AB^2 - BD^2$$
$$= x^2 - \left(\frac{x}{2}\right)^2 = x^2 - \frac{x^2}{4}$$
$$= \frac{4x^2 - x^2}{4} = \frac{3x^2}{4}.$$
$$\therefore \quad AD = \frac{\sqrt{3}}{2}x$$

As ΔABC and ΔADE are both equilateral Δ

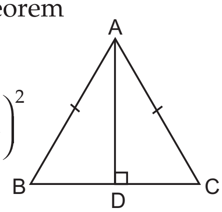
$\Rightarrow$ $\Delta ABC \sim \Delta ADE$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{AB^2}{AD^2} = \frac{x^2}{\left(\frac{\sqrt{3}}{2}x\right)^2}$$
$$= \frac{x^2}{\frac{3}{4}x^2} = \frac{4}{3}.$$

7. As AB = BC = AC

$\therefore$ $AD \perp BC \Rightarrow BD = \frac{1}{2}BC$

$\therefore$ Using Pythagoras Theorem



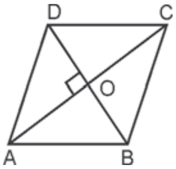
$$AB^2 = AD^2 + BD^2$$
$$\Rightarrow \quad AD^2 = AB^2 - \left(\frac{1}{2}BC\right)^2$$
$$= \frac{3AB^2}{4}$$
$$\Rightarrow \quad 4AD^2 = 3AB^2.$$

Hence proved

OR

Let ABCD be a rhombus

Since, diagonals of a rhombus bisect each other at right angles,



$\therefore \quad AO = CO, BO = DO,$

$\angle AOD = \angle DOC$

$= \angle COB = \angle BOA = 90°$

Now, in ΔAOD

$AD^2 = AO^2 + OD^2$ ...(*i*)

Similarly, $DC^2 = DO^2 + OC^2$ ...(*ii*)

$CB^2 = CO^2 + BO^2$ ...(*iii*)

and $BA^2 = BO^2 + AO^2$ ...(*iv*)

Adding equations (*i*), (*ii*), (*iii*) and (*iv*), we have

$AD^2 + DC^2 + CB^2 + BA^2$

$= 2(DO^2 + CO^2 + BO^2 + AO^2)$

$= 2\left(\frac{BD^2}{4}+\frac{AC^2}{4}+\frac{BD^2}{4}+\frac{CA^2}{4}\right)$

$= BD^2 + CA^2.$ **Hence proved**

8. Hint: $BD = DE = EC = \frac{1}{3}BC$

Use Pythagoras Theorem.

9. Statement: In a triangle, if square of the largest side is equal to the sum of the squares of the other two sides, then the angle opposite to the largest side is a right angle.

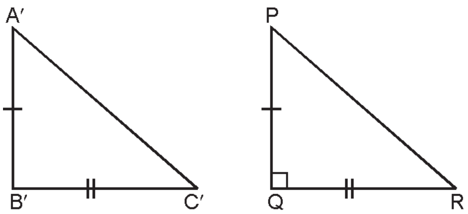
Proof: We are given a triangle ABC with

$A'C'^2 = A'B'^2 + B'C'^2$ ...(*i*)

We have to prove that $\angle B' = 90°$

Let us construct a ΔPQR with $\angle Q = 90°$ such that

$PQ = A'B'$ and $QR = B'C'$ ...(*ii*)



In ΔPQR,

$PR^2 = PQ^2 + QR^2$

(Pythagoras Theorem)

$= A'B'^2 + B'C'^2$ ...(*iii*)

[From (*ii*)]

But $A'C'^2 = A'B'^2 + B'C'^2$ ...(*iv*)

[From (*i*)]

From equations (*iii*) and (*iv*), we have

$PR^2 = A'C'^2$

$\Rightarrow \quad PR = A'C'$ ...(*v*)

Now, in $\Delta A'B'C'$ and ΔPQR,

$A'B' = PQ$ [From (*ii*)]

$B'C' = QR$ [From (*ii*)]

$A'C' = PR$ [From (*v*)]

Therefore, $\Delta A'B'C' \cong \Delta PQR$

(SSS congruence rule)

$\Rightarrow \quad \angle B' = \angle Q$ (CPCT)

But $\angle Q = 90°$

$\therefore \quad \angle B' = 90°.$ **Hence proved.**

2nd Part

In ΔADC, $\angle D = 90°$

$\therefore \quad AC^2 = AD^2 + DC^2 = 6^2 + 8^2$

$= 36 + 64 = 100$

In ΔABC,

$AB^2 + AC^2 = 24^2 + 100 = 676$

and $BC^2 = 26^2 = 676$

Clearly, $BC^2 = AB^2 + AC^2$

Hence, by converse of Pythagoras Theorem, in ΔABC,

$\angle BAC = 90°$

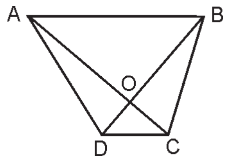
$\Rightarrow \Delta ABC$ is a right triangle.

WORKSHEET – 85

1. $\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$

$\Rightarrow \quad ar(\Delta ADE) = \frac{\left(\frac{2}{3}BC\right)^2}{BC^2} \times 81 = 36 \text{ cm}^2.$

2. $\Delta OAB \sim \Delta OCD$



$\Rightarrow \quad \frac{OA}{OC} = \frac{OB}{OD}$

$\Rightarrow \quad OB = 4 \times \frac{3}{2} = 6 \text{ cm}.$

3. In ΔABC, to make $DE \parallel AB$, we have to take

$$\Rightarrow \quad \frac{AD}{DC} = \frac{BE}{EC} \Rightarrow \frac{3x+19}{x+3} = \frac{3x+4}{x}$$

$\Rightarrow \quad 3x^2 + 19x = 3x^2 + 4x + 9x + 12$

$\Rightarrow \quad 6x = 12 \Rightarrow x = 2.$

4. No,

$\because \quad \Delta FED \sim \Delta STU$

Corresponding sides of the similar triangles are in equal ratio.

$$\therefore \quad \frac{DE}{TU} = \frac{EF}{ST}$$

$$\therefore \quad \frac{DE}{ST} \neq \frac{EF}{TU}.$$

5. $AB \parallel PQ \Rightarrow \frac{AP}{AO} = \frac{BQ}{BO}$ …(*i*)

$AC \parallel PR \Rightarrow \frac{AP}{AO} = \frac{CR}{CO}$ …(*ii*)

From (*i*) and (*ii*), $\frac{BQ}{BO} = \frac{CR}{CO}$

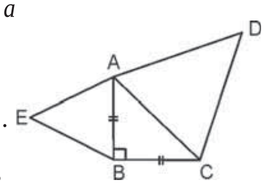
$\Rightarrow BC \parallel QR.$ (By converse of BPT)

6. 1 : 2.

Hint: Let $AB = BC = a$

$\therefore \quad AC = \sqrt{2}\,a$

$$\therefore \quad \frac{ar(\Delta ABE)}{ar(\Delta ACD)} = \frac{AB^2}{AC^2}.$$



7. In ΔABC and ΔAMP,

$\angle A = \angle A$ (Common)

$\angle ABC = \angle AMP = 90°$

(*i*) $\therefore \quad \Delta ABC \sim \Delta AMP,$ (AA criterion)

(*ii*) $\therefore \quad \frac{CA}{PA} = \frac{BC}{MP}.$

($\because$ Corresponding sides of similar triangles are proportional.)

8. Hint:

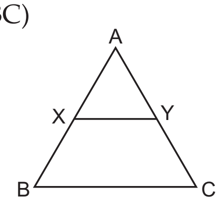
$ar(\Delta AXY) = ar(BXYC)$

$\Rightarrow 2.ar(\Delta AXY) = ar(BXYC) + ar(\Delta AXY)$

$= ar(\Delta ABC)$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta AXY)} = \frac{2}{1}$$

As $\quad \Delta ABC \sim \Delta AXY$



$$\therefore \quad \left(\frac{AB}{AX}\right)^2 = \frac{ar(\Delta ABC)}{ar(\Delta AXY)} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{AB}{AX} = \frac{\sqrt{2}}{1} \Rightarrow \frac{BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}.$$

9. Hint: Prove converse of Pythagoras Theorem.

WORKSHEET – 86

1. $AB^2 = \left(6\sqrt{3}\right)^2 = 108,\ BC^2 = 6^2 = 36$

and $\quad AC^2 = 12^2 = 144$

Now, $\ 108 + 36 = 144$

$\Rightarrow \quad AB^2 + BC^2 = AC^2$

$\Rightarrow \quad \Delta ABC$ is a right-angled triangle, right-angled at B.

2. Since $\Delta ABC \sim \Delta DEF$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} \quad \text{(CPCT)}$$

$$\Rightarrow \quad \frac{4}{DE} = \frac{3.5}{EF} = \frac{2.5}{7.5} = \frac{1}{3}$$

$\Rightarrow \quad DE = 12$ cm

and $\quad EF = 10.5$ cm

Then, perimeter of ΔDEF

$= DE + EF + FD$

$12 + 10.5 + 7.5 = 30$ cm.

3. $$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{EF^2}{BC^2}$$

$$\Rightarrow \quad ar(\Delta DEF) = 54 \times \frac{16}{9} = 96 \text{ cm}^2.$$

4. In ΔABC and ΔADE,

$\angle BAC = \angle DAE$ (Common angle)

$\angle ACB = \angle AED$ (Each 90°)

$\therefore \quad \Delta ABC \sim \Delta ADE$ (AA criterion)

$AB = \sqrt{AC^2 + BC^2} = \sqrt{25 + 144} = 13$ cm

$$\text{Now,} \quad \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\Rightarrow \quad \frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$$

$$\Rightarrow \quad DE = \frac{36}{13} \text{ cm and } AE = \frac{15}{13} \text{ cm.}$$

5. No.

Ratio of areas of two similar triangles

= Square of ratio of their corresponding altitudes

$$= \left(\frac{3}{5}\right)^2 = \frac{9}{25} \neq \frac{6}{5}.$$

Hence, it is not correct to say that ratio of areas of the triangles is $\frac{6}{5}$.

6.
$$AE^2 = AC^2 + EC^2 \quad \ldots(i)$$
$$BD^2 = DC^2 + BC^2 \quad \ldots(ii)$$

Adding (*i*) and (*ii*), we get

$$AE^2 + BD^2 = AC^2 + EC^2 + DC^2 + BC^2$$
$$= (AC^2 + BC^2) + (EC^2 + DC^2)$$
$$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2.$$

Hence proved.

7. In ΔAQO and ΔBPO,

$\angle QAO = \angle PBO$ (Each 90°)

$\angle AOQ = \angle BOP$ (Vertical opposite angles)

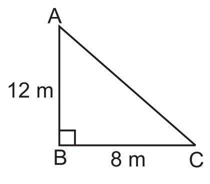
So, by AA rule of similarity,

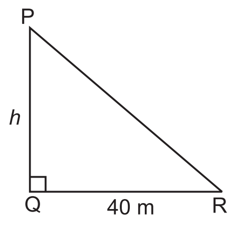
$\Delta AQO \sim \Delta BPO$

$$\Rightarrow \frac{AQ}{BP} = \frac{AO}{BO}$$
$$\Rightarrow \frac{AQ}{9} = \frac{10}{6} \Rightarrow AQ = \frac{10 \times 9}{6}$$
$$\Rightarrow AQ = 15 \text{ cm.}$$

OR

Let the height of the tower be *h* metres

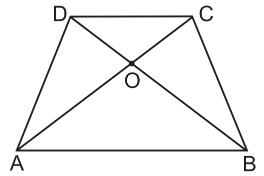




$\Delta ABC \sim \Delta PQR$.

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{12}{h} = \frac{8}{40}$$
$$\Rightarrow h = \frac{12 \times 40}{8} = 60 \text{ metres.}$$

8. Hint: As $\Delta AOB \sim \Delta COD$



$$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2} = \frac{4}{1}.$$

9. Hint: Prove Pythagoras Theorem.

For 2nd Part:

$\therefore \quad AB^2 = AD^2 + BD^2 \quad \ldots(i)$

Also $\quad AC^2 = AD^2 + CD^2 \quad \ldots(ii)$

From (*i*) and (*ii*),

$\Rightarrow AB^2 - AC^2 = BD^2 - CD^2$

$\Rightarrow AB^2 + CD^2 = AC^2 + BD^2.$

Hence proved.

WORKSHEET – 87

1. $\angle M = 180° - (\angle L + \angle N)$ (ASP)

$= 180° - (50° + 60°) = 70°$

$\because \quad \Delta LMN \sim \Delta PQR$

$\therefore \quad \angle M = \angle Q \Rightarrow \angle Q = 70°.$

2. In ΔKMN, as $PQ \parallel MN$,

$$\frac{KP}{PM} = \frac{KQ}{QN}$$
$$\Rightarrow \frac{KP}{PM} = \frac{KQ}{KN - KQ}$$
$$\Rightarrow \frac{KN}{KQ} - 1 = \frac{PM}{KP}$$
$$\Rightarrow \frac{20.4}{KQ} - 1 = \frac{13}{4}$$
$$\Rightarrow \frac{20.4}{KQ} = 1 + \frac{13}{4} = \frac{17}{4}$$
$$\Rightarrow KQ = \frac{20.4 \times 4}{17}$$
$$\Rightarrow KQ = 4.8 \text{ cm.}$$

3. $\Delta ABC \sim \Delta DEF$.

4. $\because \Delta ABC \sim \Delta PQR$

$$\therefore \frac{ar(\Delta PRQ)}{ar(\Delta BCA)} = \frac{QR^2}{BC^2} = \left(\frac{3}{1}\right)^2 = \frac{9}{1} = 9:1.$$

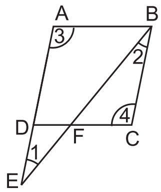
5. True

Hint: Use Basic Proportionality Theorem

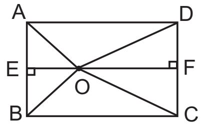
6. Hint:

Use: $\angle 1 = \angle 2$

$\angle 3 = \angle 4$.



7. Draw EOF ∥ AD



$\therefore \quad OB^2 = EO^2 + EB^2$

$OD^2 = OF^2 + DF^2$

$\therefore \quad OB^2 + OD^2 = EO^2 + EB^2 + OF^2 + DF^2$

$= EO^2 + CF^2 + OF^2 + AE^2$

$[\because DF = AE, EB = CF]$

$= (EO^2 + AE^2) + (CF^2 + OF^2)$

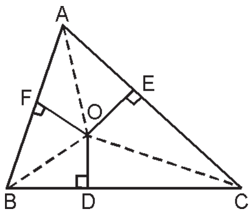
$\Rightarrow OB^2 + OD^2 = OA^2 + OC^2$.

OR

Join OA, OB and OC

In right ΔAOF,

$AO^2 = AF^2 + OF^2 \quad ...(i)$



In right ΔAOE,

$AO^2 = AE^2 + OE^2 \quad ...(ii)$

From equations (*i*) and (*ii*), we have

$AF^2 + OF^2 = AE^2 + OE^2 \quad ...(iii)$

Similarly, we can find out that

$BD^2 + OD^2 = BF^2 + OF^2 \quad ...(iv)$

and $\quad CE^2 + OE^2 = CD^2 + OD^2 \quad ...(v)$

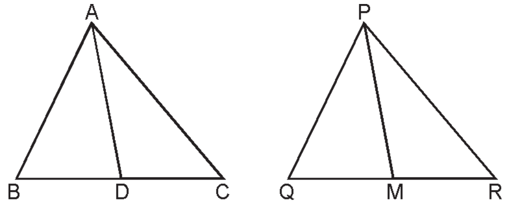
Adding equations (*iii*), (*iv*) and (*v*), we arrive

$AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$.

Hence the result.

8. $\Delta ABC \sim \Delta PQR$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$



$\Rightarrow \quad \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$ and $\angle B = \angle Q$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BP}{QM}$ and $\angle B = \angle Q$

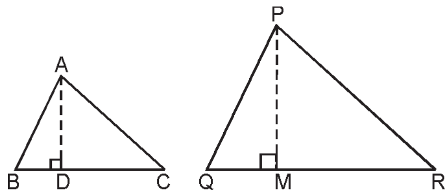
$(\because BD = DC \text{ and } QM = MR)$

$\Rightarrow \quad \Delta ABD \sim \Delta PQM$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM}$. **Hence proved.**

9. Let the two given triangles be ABC and PQR such that $\Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} \quad ...(i)$



Let us draw perpendiculars AD and PM from A and P to BC and QR respectively.

$\therefore \quad \angle ADB = \angle PMQ = 90° \quad ...(ii)$

Now, in ΔABD and ΔPQM,

$\angle B = \angle Q \quad (\because \Delta ABC \sim \Delta PQR)$

$\angle ADB = \angle PMQ \quad$ [From (*ii*)]

So, by AA rule of similarity, we have

$\Delta ABD \sim \Delta PQM$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} \quad ...(iii)$

From equations (*i*) and (*iii*), we get

$\frac{BC}{QR} = \frac{AD}{PM} \quad ...(iv)$

Now, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PM}$

$$= \frac{\frac{1}{2} \times BC \times BC}{\frac{1}{2} \times QR \times QR}$$

[Using (*iv*)]

$$= \left(\frac{BC}{QR}\right)^2 \quad ...(v)$$

Similarly, we can prove that

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 \quad ...(vi)$$

and $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AC}{PR}\right)^2 \quad ...(vii)$$

From equations (*v*), (*vi*) and (*vii*), we obtain

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

$$= \left(\frac{AC}{PR}\right)^2.$$

Hence, the theorem.

Further, in the question,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{15.4 \times 15.4}$$

$$\Rightarrow \quad BC = \sqrt{\frac{64 \times 15.4 \times 15.4}{121}}$$

$$= \frac{8}{11} \times 15.4 = 11.2 \text{ cm.}$$

WORKSHEET – 88

1. Ratio of areas of two similar triangles
= Ratio of squares of their corresponding sides.
$= 4^2 : 9^2 = 16 : 81.$

2. $\angle M = \angle Q = 35°$ (Corresponding angles)

$$\frac{PQ}{ML} = \frac{QR}{MN}$$ (Ratio of corresponding sides)

$$\Rightarrow \quad MN = 5 \times \frac{12}{6} = 10 \text{ cm.}$$

3. In ΔABC, $DE \parallel BC$

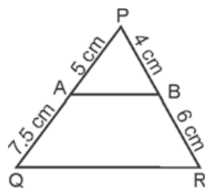
$$\Rightarrow \quad \frac{AB}{AC} = \frac{AD}{AE}$$

$$\Rightarrow \quad AB = 21 \times \frac{5}{7} = 15 \text{ cm.}$$

4. Yes.

$$\frac{AP}{AQ} = \frac{5}{7.5} = \frac{2}{3}$$

$$\frac{BP}{BR} = \frac{4}{6} = \frac{2}{3}$$



Here, $\frac{AP}{AQ} = \frac{BP}{BR}$

Hence, due to the converse of Basic Proportionality Theorem, $AB \parallel QR$.

5. $\because$ $DB \perp BC$ and $AC \perp BC$
$\therefore$ $DB \parallel AC$
Now, $\angle DBA = \angle BAC$ (Alternate angles)
And, $\angle DEB = \angle ACB$ (Each 90°)
$\therefore$ $\Delta BDE \sim \Delta ABC$ (AA similarity)

$$\frac{BE}{AC} = \frac{DE}{BC}$$ (Corresponding sides)

$$\Rightarrow \quad \frac{BE}{DE} = \frac{AC}{BC}.$$ **Hence proved.**

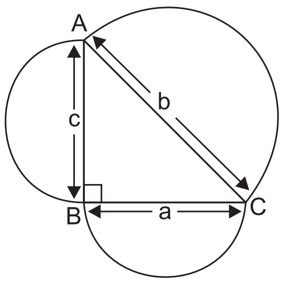
6. $\frac{AX}{AB} = \frac{2-\sqrt{2}}{2}$

Hint: ***See Worksheet–85, Sol. 8.***

7. **Hint:** $AM = \frac{1}{2}AB$; $AL = \frac{1}{2}AC$

Use Pythagoras Theorem.

8. Let ABC be a right-angled triangle such that: $\angle B = 90°$ and $BC = a$; $AB = c$; $AC = b$.



Let semicircles are drawn on side AB, BC and AC of ΔABC

$\therefore$ radius of semicircle drawn on AC = $\frac{b}{2}$

radius of semicircle drawn on AB = $\frac{c}{2}$

radius of semicircle drawn on BC = $\frac{a}{2}$

$\therefore$ Area (A_1) of semicircle with radius

$$\frac{b}{2} = \frac{1}{2}\pi\left(\frac{b}{2}\right)^2 = \frac{\pi b^2}{8}$$

and Area (A_2) of semicircle with radius

$$\frac{c}{2} = \frac{1}{2}\pi\left(\frac{c}{2}\right)^2 = \frac{\pi c^2}{8}$$

also Area (A_3) of semicircle with radius

$$\frac{a}{2} = \frac{1}{2}\pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{8}$$

As ΔABC is a right-angled triangle

$\therefore$ Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad b^2 = c^2 + a^2 \quad ...(i)$$

$$\therefore \quad A_1 = \frac{\pi b^2}{8}$$

$$= \frac{\pi}{8}[c^2 + a^2] \quad \{\because \text{ Using } (i)\}$$

$$= \frac{\pi}{8}c^2 + \frac{\pi}{8}a^2$$

$$A_1 = A_2 + A_3$$

Hence Proved.

9. As D and F are mid-points of AB and AC respectively.

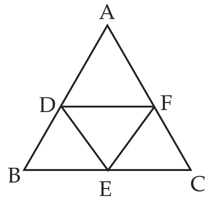
$$\Rightarrow DF \parallel BC \text{ and } DF = \frac{1}{2}BC. \Rightarrow \frac{DF}{BC} = \frac{1}{2}$$

Also, as ΔADF ~ ΔABC

$$\Rightarrow \frac{ar(\Delta ADF)}{ar(\Delta ABC)} = \frac{(DF)^2}{(BC)^2} = \frac{1}{4} \quad ...(i)$$

As $ar(\Delta ADF) = ar(\Delta BDE) = ar(\Delta CFE) = ar(\Delta DEF)$

$$\therefore (i) \Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$



(*ii*) Yes, as DE ∥ AC and DE = $\frac{1}{2}$AC

(Using mid-point theorem)

But as AB = AC = BC

$$DE = \frac{1}{2}BC.$$

(*iii*) Concept of similarity of two triangles and mid-point theorem.

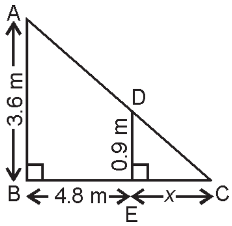
(*iv*) His **ability to think rationally and taking unbiased decision.**

WORKSHEET – 89

1. Let the length of shadow is x metres.

BE = 1.2 × 4 = 4.8 m

ΔABC ~ ΔDEC



$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\Rightarrow \frac{3.6}{0.9} = \frac{4.8 + x}{x}$$

$$3.6x = 4.32 + 0.9x.$$

$$\Rightarrow \quad x = \frac{4.32}{2.7} = 1.6 \text{ m.}$$

2. Here, $(a)^2 + \left(\sqrt{3}a\right)^2 = a^2 + 3a^2$
$= 4a^2 = (2a)^2$

According to the converse of Pythagoras Theorem, the angle opposite to longest side is of measure 90°.

3. $$\frac{AD}{DB} = \frac{2}{3} \Rightarrow \frac{AB - AD}{AD} = \frac{3}{2}$$

$$\Rightarrow \frac{AB}{AD} - 1 = \frac{3}{2} \Rightarrow \frac{AB}{AD} = \frac{5}{2}$$

DE ∥ BC ⇒ ΔABC ~ ΔADE

$$\therefore \quad \frac{BC}{DE} = \frac{AB}{AD} = \frac{5}{2}.$$

4. No.

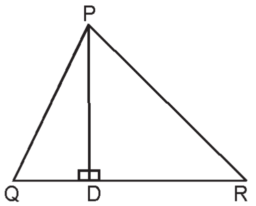
In ΔPQD and ΔRPD,

$\angle PDQ = \angle PDR = 90^\circ$

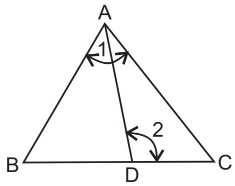
But neither $\angle PQD = \angle RPD$

nor $\angle PQD = \angle PRD$

Therefore, ΔPQD is not similar to ΔRPD.



5. Hint: $\Delta BAC \sim \Delta ADC$



$\Rightarrow \dfrac{BA}{AD} = \dfrac{AC}{DC} = \dfrac{BC}{AC}$

$\Rightarrow CA^2 = BC \times CD.$

6. $\dfrac{AD}{DB} = \dfrac{5}{4} \Rightarrow \dfrac{AD}{AB-AD} = \dfrac{5}{4}$

$\Rightarrow 5AB - 5AD = 4AD \Rightarrow \dfrac{AD}{AB} = \dfrac{5}{9}$ ...(*i*)

As DE || BC,

$\Delta ADE \sim \Delta ABC$

$\therefore \dfrac{DE}{BC} = \dfrac{AD}{AB} = \dfrac{5}{9}$ ...(*ii*)

[Using (*i*)]

$\because$ DE || BC and DC is a transversal

$\therefore \angle EDC \sim \angle BCD$

(Alternate interior angles)

i.e., $\angle EDF = \angle BCF$ ...(*iii*)

Similarly,

$\angle DEF = \angle CBF$ ...(*iv*)

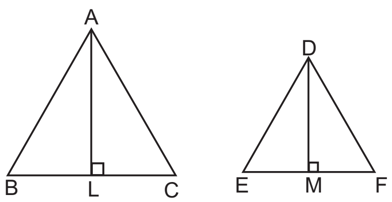
From equations (*iii*) and (*iv*), we have

$\Delta DEF \sim \Delta CBF$ (AA similarity)

$\Rightarrow \dfrac{ar(\Delta DEF)}{ar(\Delta CFB)} = \left(\dfrac{DE}{BC}\right)^2 = \dfrac{25}{81}.$

[Using equation (*ii*)]

7.



AB = AC; DE = DF

$\therefore \dfrac{AB}{AC} = \dfrac{DE}{DF} = 1$

$\Rightarrow \dfrac{AB}{DE} = \dfrac{AC}{DF}$ also $\angle A = \angle D$

$\Rightarrow \Delta ABC \sim \Delta DEF$

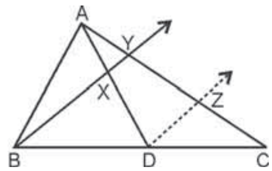
$\therefore \dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AL^2}{DM^2}$

$\Rightarrow \dfrac{AL}{DM} = \dfrac{4}{5}$

$\therefore$ Ratio of corresponding heights is 4:5.

OR

Proof: Draw a ray DZ parallel to the ray XY.



In ΔADZ, XY||DZ

$\therefore \dfrac{AY}{YZ} = \dfrac{AX}{XD} = \dfrac{2}{3}$

$\Rightarrow 2YZ = 3AY$ ...(*i*)

In ΔYBC, BY||DZ

$\therefore \dfrac{YZ}{ZC} = \dfrac{BD}{DC} = \dfrac{1}{1}$ ($\because$ BD = DC)

$\Rightarrow 2YZ = 2ZC$ ...(*ii*)

From (*i*) and (*ii*),

$2ZC = 3AY$ ...(*iii*)

Now, AC = AY + YZ + ZC

$= AY + \dfrac{3}{2}AY + \dfrac{3}{2}AY = \dfrac{8}{2}AY$

$= 4AY$

Therefore, AC: AY = 4:1. **Hence proved.**

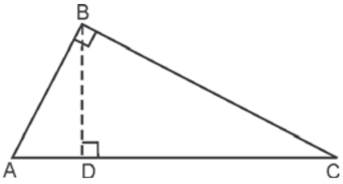
8. $2\sqrt{5}$ cm

Hint: $BD = \dfrac{1}{2}BC$; $EB = \dfrac{1}{2}AB$

Use Pythagoras Theorem.

9. Statement: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Proof: We are given a right triangle ABC right angled at B.



We need to prove that $AC^2 = AB^2 + BC^2$

Let us draw $BD \perp AC$.

Now, $\Delta ADB \sim \Delta ABC$

So, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$ (Sides are proportional)

or $AD \cdot AC = AB^2$ ...(*i*)

Also, $\Delta BDC \sim \Delta ABC$

So, $\dfrac{CD}{BC} = \dfrac{BC}{AC}$

or $CD \cdot AC = BC^2$ ...(*ii*)

Adding equations (*i*) and (*ii*), we get

$$AD \cdot AC + CD \cdot AC = AB^2 + BC^2$$

$$\Rightarrow \quad AC(AD + CD) = AB^2 + BC^2$$

$$\Rightarrow \quad AC \cdot AC = AB^2 + BC^2$$

$$\Rightarrow \quad AC^2 = AB^2 + BC^2.$$

Hence proved.

2nd Part:

$$AB^2 = AD^2 + BD^2$$

$$= AD^2 + (3CD)^2$$

$$= AD^2 + 9CD^2$$

$$= AD^2 + CD^2 + 8CD^2$$

$$= AC^2 + 8CD^2$$

$$= AC^2 + 8\left(\frac{1}{4}BC\right)^2$$

$$\left[\because CD = \frac{1}{4}BC\right]$$

$\therefore \quad 2AB^2 = 2AC^2 + BC^2.$ **Hence proved.**

WORKSHEET – 90

1. $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

($\because \ \Delta ABC \sim \Delta PQR$)

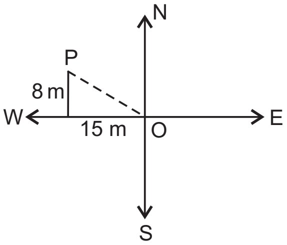
$$\Rightarrow \quad \frac{12}{9} = \frac{7}{x} = \frac{10}{y}$$

$$\therefore \quad x = \frac{7 \times 9}{12} = \frac{21}{4}$$

and $$y = \frac{9 \times 10}{12} = \frac{15}{2}.$$

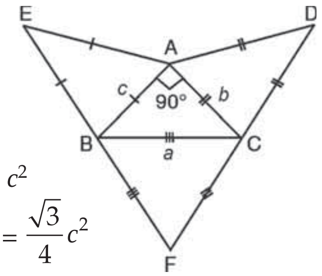
2. Required ratio $= \sqrt{\dfrac{16}{25}} = \dfrac{4}{5} = 4:5.$

3. 17 m

Hint: 

Use Pythagoras Theorem and find OP.

4. **Hint:**



Let $AB = c$

$AC = b$

$BC = a$

$\therefore \quad a^2 = b^2 + c^2$

Also, $ar(\Delta ABE) = \dfrac{\sqrt{3}}{4}c^2$

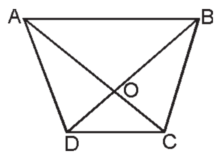
$ar(\Delta BCF) = \dfrac{\sqrt{3}}{4}a^2$

$ar(\Delta ACD) = \dfrac{\sqrt{3}}{4}b^2.$

5. ***See Worksheet – 86, Sol. 6.***

6. Let ABCD be a quadrilateral of which diagonals intersect each other at O.

It is given that



$$\frac{AO}{CO} = \frac{BO}{DO}$$

or $\dfrac{AO}{BO} = \dfrac{CO}{DO}$ ...(*i*)

In ΔAOB and ΔCOD,

$\angle AOB = \angle COD$

(Vertically opposite angles)

$\dfrac{AO}{BO} = \dfrac{CO}{DO}$ [From (*i*)]

Hence, by SAS rule of similarity, we obtain

$\Delta AOB \sim \Delta COD$

$\Rightarrow \angle BAO = \angle DCO$

i.e. $\angle BAC = \angle DCA$

These are alternate angles.

Therefore, AB ∥ CD and AC is transversal

$\Rightarrow$ ABCD is a trapezium. **Hence proved**

OR

Hint:

As $\angle BAC = \angle EFG$; $\angle ABC = \angle FEG$ and $\angle ACB = \angle FGE$

$\therefore \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

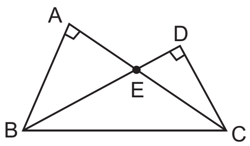
$\therefore \quad \angle ACD = \angle FGH$

and $\quad \angle DCB = \angle HGE$

$\therefore \quad \Delta DCA \sim \Delta HGF$

Similarly, $\Delta DCB \sim \Delta HGE$.

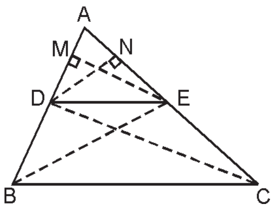
7. Hint:



Prove that $\Delta AEB \sim \Delta DEC$.

8. Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio.

Proof: ABC is a given triangle in which DE ∥ BC. DE intersects AB and AC at D and E respectively.



We have to prove

$$\frac{AD}{BD} = \frac{AE}{CE}$$

Let us draw EM ⊥ AB and DN ⊥ AC. Join BE and CD.

Now, $\quad ar(\Delta ADE) = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times AD \times EM \quad ...(i)$$

Also, $\quad ar(\Delta ADE) = \frac{1}{2} \times AE \times DN \quad ...(ii)$

$$ar(\Delta BDE) = \frac{1}{2} \times BD \times EM \quad ...(iii)$$

$$ar(\Delta CDE) = \frac{1}{2} \times CE \times DN \quad ...(iv)$$

Dividing equation (*i*) by equation (*iii*) and equation (*ii*) by equation (*iv*), we have

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{BD} \quad ...(v)$$

and $\quad \frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{CE} \quad ...(vi)$

But $\quad ar(\Delta BDE) = ar(\Delta CDE) \quad ...(vii)$

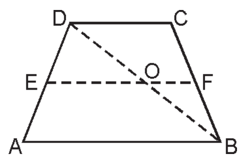
(Triangles are on the same base DE and between the same parallels BC and DE)

Comparing equations (*v*), (*vi*) and (*vii*), we have

$$\frac{AD}{BD} = \frac{AE}{CE}.$$

2nd Part

Join EF and join BD to intersect EF at O.



$\because$ AB ∥ DC, and EF ∥ AB,

$\therefore$ AB ∥ DC ∥ EF

In ΔABD, EO ∥ AB,

$$\frac{DE}{AE} = \frac{DO}{BO} \quad ...(viii)$$

(Basic Proportionality Theorem)

Similarly, in ΔBCD,

$$\frac{DO}{BO} = \frac{CF}{BF} \quad ...(ix)$$

Using equations (*viii*) and (*ix*), we obtain the required result, *i.e.*,

$$\frac{AE}{ED} = \frac{BF}{FC}.$$

WORKSHEET – 91

1. $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$
(*i*) (*ii*) (*iii*)

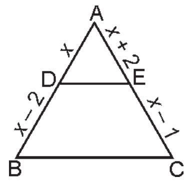
$= \frac{DE + EF + DF}{AB + BC + CA}$
(*iv*)

$\Rightarrow \quad \frac{4}{2} = \frac{\text{Perimeter of } \Delta DEF}{3 + 2 + 2.5}$

[Taking (*ii*) and (*iv*)]

$\Rightarrow$ Perimeter of ΔDEF = 15 cm.

2. DE || BC



$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1}$

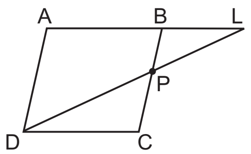
$\Rightarrow \quad x^2 - 4 = x^2 - x$

$\Rightarrow \quad x = 4.$

3. ΔKNP ~ ΔKML

$\Rightarrow \frac{x}{a} = \frac{c}{b+c} \quad \therefore \quad x = \frac{ac}{b+c}.$

4. Hint:



Prove that ΔADL ~ ΔCPD.

5. Hint: 2AP = PC $\Rightarrow$ AP = $\frac{1}{3}$ AC

Similarly, BQ = $\frac{1}{3}$ BC

Use Pythagoras Theorem.

6. PQ || BC $\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC} = \frac{1}{2}$

$\therefore$ ΔAPQ ~ ΔABC

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta APQ)} = \left(\frac{AB}{AP}\right)^2 = (3)^2 = 9$

$\left[\because \frac{AB}{AP} = 3\right]$

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta APQ)} - 1 = 8$

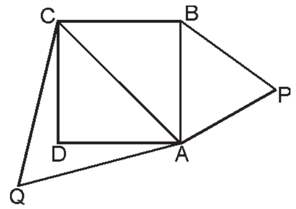
$\Rightarrow \frac{ar(\square BPQC)}{ar(\Delta APQ)} = \frac{8}{1} \Rightarrow \frac{ar(\Delta APQ)}{ar(\square BPQC)} = \frac{1}{8}.$

$\therefore$ Ratio of areas of ΔAPQ and trapezium BPQC is 1 : 8.

OR

Let the given square be ABCD.

Let us draw an equilateral triangle APB and another equilateral triangle AQC on the side AB and on the diagonal AC respectively.



We need to prove

$ar(\Delta APB) = \frac{1}{2}\ ar(\Delta AQC)$

In right ΔABC,

$AC = \sqrt{AB^2 + BC^2}$

$= AB\sqrt{2}$ ...(*i*)

($\because$ AB = BC)

Now, $ar(\Delta APB) = \frac{\sqrt{3}}{4} AB^2$ ...(*ii*)

And $ar(\Delta AQC) = \frac{\sqrt{3}}{4} AC^2$

$= \frac{\sqrt{3}}{4}\left(AB\sqrt{2}\right)^2$

$= \frac{\sqrt{3}}{2} AB^2$ ...(*iii*)

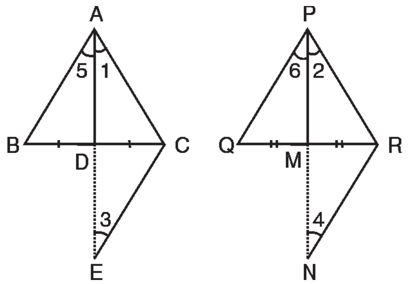
Dividing equation (*ii*) by equation (*iii*), we obtain

$\frac{ar(\Delta APB)}{ar(\Delta AQC)} = \frac{\frac{\sqrt{3}}{4} AB^2}{\frac{\sqrt{3}}{2} AB^2} = \frac{1}{2}$

$\Rightarrow \quad ar(\Delta APB) = \frac{1}{2}\ ar(\Delta AQC).$

Hence proved.

7. Hint:



Extend AD till E such that AD = DE and similarly, PM = MN

Prove that $\Delta ACE \sim \Delta PRN$

$\angle 1 = \angle 2$ ...(*i*)

But $\angle 3 = \angle 5$, (CPCT)

$\angle 3 = \angle 4$ and $\angle 4 = \angle 6$

$\therefore$ $\angle 5 = \angle 6$ ...(*ii*)

Adding (*i*) and (*ii*),

$\angle 1 + \angle 5 = \angle 2 + \angle 6$

$\Rightarrow$ $\angle BAC = \angle QPR$

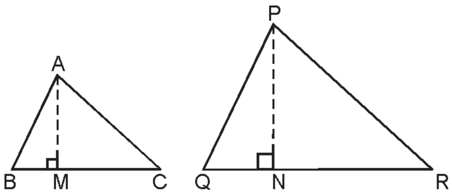
$\therefore$ $\Delta ABC \sim \Delta PQR$. (By SAS)

8. Let us take two similar triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$.

$\therefore$ $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$ ...(*i*)

We need to prove

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$



Let us draw $AM \perp BC$ and $PN \perp QR$.

$\because$ $\Delta ABC \sim \Delta PQR$

$\therefore$ $\angle B = \angle Q$ ...(*ii*)

In ΔABM and ΔPQN,

$\angle B = \angle Q$ [From (*ii*)]

and $\angle M = \angle N$ (Each 90°)

$\therefore$ $\Delta ABM \sim \Delta PQN$ (AA criterion)

$\therefore$ $\dfrac{AB}{PQ} = \dfrac{AM}{PN}$ ...(*iii*)

From equations (*i*) and (*iii*), we have

$\dfrac{AM}{PN} = \dfrac{BC}{QR}$ ...(*iv*)

Now, $ar(\Delta ABC) = \dfrac{1}{2} \times \text{base} \times \text{height}$

$= \dfrac{1}{2} \times BC \times AM$

And $ar(\Delta PQR) = \dfrac{1}{2} \times QR \times PN$

Therefore, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC \times AM}{QR \times PN}$

$= \dfrac{BC^2}{QR^2}$ ...(*v*)

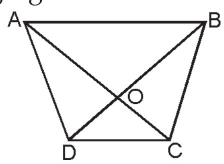
[Using (*iv*)]

From results (*i*) and (*v*), we arrive

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}.$$

Hence the result.

Further, consider the question in the following figure.



$\angle ABO = \angle CDO$ and $\angle BAO = \angle DCO$ (Alternate angles)

$\Rightarrow$ $\Delta AOB \sim \Delta COD$ (AA rule)

$\Rightarrow$ $\dfrac{ar(\Delta AOB)}{ar(\Delta COD)} = \dfrac{AB^2}{CD^2}$

$\Rightarrow$ $ar(\Delta COD) = 84 \times \left(\dfrac{1}{2}\right)^2$ $\left(\because \dfrac{CD}{AB} = \dfrac{1}{2}\right)$

$= 21\,cm^2$.

WORKSHEET – 92

1 . $294\ cm^2$

Hint: Prove that $\Delta OBP \sim \Delta OAQ$.

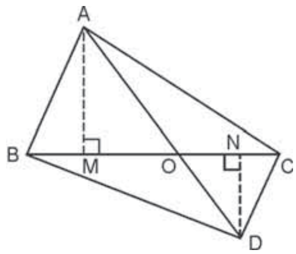
2. 6 cm

Hint: Use AA-similarity to prove $\Delta AOB \sim \Delta COD$.

3. **Hint:** Draw $AM \perp BC$ and $DN \perp BC$

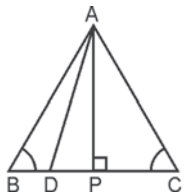
As $\Delta AOM \sim \Delta DON$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN} = \frac{AM}{DN} = \frac{AO}{OD}.$$



4. Hint: Use concept of similarity.

5. Draw $AP \perp BC$



$\therefore \quad AB^2 = AP^2 + BP^2$

$= AP^2 + (BD + DP)^2$

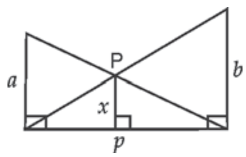
$\Rightarrow \quad AB^2 = AP^2 + BD^2 + DP^2 + 2BD.DP$

$= AD^2 + BD\,(BD + 2DP)$

$\Rightarrow \quad AB^2 - AD^2 = BD \times CD. \quad [\because BP = PC]$

Hence proved.

6. Hint:



From figure, show $x = \dfrac{ab}{a+b}$.

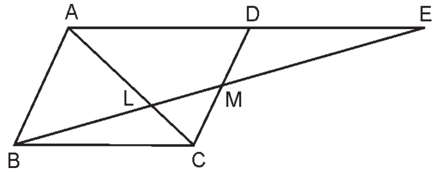
7. In ΔMDE and ΔMCB,

$\angle MDE = \angle MCB$ (Alternate angles)

$MD = MC$ (M is mid-point of CD)

$\angle DME = \angle CMB$

(Vertically opposite angles)



$\therefore \quad \Delta MDE \cong \Delta MCB,$ (ASA criterion)

$\Rightarrow \quad DE = CB$ (CPCT)

$\Rightarrow \quad AE - AD = BC$

$\Rightarrow \quad AE = 2BC \quad ...(i) \; (\because BC = AD)$

Now, in ΔLAE and ΔLCB,

$\Rightarrow \quad \angle LAE = \angle LCB$ (Alternate angles)

$\Rightarrow \quad \angle ALE = \angle CLB$

(Vertically opposite angles)

$\therefore \quad \Delta LAE \sim \Delta LCB$ (AA criterion)

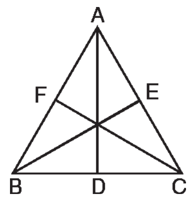
$\Rightarrow \quad \dfrac{AE}{BC} = \dfrac{LE}{BL}$ (Corresponding sides)

$\Rightarrow \quad \dfrac{2BC}{BC} = \dfrac{EL}{BL}$ [Using equation (*i*)]

$\Rightarrow \quad EL = 2BL.$ **Hence proved.**

OR

Hint:



As AD is median

so, $\quad AB^2 + AC^2 = 2(AD^2 + BD^2)$

$\Rightarrow \quad AB^2 + AC^2 = 2\left\{AD^2 + \dfrac{BC^2}{4}\right\}$

$\Rightarrow \quad 2(AB^2 + AC^2) = 4AD^2 + BC^2 \quad ...(i)$

Similarly,

$2(AB^2 + BC^2) = 4BE^2 + AC^2 \quad ...(ii)$

$2(AC^2 + BC^2) = 4CF^2 + AB^2 \quad ...(iii)$

Add (*i*), (*ii*) and (*iii*),

$3(AB^2 + AC^2 + BC^2)$

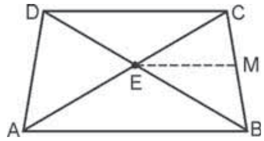
$= 4(AD^2 + BE^2 + CF^2).$

8. ***See Worksheet–91, Sol. 9 (1st part).***

2nd Part: Draw EM || AB

M is a point on CB

$\therefore \quad EM \parallel AB$



In ΔABC,

$\Rightarrow \quad \dfrac{CE}{AE} = \dfrac{CM}{MB} \quad ...(i)$

Also in ΔBCD,

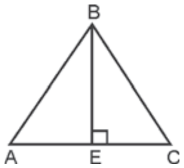
$$\frac{DE}{EB} = \frac{CM}{MB} \quad ...(ii)$$

From (*i*) and (*ii*),

$$\frac{CE}{AE} = \frac{DE}{EB}.$$

WORKSHEET – 93

1. $BE^2 = \frac{3}{4}a^2 \Rightarrow a^2 = \frac{4}{3}BE^2$



$\therefore$ $AB^2 + BC^2 + CA^2$

$= a^2 + a^2 + a^2$

$= 3a^2$

$= 3 \times \left(\frac{4}{3}BE^2\right) = 4\,BE^2.$

2. $\frac{60}{13}$ cm

Hint: Use Pythagoras Theorem.

3. $x = 4$

Hint: Use Basic Proportionality Theorem.

4. Hint: In ΔACD and ΔABC,

$\angle A = \angle A$

$\angle ADC = \angle ACB = 90°$

$\Rightarrow$ $\Delta ACD \sim \Delta ABC$

$\Rightarrow$ $AC^2 = AB \cdot AD$ …(*i*)

$\Delta BCD \sim \Delta BAC$

$\Rightarrow$ $BC^2 = BA \cdot BD$ …(*ii*)

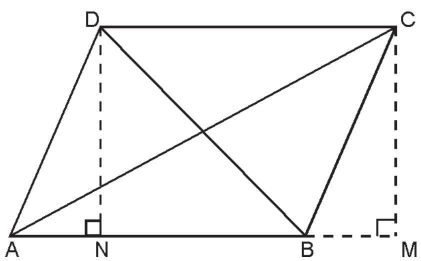
$\therefore$ Applying (*ii*) ÷ (*i*) gives the result.

5. Let the given parallelogram be ABCD

We need to prove that

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

Let us draw perpendiculars DN on AB and CM on AB produced as shown in figure.



In ΔBMC and ΔAND,

BC = AD (Opposite sides of a ||^{gm})

$\angle BMC = \angle AND$ (Each 90°)

CM = DN (Distance between same parallels)

$\therefore$ ΔBMC ≅ ΔAND (RHS criterion)

$\Rightarrow$ BM = AN ...(*i*) (CPCT)

In right triangle ACM,

$AC^2 = AM^2 + CM^2$

$= (AB + BM)^2 + BC^2 - BM^2$

$= AB^2 + 2AB \cdot BM + BM^2 + BC^2 - BM^2$

$= AB^2 + BC^2 + 2AB \cdot BM$ ...(*ii*)

In right triangle BDN,

$BD^2 = BN^2 + DN^2$

$= (AB - AN)^2 + (AD^2 - AN^2)$

$= AB^2 - 2AB \cdot AN + AN^2 + AD^2 - AN^2$

$BD^2 = AB^2 + DA^2 - 2AB \cdot AN$

$\Rightarrow$ $BD^2 = CD^2 + DA^2 - 2AB \cdot BM$ ...(*iii*)

[Using (*i*) and AB = CD]

Adding equations (*ii*) and (*iii*), we have

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2.$

Hence proved.

6. Hint: AP || QB || RC

Use Basic Proportionality Theorem.

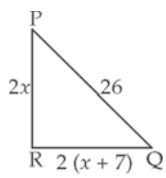
7. (*i*) $PQ^2 = PR^2 + QR^2$

$\Rightarrow$ $(26)^2 = (2x)^2 + \{2(x + 7)\}^2$

$\Rightarrow$ $676 = 4x^2 + 4(x^2 + 49 + 14x)$

$\Rightarrow$ $676 = 4x^2 + 4x^2 + 196 + 56x$

$\Rightarrow$ $8x^2 + 56x - 480 = 0$



$\Rightarrow$ $x^2 + 7x - 60 = 0$

$\Rightarrow$ $x^2 + 12x - 5x - 60 = 0$

$\Rightarrow$ $x(x + 12) - 5(x + 12) = 0$

$\Rightarrow$ $(x - 5)(x + 12) = 0$

$\Rightarrow$ $x = 5$ or $x = -12$ (reject it)

$\Rightarrow$ $x = 5$

$\therefore$ $PR = 2 \times 5 = 10$ km

$QR = 2(5 + 7) = 24$ km

$\therefore$ Before construction of the highway the distance travelled $= 10 + 24 = 34$ km

After construction of the highway the distance travelled = 26 km

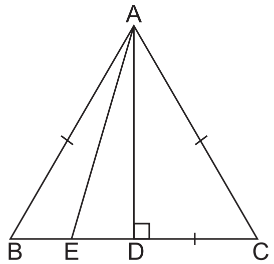
$\therefore$ Distance saved $= 34 - 26 = 8$ km.

(*ii*) Pythagoras theorem

(*iii*) Yes: as it will save time and fuel. Ravi is innovative in his thoughts, so his **rationality and social responsibility is reflected here.**

8. Let ABC be equilateral triangle

Such that $AB = BC = AC$



draw $AD \perp BC$.

also let E be a point on BC such that:

$$BE = \frac{1}{4}BC.$$

Now as ΔADB is a right-angled triangle

$\therefore$ Using pythagoras theorem.

$$AB^2 = AD^2 + BD^2$$

$$= AD^2 + \left(\frac{1}{2}BC\right)^2$$

$$\left(\because AD \perp BC \Rightarrow BD = DC = \frac{1}{2}BC\right)$$

$$= AD^2 + \frac{1}{4}BC^2$$

$$= AD^2 + \frac{1}{4}AB^2$$

$$\Rightarrow AB^2 - \frac{1}{4}AB^2 = AD^2$$

$$\Rightarrow \frac{3}{4}AB^2 = AD^2$$

$$\Rightarrow 3AB^2 = 4AD^2$$

$$= 4[AE^2 - ED^2] \quad (\because \Delta ADE \text{ is rt } \angle\Delta)$$

$$= 4[AE^2 - (BD - BE)^2]$$

$$= 4\left[AE^2 - \left(\frac{1}{2}BC - \frac{1}{4}BC\right)^2\right]$$

$$\left(\because BD = \frac{1}{2}BC\right)$$

$$= 4\left[AE^2 - \left(\frac{1}{4}BC\right)^2\right]$$

$$= 4\left[AE^2 - \frac{1}{16}BC^2\right]$$

$$= 4AE^2 - \frac{1}{4}BC^2$$

$$= 4AE^2 - \frac{1}{4}AB^2 \qquad (\because BC = AB)$$

$$\Rightarrow 3AB^2 + \frac{1}{4}AB^2 = 4AE^2$$

$$\frac{13AB^2}{4} = 4AE^2$$

$$\Rightarrow 13AB^2 = 16\ AE^2$$

$$\Rightarrow 16AE^2 = 13AB^2$$ **Hence Proved.**

OR

ΔABE, ΔACE and ΔADE are right angled triangles right angle at E each.

$$\therefore \quad AB^2 = AE^2 + BE^2 \qquad ...(i)$$

$$AC^2 = AE^2 + CE^2 \qquad ...(ii)$$

$$\text{and} \quad AD^2 = AE^2 + DE^2 \qquad ...(iii)$$

Adding equations (*i*) and (*ii*), we get

$$AB^2 + AC^2 = 2AE^2 + BE^2 + CE^2$$

$$= 2AE^2 + (BD - DE)^2 + (CD + DE)^2$$

$$= 2AE^2 + BD^2 - 2BD \times DE + DE^2 + CD^2 + 2CD \times DE + DE^2$$

$$= 2AE^2 + BD^2 - 2BD \times DE + DE^2 + BD^2 + 2BD \times DE + DE^2$$

$$(\because BD = CD)$$

$$= 2AE^2 + 2DE^2 + 2BD^2$$

$$= 2(AE^2 + DE^2) + 2\left(\frac{BC}{2}\right)^2$$

(D is a mid-point of BC)

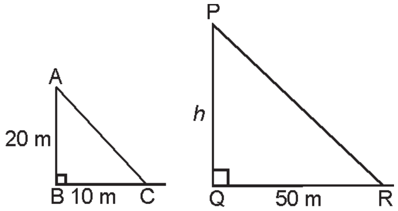
$$= 2AD^2 + \frac{1}{2}BC^2 \qquad [\text{Using } (iii)]$$

Hence proved.

WORKSHEET – 94

1. $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \quad \frac{AP}{AO} = \frac{10}{50}$$



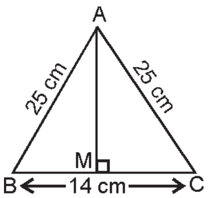
$$\Rightarrow \quad h = \frac{50 \times 20}{10} = 100 \text{ m.}$$

2. The ratio of similar triangles is equal to the ratio of squares of their corresponding altitudes.

$$\therefore \quad \frac{100}{49} = \frac{5^2}{h^2} \Rightarrow h^2 = \frac{25 \times 49}{100}$$

$$\Rightarrow \quad h = \sqrt{\frac{25 \times 49}{100}} \Rightarrow h = \frac{5 \times 7}{10} = 3.5 \text{ cm.}$$

3. Altitude AM divides base BC in two equal parts. That is BM = MC = 7 cm.
Using Pythagoras Theorem
In right ΔABM,



$$AM = \sqrt{25^2 - 7^2} = \sqrt{(25+7)(25-7)}$$

$$= \sqrt{32 \times 18} = 24 \text{ cm.}$$

4. (*i*) We know that diagonal of a square $= \sqrt{2} \times \text{side}$

In square AEFG, $AF = \sqrt{2}\, AG$ ...(*i*)

In square ABCD, $AC = \sqrt{2}\, AD$ ...(*ii*)

Using equations (*i*) and (*ii*), we obtain

$$\frac{AF}{AG} = \frac{AC}{AD}. \quad ...(iii)$$

(*ii*) $\angle GAF = \angle DAC$ (Each 45°)

$\Rightarrow \angle GAF - \angle GAC = \angle DAC - \angle GAC$

$\Rightarrow \angle CAF = \angle DAG$ ...(*iv*)

From equations (*iii*) and (*iv*), we have

$\Delta ACF \sim \Delta ADG$.

(SAS criterion)

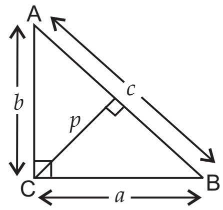
5. **Hint:** $\because \angle 1 = \angle 2$

$\therefore PQ = PR$

$$\therefore \frac{QR}{QS} = \frac{QT}{PQ}.$$

6. **Hint:** Draw $AM \perp BC$ and $DN \perp BC$.

7. **Hint:** Fig.



8. **Hint: For 1st part:** Prove Pythagoras Theorem.

For 2nd part: $AC^2 - AB^2$

$$= (AD^2 + CD^2) - (AD^2 + BD^2)$$

9. **Hint:** Let the $DC = AB = x$

Then $QC = \frac{4}{5}x$ and $AP = \frac{3}{5}x$

$\Delta QRC \sim \Delta PRA$.

OR

See Worksheet – 93, Sol. 5.

WORKSHEET – 95

1. (C) In ΔABC, $PQ \parallel BC$

$$\therefore \quad \frac{\sqrt{3}}{4}c^2 = \frac{AQ}{QC}$$

$$\therefore \quad \frac{2.4}{BP} = \frac{2}{3} \Rightarrow BP = 3.6 \text{ cm}$$

$\therefore \quad AB = AP + BP = 2.4 + 3.6 = 6$ cm.

2.
$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

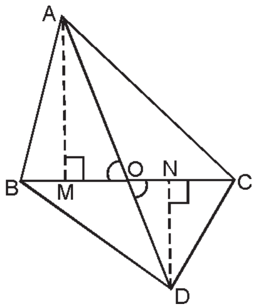
$$\Rightarrow \quad \frac{9}{4} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \quad \frac{BC}{EF} = \frac{3}{2}.$$

3. Draw $AM \perp BC$ and $DN \perp BC$

$\because \quad \angle AMO = \angle DNO = 90°$

and $\quad \angle AOM = \angle DON$



$\therefore \quad \Delta AMO \sim \Delta DNO$

(AA similarity)

$$\therefore \quad \frac{AM}{DN} = \frac{AO}{DO} \quad ...(i)$$

$$\text{Now, } \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$$

$$= \frac{AO}{DO}. \quad \text{[Using } (i)\text{]}$$

$$\text{Therefore } \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO} \quad \textbf{Hence proved.}$$

4. **True**, because $\Delta BCD \sim \Delta CAD$

$\Rightarrow \quad CD^2 = BD \cdot AD.$

5. **Hint:** BMDN is a rectangle.

$\Delta BMD \sim \Delta DMC$

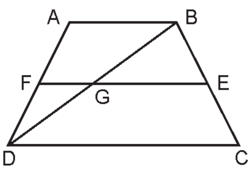
$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \Rightarrow DM^2 = DN \times MC$$

Also, $\Delta BND \sim \Delta DNA$.

$$\Rightarrow \frac{DM}{DN} = \frac{DN}{AN} \Rightarrow DN^2 = DM \times AN.$$

6. Let $BE = 3x$ and $EC = 4x$.

In ΔBCD, $GE \parallel DC$



$\therefore \quad \Delta BGE \sim \Delta BDC$

$$\therefore \quad \frac{BE}{BC} = \frac{GE}{DC}$$

$$\Rightarrow \frac{3x}{3x+4x} = \frac{GE}{2AB} \quad (\because \; DC = 2\,AB)$$

$$\Rightarrow \quad GE = \frac{6}{7}\,AB \quad ...(i)$$

Similarly, $\Delta DGF \sim \Delta DBA$

$$\Rightarrow \quad \frac{FG}{AB} = \frac{4}{7} \Rightarrow FG = \frac{4}{7}\,AB \quad ...(ii)$$

Adding equations (*i*) and (*ii*), we get

$$GE + FG = \frac{6}{7}AB + \frac{4}{7}AB$$

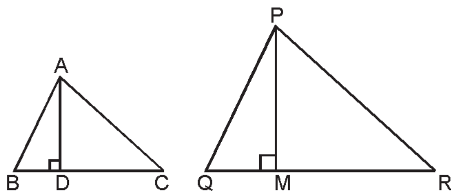
$$\Rightarrow \quad EF = \frac{10}{7}AB$$

$\Rightarrow \quad 7\,EF = 10AB.$ **Hence proved.**

7. ***See Worksheet–91, Sol. 9 (1st part).***

8. We are given two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$.

Draw perpendiculars AD and PM on BC and QR respectively.



We need to prove

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$$

In ΔABD and ΔPQM,

$\angle ADB = \angle PMQ = 90°$

$\angle ABD = \angle PQM \; (\because \Delta ABC \sim \Delta PQR)$

$\therefore \quad \Delta ABD \sim \Delta PQM$

(AA criterion of similarity)

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} \quad ...(i)$$

(Corresponding sides)

We know that the ratio of areas of two similar triangles is equal to ratio of squares of their corresponding sides

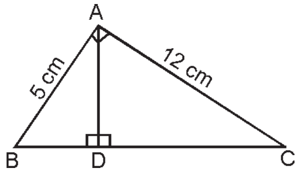
$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}. \quad \textbf{Hence proved.}$$

CHAPTER TEST

1. $BC = \sqrt{5^2 + 12^2} = 13$ cm



$\Delta ABD \sim \Delta CBA$

$\Rightarrow \frac{AB}{BC} = \frac{AD}{AC}$

$\Rightarrow AD = \frac{5 \times 12}{13} = \frac{60}{13}$ cm.

2. $\frac{\Delta_1}{\Delta_2} = \frac{P_1^2}{P_2^2} = \frac{40^2}{50^2} = \frac{16}{25}$

$\Rightarrow \Delta_1 : \Delta_2 = 16 : 25.$

3. $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

$\Rightarrow EC = \frac{3}{1.5} = 2$ cm.

4. Yes.

$MQ = PQ - PM$
$= 15.2 - 5.7 = 9.5$ cm

$NR = PR - PN$
$= 12.8 - 4.8 = 8$ cm

Now, $\frac{PM}{MQ} = \frac{5.7}{9.5} = 0.6$

and $\frac{PN}{NR} = \frac{4.8}{8} = 0.6$

Clearly, $\frac{PM}{MQ} = \frac{PN}{NR}$

$\Rightarrow MN \parallel QR.$

5. $\Delta AOB \sim \Delta COD$ (AAA criterion of similarity)

$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$ (Corresponding sides)

$\Rightarrow \frac{7x-9}{2x-1} = \frac{9x-8}{3x}$

$\Rightarrow 21x^2 - 27x = 18x^2 - 16x - 9x + 8$

$\Rightarrow 3x^2 - 2x - 8 = 0 \Rightarrow (x-2)(3x+4)$

$\Rightarrow x = 2$ or $x = -\frac{4}{3}$

$\Rightarrow x = 2.$ (Negative value rejected)

6. $\therefore \Delta ABE \cong \Delta ACD$

$\therefore AB = AC$ and $AE = AD$ (CPCT)

Consider $AB = AC$

$\Rightarrow AD + DB = AE + EC$

$\Rightarrow DB = EC$ ...(*i*) ($\because AE = AD$)

Also $AD = AE$ ...(*ii*) (Proved above)

Dividing equation (*ii*) by equation (*i*), we have

$\frac{AD}{DB} = \frac{AE}{EC}$ ...(*iii*)

Hence, in ΔABC

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow DE \parallel BC$ (Converse of Basic Proportionality Theorem)

$\Rightarrow \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$

$\Rightarrow \Delta ADE \sim \Delta ABC.$

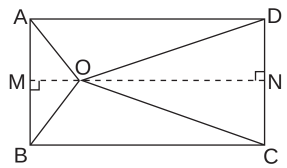
7. Hint:

$\Delta PAC \sim \Delta QBC \Rightarrow \frac{x}{z} = \frac{AC}{BC}$

$\Delta RCA \sim \Delta QBA \Rightarrow \frac{y}{z} = \frac{AC}{AB}.$

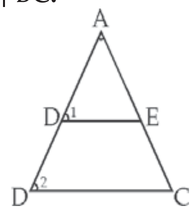
8. Hint:

Draw $MN \parallel AD$, passing through O to intersect AB at M and DC at N.



Use Pythagoras Theorem for ΔAOM, ΔBOM, ΔCON and ΔDON.

9. (*i*) As DE ∥ BC.



$\therefore \quad \angle 1 = \angle 2$ (Corresponding angles)

$\angle A = \angle A$ (Common)

$\Rightarrow \quad \Delta ADE \sim \Delta ABC$ (AA-criterion)

(*ii*) As $\Delta ADE \sim \Delta ABC$

$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{(AD)^2}{(AB)^2} \Rightarrow \frac{1}{2} = \frac{(AD)^2}{(AB)^2}$$

[$\because$ $ar(\Delta ADE) = ar(DECB) \Rightarrow 2\, ar(\Delta ADE) = ar(DECB) + ar(\Delta ADE) \Rightarrow 2\, ar(\Delta ADE) = ar(\Delta ABC) \Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{1}{2}$]

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{AD}{AB} \Rightarrow \frac{1}{\sqrt{2}} - 1 = \frac{AD}{AB} - 1$$

$$\Rightarrow \quad \frac{1-\sqrt{2}}{\sqrt{2}} = \frac{AD - AB}{AB}$$

$$= -\left(\frac{AB - AD}{AB}\right) = -\frac{BD}{AB}$$

$$\Rightarrow \quad \frac{BD}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

Hence proved.

(*iii*) Concept of similarity of two triangles.

(*iv*) **Honesty and rationality** to divide his land equally between his two children.

❑❑

Chapter 10

CIRCLES

WORKSHEET – 98

1. BC = BQ + QC

as BQ = BP = 3 cm

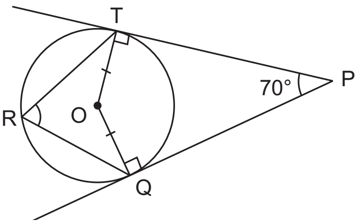
and QC = RC = AC – AR

= 11 – AP {$\because$ AR = AP}

= 11 – 4 = 7 cm

$\therefore$ BC = 3 + 7 = 10 cm.

2. Join OT and OQ.



As OT = OQ = radius

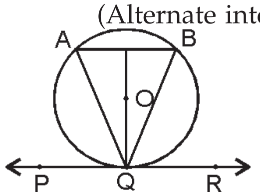
$\therefore$ $\angle TOQ = 180° - 70° = 110°$

$\therefore$ $\angle TRQ = \frac{1}{2} \times \angle TOQ = \frac{1}{2} \times 110° = 55°$.

3. As AB ∥ PR

$\Rightarrow$ $\angle BQR = \angle ABQ = 70°$

(Alternate interior angles)



Also $\angle ABQ = \angle BAQ = 70°$

{$\because$ $\Delta AMQ \cong \Delta BMQ$}

$\therefore$ In ΔAQB, using Angle sum property

$\angle AQB = 180° - 70° - 70° = 40°$.

4. False

Perimeter of ΔABC = AB + BC + AC

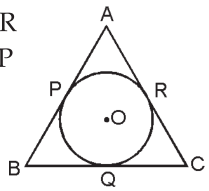
= AB + BQ + CQ + CR + AR

= AB + BP + CQ + CQ + AP

= AB + (BP + AP) + 2CQ

= 2(AB + CQ)

= 2(8) = 16 cm.



5. LHS

= AB + CD

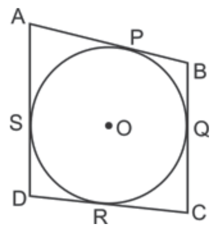
= (AP + PB) + (CR + RD)

= AS + BQ + CQ + DS

= (AS + DS) + (BQ + CQ)

= AD + BC

= RHS.



6. ***See solved example 4.***

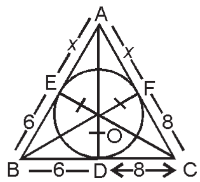
7. AB = 13 cm, AC = 15 cm

Hint:

Use

$ar(\Delta OBC + \Delta OAC + \Delta OAB)$

$= ar(\Delta ABC)$.



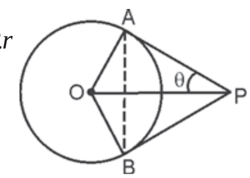
8. Hint: Join AB

Let OA = r $\Rightarrow$ OP = $2r$

In ΔOAP,

$\sin\theta = \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2}$

$\theta = 30°$



$\therefore$ $\angle APB = 2 \times 30° = 60°$

In ΔABP, AP = BP

$\therefore$ $\angle A = \angle B$

But, $\angle A + \angle B = 180° - \angle APB = 120°$

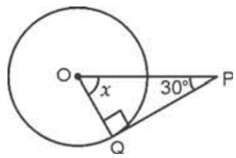
$\Rightarrow$ $\angle A = \angle B = 60°$

$\therefore$ ΔAPB is an equilateral triangle.

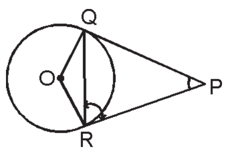
WORKSHEET – 99

1. As $\angle OQP = 90°$

$\therefore$ $x = 90° - 30° = 60°$.



2. $\angle QOR = 180° - 30° = 150°$



$\angle PRQ = \frac{1}{2} \times 150° = 75°$.

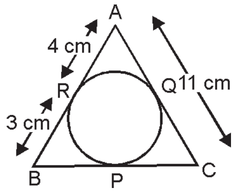
3. BC = BP + PC

= BR + CQ

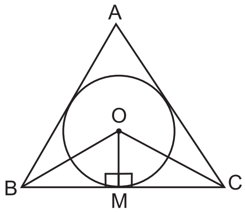
= 3 + [AC – AQ]

= 3 + [11 – 4]

= 10 cm.



4. True.



Let M be the point of contact and O be the centre of the circle.

$\angle ABM = \angle ACM$

($\because$ AB = AC)

$\frac{1}{2}\angle ABM = \frac{1}{2}\angle ACM$

$\angle OBM = \angle OCM$ ...(*i*)

$\angle BMO = \angle CMO$ ...(*ii*) (Each 90°)

OM = OM ...(*iii*) (Common)

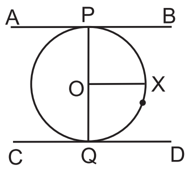
Using equations (*i*), (*ii*) and (*iii*) in ΔBMO and ΔCMO, we have

$\Delta BMO \cong \Delta CMO$ (AAS corollory)

$\therefore$ BM = CM (CPCT)

$\Rightarrow$ BC is bisected at the point of contact.

5. Let AB and CD be two parallel tangents to a circle with centre O.



Join OP and OQ.

Draw OX parallel PB and QD.

$\Rightarrow$ $\angle BPO + \angle XOP = 180°$

[Sum of the angles on the same side of a transversal is 180°]

$\Rightarrow$ $90° + \angle XOP = 180°$

[$\therefore \angle BPO$ = angle between a tangent and radius = 90°]

$\Rightarrow$ $\angle XOP = 90°$

Similarly, $\angle XOQ = 90°$

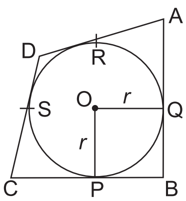
$\therefore$ $\angle XOP + \angle XOQ = 90° + 90° = 180°$

Hence, POQ is a straight line passing through O.

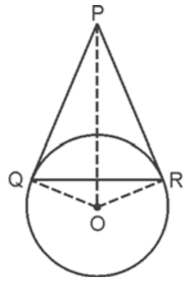
6. 11 cm

Hint: OQBP is a square

$\therefore$ OQ = BP = 11 cm.



7. Let the tangents be PQ and PR corresponding to the chord QR of the circle with centre O.



Join OQ, OR and OP.

In ΔPQO and ΔPRO,

$\angle PQO = \angle PRO = 90°$

(Angles formed between tangent and corresponding radius)

PO = PO (Common)

QO = RO (Radii of same circle)

Therefore, we arrive at

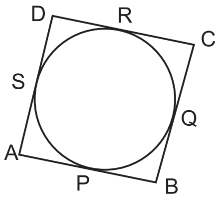
$\Delta PQO \cong \Delta PRO$(RHS axiom of congruence)

So, PQ = PR ($\therefore$ CPCT)

Thus, ΔPQR is an isosceles triangle.

$\therefore$ $\angle PQR = \angle PRQ$. **Hence proved.**

8. Let the given parallelogram be ABCD whose sides touches a circles at P, Q, R and S as shown in the adjoining figure.



Since, length of two tangents drawn from an external point to a circle are equal.

$\therefore$ AP = AS ...(*i*)

Similarly, we have

PB = BQ ...(*ii*)

DR = SD ...(*iii*)

RC = QC ...(*iv*)

Adding these four equations, we have

AP + PB + DR + RC = AS + BQ + SD + QC

$\Rightarrow$ (AP + PB) + (DR + RC)

= (AS + SD) + (BQ + QC)

$\Rightarrow$ AB + DC = AD + BC

$\because$ AB = DC and AD = BC

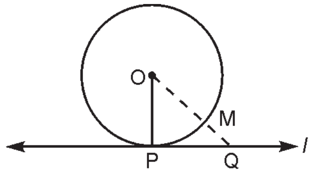
(ABCD is a parallelogram)

$\therefore$ AB = BC

Thus, AB = BC = CD = DA

Hence, ABCD is a rhombus.

9. Let line *l* be the tangent at a point P to the circle with centre O. Let us take any point Q on the tangent *l* as shown in the figure.



Join OQ to meet the circle at M.

We know that if a point is met with the different points of a line, then the shortest line segment is the perpendicular on that line. Consider the adjoining figure:

OM = OP (Radii of same circle)

OQ = OM + MQ

$\Rightarrow$ OQ = OP + MQ

$\Rightarrow$ OQ > OP

i.e., OP < OQ

Clearly, OP is the shorter than OQ. Similarly, we can prove that OP is the shortest all OV, V being a variable point on the line other than P. Therefore, OP is the perpendicular to line *l*.

Hence, tangent $l \perp$ radius OP.

2nd Part: Join OY

$\angle OYX = 90^\circ$

and $\angle OAY = b + a = \angle OYA$

[$\because$ OA = OY = radius]

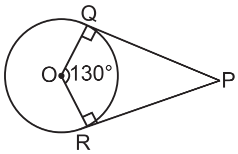
$\Rightarrow$ $b + a = 90^\circ - a$

$\Rightarrow$ $b + 2a = 90^\circ$.

WORKSHEET – 100

1. $\angle Q = \angle R = 90^\circ$

In quadrilateral PQOR,



$\angle P = 360^\circ - (90^\circ + 130^\circ + 90^\circ) = 50^\circ$

2. Join CA and CB

as CA $\perp$ AP

CB $\perp$ PB

and AP $\perp$ PB $\Rightarrow$ CA $\perp$ CB

$\therefore$ CA = CB and AP = PB

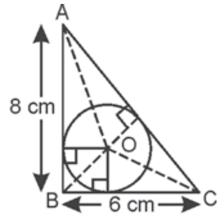
$\Rightarrow$ CAPB is a square

$\therefore$ AP = PB = CA = CB = 4 cm

3. Hint: AC = 10 cm

$\therefore$ $ar(\Delta ABC)$

$= ar(\Delta AOB) + ar(\Delta BOC) + ar(\Delta AOC)$



$\Rightarrow$ $24 = \frac{1}{2} \times (8 \times r + 6 \times r + 10 \times r)$

$\Rightarrow$ $48 = r \times 24$

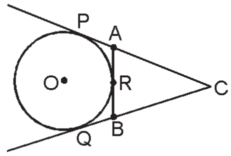
$\Rightarrow$ $r = 2$ cm.

4. CP = CQ = 11 cm

BQ = CQ – CB

= 11 – 7 = 4 cm

$\therefore$ BR = QB = 4 cm.

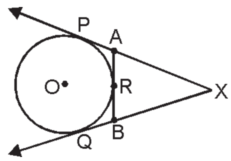


5. Hint: XP = XQ

$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR

$\left\{ \begin{array}{l} \because \text{ AP = AR} \\ \text{and BQ = BR} \end{array} \right.$.



6. ***See Worksheet–99, Sol. 8.***

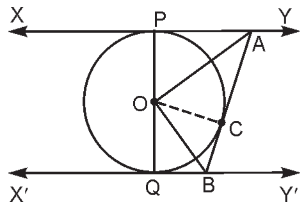
7. Hint:

As PA = AC

$\therefore$ $\Delta PAO \cong \Delta CAO$ (SSS)

$\Rightarrow$ $\angle PAO = \angle CAO$ (CPCT)

$\Rightarrow$ $\angle PAC = 2\angle CAO$



Similarly,

$\angle CBQ = 2\angle CBO$

As $\angle PAC + \angle CBQ = 180^\circ$

$\Rightarrow$ $\frac{1}{2}\angle PAC + \frac{1}{2}\angle CBQ = \frac{1}{2} \times 180^\circ$

$\Rightarrow$ $\angle CAO + \angle CBO = 90^\circ$

$\therefore$ $\angle AOB = 90^\circ$.

8. (*i*) $PA \cdot PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$

(As AN = BN)

$= PN^2 = AN^2$

(*ii*) $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$
(As $ON \perp PN$)
$= OP^2 - (ON^2 + AN^2)$
$= PO^2 = OA^2$ (As $ON \perp AN$)
$= OP^2 - OT^2$ (As $OA = OT$)

(*iii*) From (*i*) and (*ii*)
$PA \cdot PB = OP^2 - OT^2$
$= PT^2$ (As $\angle OTP = 90°$)

WORKSHEET – 101

1. Let P is point of contact as $OP \perp AB$.

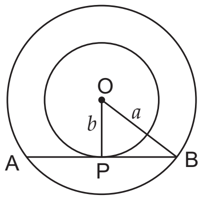
and $OP = b$, $OB = a$

$\Rightarrow$ $OB^2 = OP^2 + BP^2$

$a^2 = b^2 + BP^2$

$\Rightarrow$ $BP^2 = a^2 - b^2$

$BP = \sqrt{a^2 - b^2}$



$\therefore$ $AB = 2BP = 2\sqrt{a^2 - b^2}$.

2. $\angle QOR = 180° - 46° = 134°$

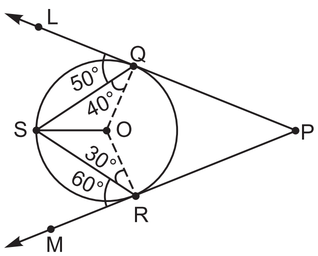
3. False.

$\because$ $\angle OQL = 90°$

$\therefore$ $\angle OQS = 90° - \angle SQL = 40°$

Similarly,

$\angle ORS = 90° - \angle SRM = 30°$



$\because$ In ΔSOQ, $\angle OSQ = \angle OQS = 40°$

And in ΔSOR, $\angle OSR = \angle ORS = 30°$

$\therefore$ $\angle QSR = OSQ + \angle OSR$
$= 40° + 30° = 70°$.

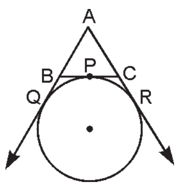
4. Perimeter of ΔABC

$= AB + BC + AC$

$= (AQ - BQ) + BC + (AR - CR)$

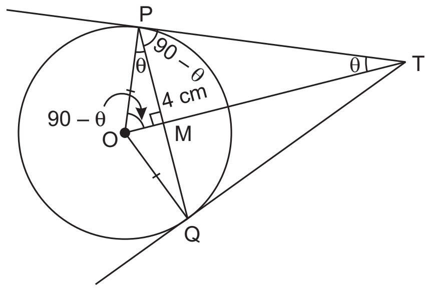
$= AQ + AR + BC - (BP + PC)$

$= 2AQ$ [$\because$ $AQ = AR$]



$\therefore$ $AQ = AR = \frac{1}{2}$(Perimeter of ΔABC).

5. Join OT and let OT intersect PQ at M.



$\therefore$ In ΔPMT and ΔQMT,

$PT = QT$

($\because$ Lengths of tangent from an external point to a circle are equal)

$TM = TM$ (Common)

$\angle PTM = \angle QTM$

($\because$ Tangents are equally inclined to line joining external point to circle)

$\therefore$ $\Delta PMT \cong \Delta QMT$ (SAS)

$\Rightarrow$ $PM = MQ = \frac{1}{2} PQ$

$\Rightarrow$ $PM = \frac{1}{2} \times 8 = 4$ cm

Also, $OM \perp PQ$

$\Rightarrow$ $\angle TMP = 90°$.

Let $\angle PTM = \theta$ ...(*i*)

$\therefore$ $\angle TPM = 90 - \theta$
...(*ii*) {$\because$ $\angle TMP = 90°$)

Also, In right ΔPMO,

$\angle OPM = \theta$ ...(*iii*)

$\therefore$ $\angle POM = 90 - \theta$ ...(*iv*)

$\therefore$ In ΔPOM and ΔTPM,

$\angle PTM = \angle MPO = \theta$
(From (*i*) and (*iii*))

and $\angle TPM = \angle POM = 90 - \theta$
(From (*ii*) and (*iv*))

$\Rightarrow$ $\Delta POM \sim \Delta TPM$ (AA)

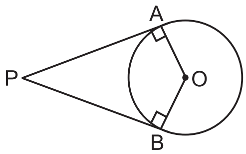
$\Rightarrow$ $\frac{PO}{TP} = \frac{OM}{PM}$ $\left\{\begin{array}{l} \because OM = \sqrt{OP^2 - PM^2} \\ = \sqrt{25 - 16} = \sqrt{9} \\ = 3 \text{ cm} \end{array}\right\}$

$\Rightarrow$ $\frac{5}{TP} = \frac{3}{4}$

$\Rightarrow$ $TP = \frac{20}{3}$ cm = TQ.

6. ***See Worksheet – 98, Sol. 5.***

7. Let the given two tangents be PA and PB to the circle with centre O.



We need to prove $\angle APB + \angle AOB = 180°$.

We know that the angle formed by a tangent to the circle and the radius passing through the point of contact is 90°.

$\therefore \quad \angle PAO = \angle PBO = 90°$

Applying angle sum property in the quadrilateral AOBP, we get

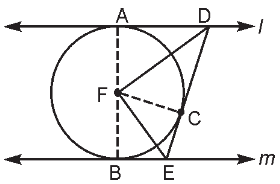
$\angle PAO + \angle AOB + \angle PBO + \angle APB = 360°$

$\Rightarrow \quad 90° + \angle AOB + 90° + \angle APB = 360°$

$\Rightarrow \quad \angle AOB + \angle APB = 180°.$

Hence proved.

8. We have given $l \parallel m$ to a circle. DE is intercept made by tangent at C, betw-een l and m.



We have to prove $\angle DEF = 90°$

Construction: Join A to F, F to B and F to C.

Proof: In a triangles ADF and DFC, we have

DA = DC
(Tangents drawn from an external point are equal in length)

DF = DF (Common)

AF = CF (Radii of the same circle)

$\therefore \quad \Delta ADF \cong \Delta CDF$ (SSS)

$\Rightarrow \quad \angle ADF = \angle CDF$ (CPCT)

$\Rightarrow \quad \angle ADC = 2\angle CDF$ ... (*i*)

Similarly, $\angle CEB = 2\angle CEF$ ... (*ii*)

Now, $\angle ADC + \angle CEB = 180°$

Sum of the interior angles on the same side of transversal is 180°.

$\Rightarrow \quad 2\angle CDF + 2\angle CEF = 180°$

$\Rightarrow \quad \angle CDF + \angle CEF = 90°$ ...(*iii*)

In ΔDEF,

$\angle DEF + \angle EDF + \angle DFE = 180°$

$\Rightarrow \quad 90° + \angle DFE = 180°$ [From (*iii*)]

$\Rightarrow \quad \angle DFE = 90°.$

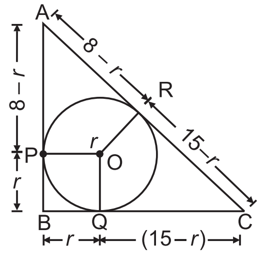
Hence proved.

WORKSHEET – 102

1. Let radius = r

$\therefore \quad PO = PB = BQ = r$

$AB = \sqrt{17^2 - 15^2} = 8$ cm



Now,

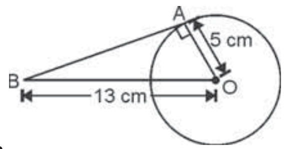
$AC = AR + RC$

$\Rightarrow \quad 17 = 8 - r + 15 - r$

$\Rightarrow \quad 2r = 23 - 17 = 6$

$\Rightarrow \quad r = 3$ cm.

2. $AB = \sqrt{OB^2 - OA^2}$

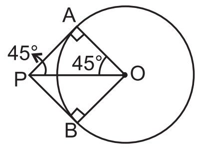


$= \sqrt{169 - 25}$

$= \sqrt{144} = 12$ cm.

3. True, because in right angled isosceles triangle AOB,

$OP = \sqrt{a^2 + a^2} = a\sqrt{2}$



4. In ΔAPQ,

$\angle PAQ = \angle AQP = \theta$ (say)

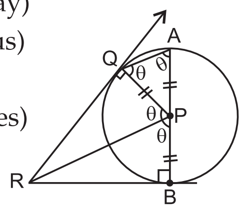
($\because$ AP = PQ = radius)

$\angle RPB = \angle QAP = \theta$

(Corresponding angles)

$\angle RPQ = \angle AQP = \theta$

(Alternate angles)



Now, in ΔRPQ and ΔRPB,

RP = RP (Common)

$\angle RPQ = \angle RPB$ (Each θ)

PQ = PB (Each radius)

So, by SAS creterion of similarity, we have

$\Delta RPQ \sim \Delta RPB$

$\therefore \quad \angle RBP = \angle RQP$

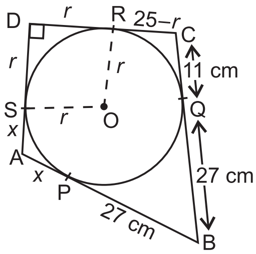
But $RQ \perp PQ$,

$\therefore \quad \angle RQP = 90°$

$\therefore \quad \angle RBP = 90°$

$\Rightarrow$ BR is tangent at B. **Hence proved.**

5. Join OR and OS.



Let $\qquad AP = x \therefore AS = x$

In quadrilateral OSDR,

$\angle O + \angle S + \angle D + \angle R = 360°$

$\Rightarrow \angle O + 90° + 90° + 90° = 360°$

($\because$ OS $\perp$ AD and OR $\perp$ CD)

$\Rightarrow \qquad \angle O = 90°$

$\Rightarrow$ OSDR is a square.

$\Rightarrow$ DR = DS = r

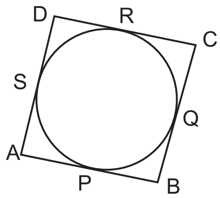
Now, ABCD is a subscribed quadrilateral

$\therefore \quad AB + CD = BC + DA$

$\Rightarrow x + 27 + 25 = 38 + r + x$

$\Rightarrow \qquad r = 14$ cm.

6. Let the given parallelogram be ABCD whose sides touches a circles at P, Q, R and S as shown in the adjoining figure.



Since, length of two tangents drawn from an external point to a circle are equal.

$\therefore \qquad AP = AS \qquad$ ...(*i*)

Similarly, we have

$PB = BQ \qquad$ ...(*ii*)

$DR = SD \qquad$ ...(*iii*)

$RC = QC \qquad$ ...(*iv*)

Adding these four equations, we have

$AP+PB+DR+RC = AS + BQ + SD + QC$

$\Rightarrow (AP + PB) + (DR + RC)$

$= (AS+SD)+(BQ+QC)$

$\Rightarrow \qquad AB + DC = AD + BC$

$\because$ AB = DC and AD = BC

(ABCD is a parallelogram)

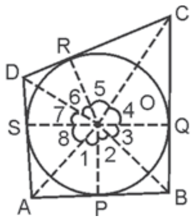
$\therefore \qquad AB = BC$

Thus, AB = BC = CD = DA

Hence, ABCD is a rhombus.

OR

Let the given quadrilateral be ABCD subscribing a circle with centre O. Let the sides AB, BC, CD and DA touch the circle at P, Q, R and S respectively (see figure).



Join OA, OB, OC, OD, OP, OQ, OR and OS.

We need to prove

$\angle AOB + \angle COD = \angle BOC + \angle DOA = 180°$.

Proof: In ΔAOP and ΔAOS,

OP = OS $\qquad$ (Radii of same circle)

AP = AS (Tangents from external points)

AO = AO $\qquad$ (Common)

$\therefore \Delta AOP \cong \Delta AOS$(SSS axiom of congruence)

$\therefore \angle 1 = \angle 8 \qquad$ ...(*i*) (CPCT)

Similarly, we can prove that

$\angle 2 = \angle 3, \angle 4 = \angle 5$ and $\angle 6 = \angle 7 \qquad$ ...(*ii*)

As, $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7$ and $\angle 8$ are subtended at a point

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 2 + \angle 2 + \angle 5 + \angle 5 + \angle 6 + \angle 6 = 360°$

Also, $\angle 8 + \angle 8 + \angle 3 + \angle 3 + \angle 4 + \angle 4 + \angle 7 + \angle 7 = 360°$

[Using results from equations (*i*) and (*ii*)]

$\Rightarrow \qquad 2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$

Also, $\quad 2(\angle 3 + \angle 4) + 2(\angle 7 + \angle 8) = 360°$

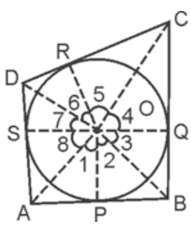
$\Rightarrow \qquad 2\angle AOB + 2\angle COD = 360°$

Also, $\qquad 2\angle BOC + 2\angle DOA = 360°$

$\Rightarrow \angle AOB + \angle COD = \angle BOC + \angle DOA = 180°$

Hence proved.

7. Let the given quadrilateral be ABCD subscribing a circle with centre O. Let the sides AB, BC, CD and DA touch the circle at P, Q, R and S respectively (see figure).



Join OA, OB, OC, OD, OP, OQ, OR and OS.

We need to prove

$\angle AOB + \angle COD = \angle BOC + \angle DOA = 180°$.

Proof: In ΔAOP and ΔAOS,

OP = OS (Radii of same circle)

AP = AS (Tangents from external points)

AO = AO (Common)

$\therefore \Delta AOP \cong \Delta AOS$ (SSS axiom of congruence)

$\therefore \angle 1 = \angle 8$ ...(*i*) (CPCT)

Similarly, we can prove that

$\angle 2 = \angle 3$, $\angle 4 = \angle 5$ and $\angle 6 = \angle 7$ ... (*ii*)

As, $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7$ and $\angle 8$ are subtended at a point

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 2 + \angle 2 + \angle 5 + \angle 5 + \angle 6 + \angle 6 = 360°$

Also, $\angle 8 + \angle 8 + \angle 3 + \angle 3 + \angle 4 + \angle 4 + \angle 7 + \angle 7 = 360°$

[Using results from equations (*i*) and (*ii*)]

$\Rightarrow 2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$

Also, $2(\angle 3 + \angle 4) + 2(\angle 7 + \angle 8) = 360°$

$\Rightarrow 2\angle AOB + 2\angle COD = 360°$

Also, $2\angle BOC + 2\angle DOA = 360°$

$\Rightarrow \angle AOB + \angle COD = \angle BOC + \angle DOA = 180°$

Hence proved.

8. ***Try yourself***

WORKSHEET – 103

1. Perimeter of ΔEOF = ED + DF + EF

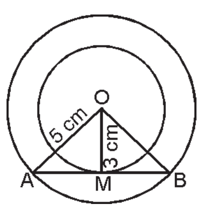
= (ED + DH) + (HF + EF)

= (ED + DK) + (FM + EF)

= EK + EM = 2EK = 2 × 9

= 18 cm.

2. Hint: AB = 2AM



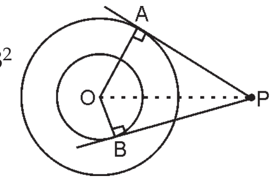
3. AP = 15 cm; OA = 8 cm; OB = 5 cm

$\therefore$ as OA ⊥ AP

$\Rightarrow \quad OP = \sqrt{OA^2 + AP^2} = \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{289} = 17$

$\therefore$ as OB ⊥ BP



$\Rightarrow \quad BP^2 = OP^2 - OB^2$

$= 17^2 - 5^2$

$= 289 - 25$

$= 264$

$\therefore \quad BP = \sqrt{264}$ cm $= 2\sqrt{66}$ cm.

4. 16 cm

Given: AP = 5 cm

$\Rightarrow$ BP = 12 − 5 = 7 cm

also AP = 5 cm = AQ

$\therefore$ QC = 14 − 5 = 9 cm

$\therefore$ BC = BR + RC

= BP + CQ = 7 + 9 = 16 cm.

5. ***See Worksheet – 102, Sol. 7.***

6. ***See Worksheet – 100, Sol. 7.***

7. ***For proof of theorem see solved example 4.***

8. Join OB, OG, OA, OH and OC.

Radius = OD = OG = OH = 4 cm

HC = DC = 6 cm

BG = BD = 8 cm

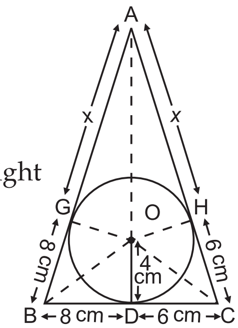
Let AG = AH = x

ar(ΔOBC)

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 14 \times 4$

$= 28\ cm^2$



$ar(\Delta OAB) = \frac{1}{2} \times (x + 8) \times 4$

$= (2x + 16)\ cm^2$

$ar(\Delta OBC) = \frac{1}{2} \times (x + 6) \times 4$

$= (2x + 12)\ cm^2$

$\therefore\ ar(\Delta ABC) = 28 + 2x + 16 + 2x + 12$

$\Rightarrow ar(\Delta ABC) = (4x + 56)\ cm^2$ ...(*i*)

In ΔABC, $s = \frac{AB + BC + CA}{2}$

$= \frac{x + 8 + 14 + x + 6}{2} = x + 14$

$\therefore\ ar(\Delta ABC) = \sqrt{s(s-AB)(s-BC)(s-CA)}$

$\Rightarrow ar(\Delta ABC) = \sqrt{(x+14)\times 6\times x\times 8}$ ...(*ii*)

Comparing equations (*i*) and (*ii*), we get

$4x + 56 = \sqrt{(x+14)\times 6\times x\times 8}$

$\Rightarrow 4^2\ (x + 14)^2 = (x + 14) \times 6 \times x \times 8$

(On squaring both sides)

$\Rightarrow 16\ (x + 14)\ (x + 14 - 3x) = 0$

$\Rightarrow x = 7$ as $x \neq -14$ $(\because\ x > 0)$

So, AB = $x + 8 = 7 + 8 = 15$ cm

and AC = $x + 6 = 7 + 6 = 13$ cm.

WORKSHEET – 104

1. QR = QP + PR

= PT + PT [∵ PT = PQ = PR]

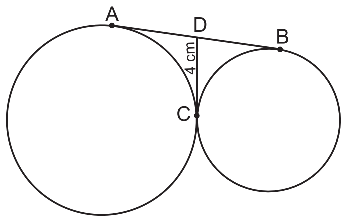
= 2 (PT) = 2 × 3.8 [∵ PT = 3.8 cm]

= 7.6 cm.

2. ∠BAT = ∠ACB = 55°.

3. AD = DC = 4

⇒ AD = 4 cm



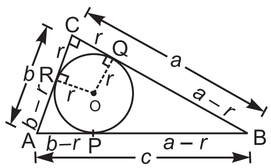
Similarly CD = DB = 4 cm

⇒ DB = 4 cm

∴ AB = AD + DB = 4 + 4 = 8 cm.

4. False, because the centres of the circles lie on the perpendicular of PQ, which passes through A.

5. Let the sides AB, BC and CA of the ΔABC touch the circle with centre O at the point P, Q and R respectively.



In quadrilateral OQCR,

∠OQC = ∠ORC = 90°

(Angles between tangent and corresponding radius)

and ∠QCR = 90° (Given)

⇒ ∠QOR = 90°

⇒ OQCR is a square

⇒ CQ = CR = r

⇒ BQ = $a - r$, AR = $b - r$,

⇒ AP = $b - r$, PB = $a - r$

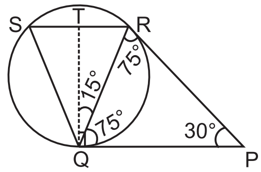
But AB = c

∴ $b - r + a - r = c$

⇒ $2r = a + b - c$

⇒ $r = \dfrac{a+b-c}{2}$. **Hence proved.**

6. Draw a line QT passing through Q and perpendicular to QP to meet SR at T.



In ΔPQR,

PQ = PR

(Tangents from an external point)

∴ ∠PRQ = ∠PQR ...(*i*)

(Angles opposite to equal sides)

∠PQR + ∠PRQ + ∠QPR = 180° ...(*ii*)

(Angle sum property for a triangle)

From equations (*i*) and (*ii*),

∠PQR + ∠PQR + 30° = 180°

⇒ ∠PQR = ∠PRQ = 75° ...(*iii*)

Now, ∠TQR + ∠PQR = 90°

(Angle between tangent)

⇒ TQR = 15° ...(*iv*) [Using (*iii*)]

∴ SR ∥ QP and QT ⊥ QP

∴ QT ⊥ SR

⇒ ST = TR ...(*v*)

(∵ TQ passess through the centre of the circle)

In ΔSTQ and RTQ,

ST = TR [From (*v*)]

∠STQ = ∠RTQ (∵ QT ⊥ SR)

TQ = TQ (Common)

∴ ΔSTQ ≅ ΔRTQ (SAS criterion)

⇒ ∠SQT = ∠RQT = 15° [Using (*iv*)]

∠SQT + ∠TQR = 15° + 15°

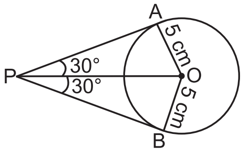
⇒ ∠RQS = 30°.

OR

$\angle APB = 120°$

$\therefore \quad \angle APO = \angle OPB = 60°$

In right-angled ΔOAP

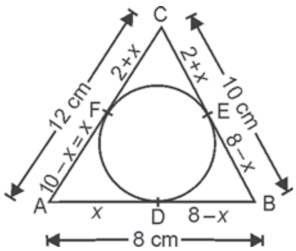


$\cos 60° = \dfrac{AP}{OP}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{AP}{OP} \Rightarrow OP = 2AP$

7. ***See Worksheet–101, Sol. 8.***

8. Let AD = x. We know that the tangents drawn from an external point to a circle are equal.



$\therefore \quad AD = AF = x$

$BD = BE$

and $CE = CF$

Now, $BD = AB - AD = 8 - x = BE$

and $\quad CE = BC - BE = 10 - (8 - x)$

$= 2 + x = CF.$

$AF = AC - CF = 12 - (2 + x)$

$= 10 - x = AD$

But $\quad AD = x$

$\therefore \quad 10 - x = x \Rightarrow x = 5$ cm, *i.e.*, AD = 5 cm

$BE = 8 - x = 8 - 5 = 3$ cm

and $\quad CF = 2 + x = 2 + 5 = 7$ cm

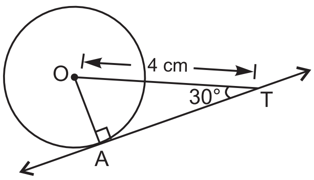
Thus, AD = 5 cm, BE = 3 cm and CF = 7 cm.

CHAPTER TEST

1. $\because$ $OA \perp AT$

$\therefore \quad \angle OAT = 90°$

In ΔOAT,



$\cos T = \dfrac{AT}{OT}$

$\Rightarrow \cos 30° = \dfrac{AT}{4}$

$\Rightarrow AT = 2\sqrt{3}$ cm.

2. As AB is diameter $\Rightarrow \angle ACB = 90°$

Also, $\quad \angle CAB = 30°$

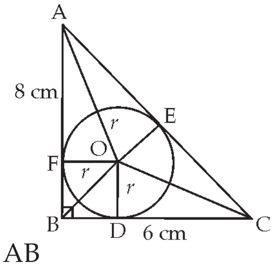
$\therefore$ In right-angle ΔACB,

$\angle ABC = 180° - (90 + \angle CAB)$

$= 180° - 90 - 30 = 60°$

$\therefore \quad \angle PCA = \angle ABC = 60°.$

3. $AC = \sqrt{8^2 + 6^2}$



$= \sqrt{100}$

$= 10$ cm

Area ΔABC

$= \dfrac{1}{2} \times BC \times AB$

$= \dfrac{1}{2} \times 6 \times 8 = 24$ cm $\quad \ldots(i)$

Also area of ΔABC = *ar* ΔAOB + *ar* ΔBOC + *ar* ΔAOC

$= \dfrac{1}{2} \times AB \times r + \dfrac{1}{2} \times BC \times r + AC \times r$

$= \dfrac{1}{2} \times r \times [AB + BC + AC]$

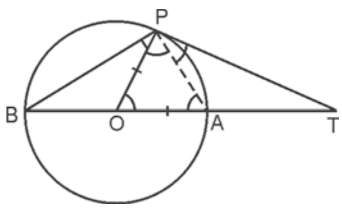
$= \dfrac{1}{2} \times r \times 24 = 12r \quad \ldots(ii)$

$\therefore$ From (i) and (ii) $\Rightarrow$ $24 = 12r$

$= r = 2$ cm.

4. **True,** as $\angle BPA = 90°$, $\quad (\because$ AB is diameter)

$\angle PAB = \angle OPA = 60° \quad (\because OP = OA)$

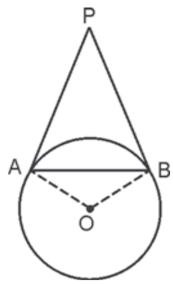


Also $\quad OP \perp PT.$

$\therefore \quad \angle APT = 30°$

and $\quad \angle PTA = 60° - 30° = 30°.$

5. Let the given chord be AB and two tangents to the circle with centre O be AP and BP.



We need to prove

$\angle PAB = \angle PBA$

Join OA and OB.

Proof:

In ΔAOB, AO = BO

$\therefore \angle ABO = \angle BAO \quad \ldots(i)$

As, the tangent is perpendicular to the radius passing through the point of contact,

$\angle PAO = \angle PBO = 90°$ ...(*ii*)

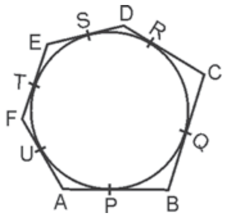
Again, $\angle PAO = \angle PBO$ [Using (*ii*)]

$\Rightarrow \angle PAB + \angle BAO = \angle PBA + \angle ABO$

$\Rightarrow \angle PAB + \angle ABO = \angle PBA + \angle ABO$ [Using (*i*)]

$\Rightarrow \angle PAB = \angle PBA.$

6. Hint: AP = AU,



BP = BQ,

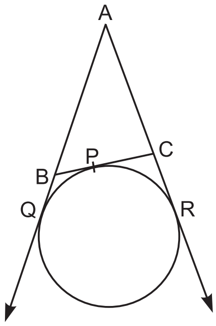
CR = CQ,

DR = DS,

ET = ES

FT = FU.

7. We know that the tangents drawn from an external point to a circle are equal in length.



$\therefore$ AQ = AR ... (*i*)

BQ = BP ... (*ii*)

and CP = CR ... (*iii*)

Now,

AQ = AB + BQ

= AB + BP [From (*ii*)]

= AB + (BC – PC)

= AB + BC – CR [From (*iii*)]

= AB + BC – (AR – AC)

= AB + BC + CA – AR

= AB + BC + CA – AQ [From (*i*)]

$\Rightarrow$ AQ + AQ = AB + BC + CA

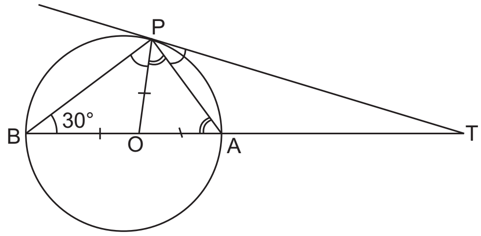
$\Rightarrow AQ = \frac{1}{2}(AB + BC + CA)$

Hence proved.

8. Join OP.

$\therefore$ As BA is diameter

$\Rightarrow \angle BPA = 90°$



$\therefore$ In ΔBPA,

$\angle BAP = 180° - (90 + 30°) = 60°$

Also as OP = OA = (Radius)

$\Rightarrow \angle OPA = \angle OAP = \angle BAP = 60°$ …(*i*)

$\therefore$ In $\angle OPT$ as $OP \perp PT$

$\Rightarrow \angle OPT = 90°$

$\Rightarrow \angle OPA + \angle APT = 90°$

$\Rightarrow 60° + \angle APT = 90°$ (Using (*i*))

$\Rightarrow \angle APT = 90° - 60 = 30°$

Also $\angle OAP = \angle APT + \angle PTA$

(Exterior angle sum property)

$\Rightarrow 60 = 30° + \angle PTA$

$\Rightarrow \angle PTA = 30°$

$\therefore \angle APT = \angle PTA = 30°$

$\Rightarrow$ AP = AT …(*ii*)

(Sides opposite to equal angles are also equal)

Now, In ΔOPA,

as $\angle OPA = \angle OAP = 60°$

$\Rightarrow \angle POA = 60°$ (Using angle sum property)

$\therefore \angle OPA$ is equilateral

$\Rightarrow$ OP = OA = PA. ...(*iii*)

Now, BA = BO + OA = OA + OA

= 2OA = 2AT

{$\because$ From (*ii*) and (*iii*), OA = AT}

$\Rightarrow \frac{BA}{AT} = \frac{2}{1}$

$\Rightarrow$ BA : AT = 2 : 1 **Hence proved.**

❑❑

Chapter 11 CONSTRUCTIONS

WORKSHEET – 106

1. Since, the angle between two radii of a circle and the angle between corresponding two tangents are supplementary.

∴ Required angle = 180°−35° = 145°.

2. 5 : 2.

As there are 5 marks at equal distance on AX and 2 marks at equal distance on BX′.

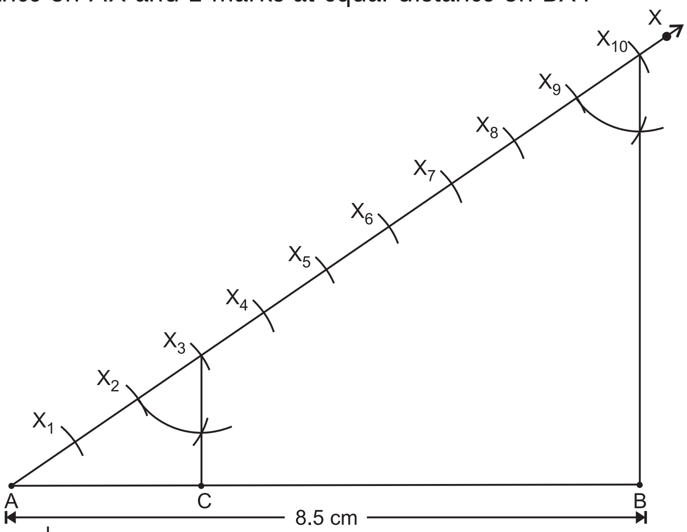
As A_5 is joined with B_2.

∴ P divides AB in the ratio 5 : 2.

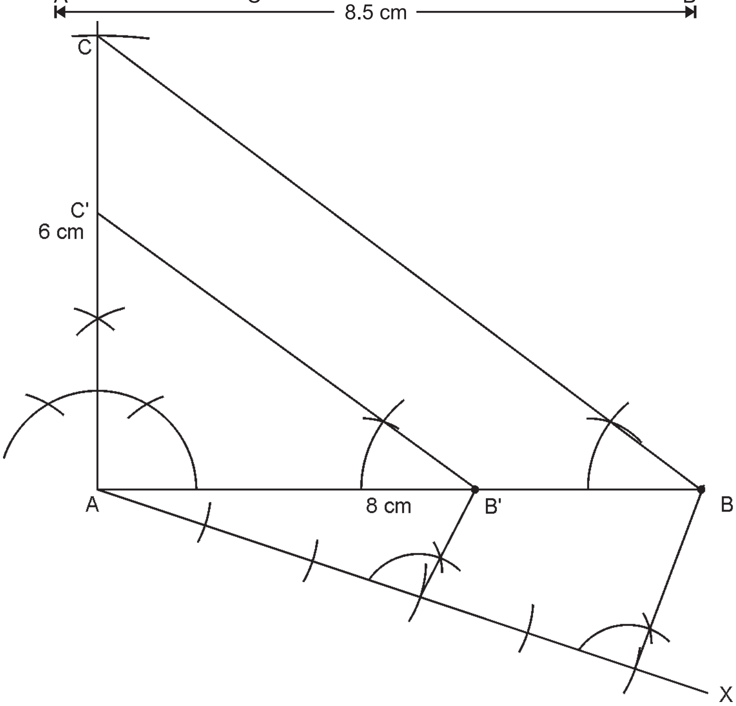
3.

AB = 8.5 cm and $\frac{AC}{CB} = \frac{3}{7}$

AC = 2.55 cm, CB = 5.95 cm.



4. ΔCAB ~ ΔC′AB′



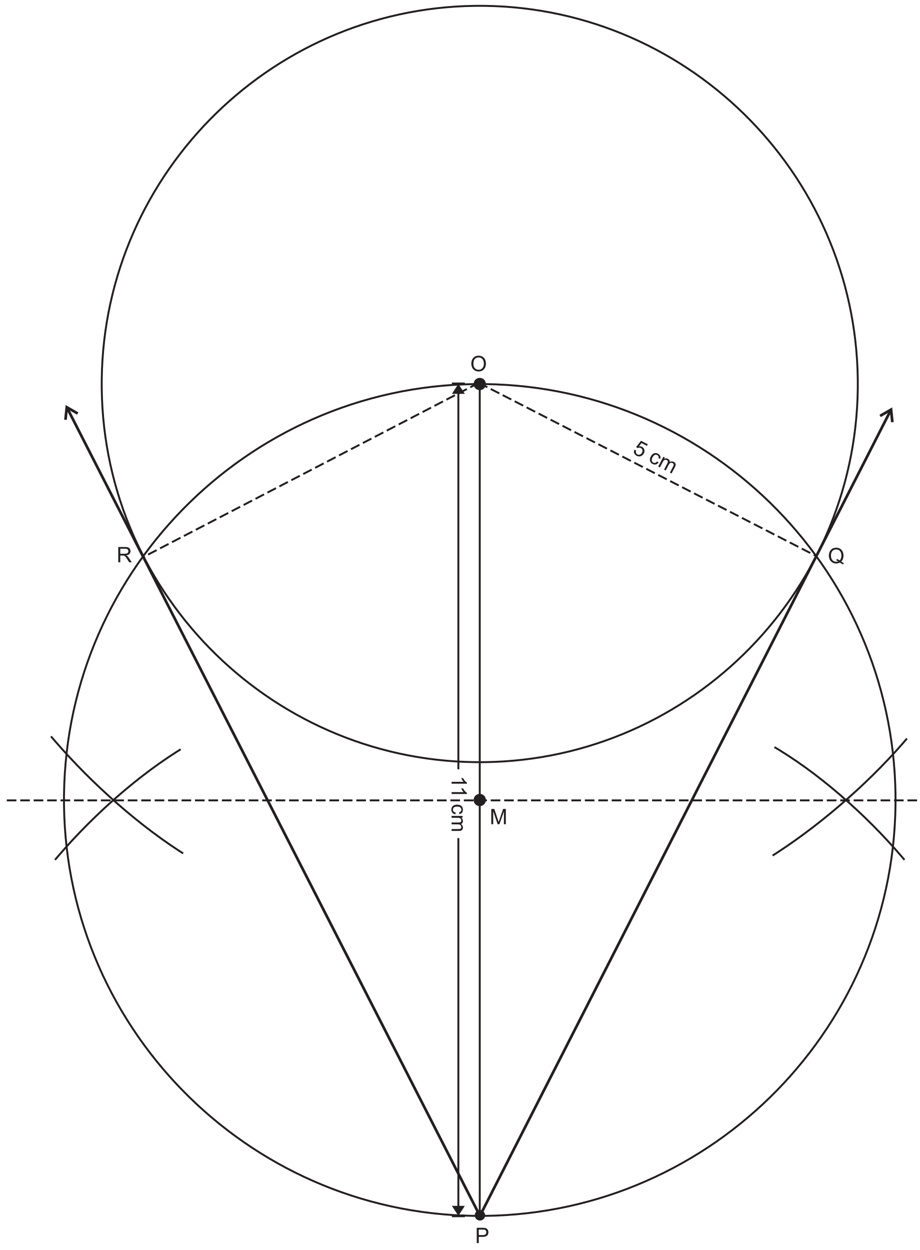
5. Steps of construction:

Step I: First, draw a circle with radius as 5 cm and centre at O. Then take a point P so that OP = 11 cm.

Step II: Bisect OP to find mid-point M of OP. Then take M as centre and MP = MO as radius, draw a circle to intersect the previous circle at Q and R.

Step III: Join PQ and PR which are the required tangents.

After measuring PQ and PR, we find PQ = PR = 9.8 cm (approximately).



Justification:

Join OQ and OR.

In ΔOPQ, $OP = 11$ cm, $OQ = 5$ cm and $PQ = 9.8$ cm

$\therefore \quad OP^2 - OQ^2 = 11^2 - 5^2 = (11+5)(11-5) = 96$

And $\quad PQ^2 = (9.8)^2 = 96.04$

Clearly, $OP^2 - OQ^2 \approx PQ^2$

$\Rightarrow \quad OP^2 = OQ^2 + PQ^2$

Also, $\quad OP^2 = OR^2 + PR^2$

Therefore, ΔPOQ and ΔPOR are right triangles with $\angle PQO = \angle PRO = 90°$.

So, tangents are perpendicular to radii passing through their respective points of contact.

i.e., $\quad PQ \perp OQ$ and $PR \perp OR$.

6. $\angle C = 180° - (\angle B + \angle A) = 180° - 150° = 30°$.

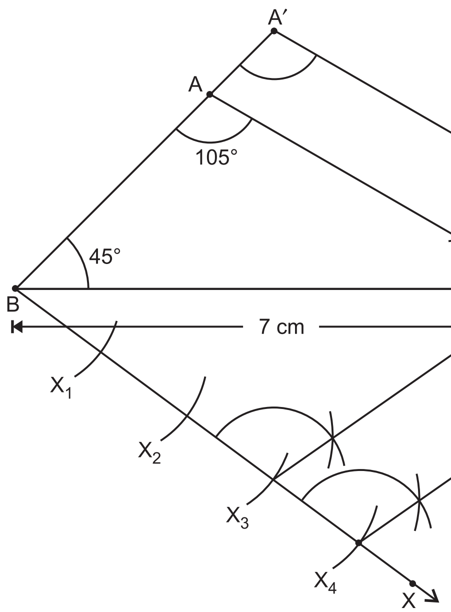
Steps of construction:

In order to construct a triangle similar to ΔABC, follow the following steps:

Step I: First, construct a ΔABC in which $BC = 7$ cm, $\angle B = 45°$ and $\angle C = 30°$.

Step II: Make an acute angle CBX such that X is on the side opposite to vertex A.

Step III: Locate four points namely X_1, X_2, X_3 and X_4 on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4$.



Step IV: Join X_3C and draw a line $X_4C' \parallel X_3C$ to intersect BC produced at C′.

Step V: Draw a line C′A′ parallel to side CA of ΔABC to intersect BA produced at A′. Then, $\Delta A'BC'$ is the required triangle.

WORKSHEET – 107

1. Since 4+7=11, therefore, B will be joined to A_{11}.

2. The required angle and the angle between the two tangents are supplementary.

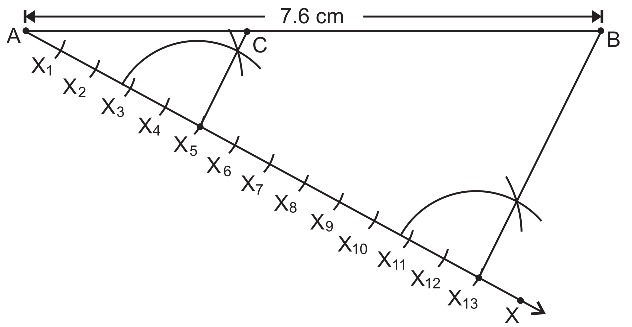
∴ Required angle = 180°–60° = 120°.

3. Here, 5+8 = 13

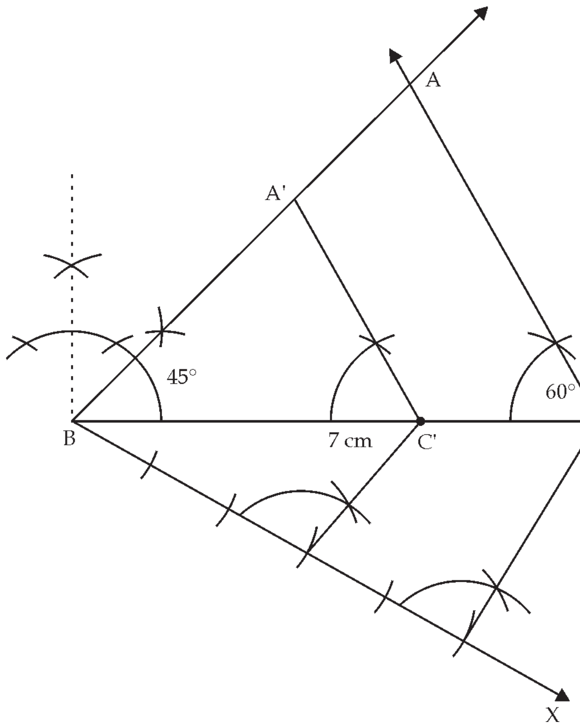
AB = 7.6 cm

AC : BC = 5:8

AC = 2.92 cm and BC = 4.68 cm



4. ΔA′BC′ ~ ΔABC

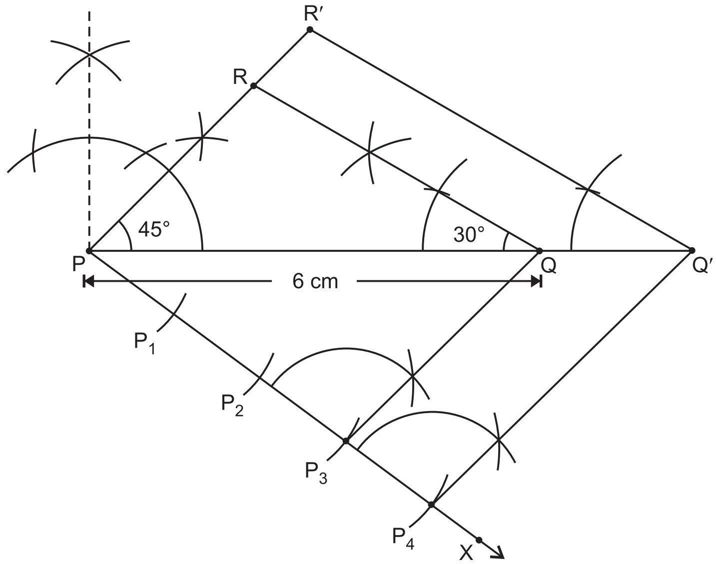


5. **Justification:** In ΔPQR and $\Delta PQ'R'$, $\angle P = 45°$ is common and $RQ \parallel R'Q'$.

$\therefore \Delta PQR \sim \Delta PQ'R'$

We have draw $PP_1 = P_1P_2$

$= P_2P_3 = P_3P_4$



$$\therefore \quad \frac{P_3P_4}{PP_3} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{P_3P_4}{PP_3} + 1 = \frac{1}{3} + 1$$

$$\Rightarrow \quad \frac{P_3P_4 + PP_3}{PP_3} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{PP_4}{PP_3} = \frac{4}{3}$$

And $P_3Q \parallel P_4Q'$

$$\therefore \quad \frac{PQ'}{PQ} = \frac{4}{3}.$$

Also, $\Delta PQR \sim \Delta PQ'R'$

Hence, $\dfrac{PQ'}{PQ} = \dfrac{PR'}{PR} = \dfrac{R'Q'}{RQ}$.

6. **Steps of construction:** In order to construct a pair of required tangents, follow the following steps:

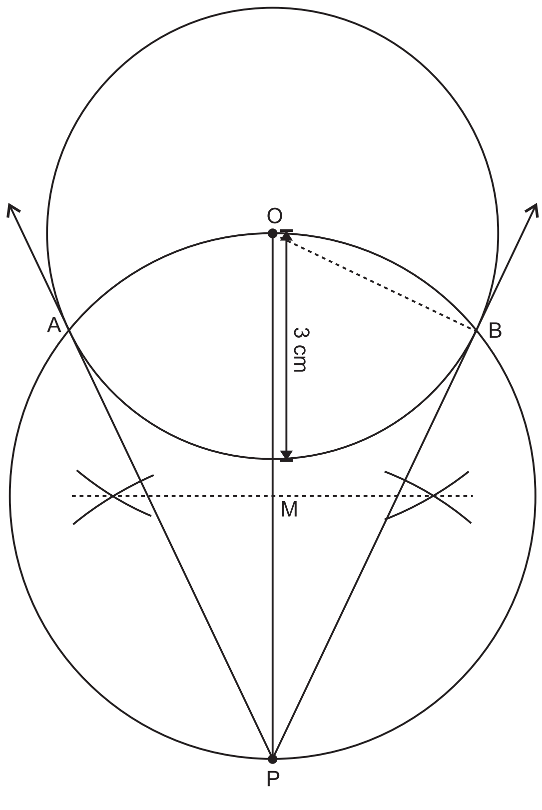
Step I: Draw a circle with radius OA = 3 cm and centre O.

Step II: Take any point P outside the circle drawn in step I and join OP.

Step III: Obtain mid-point M of OP obtained in step II and draw another circle with radius OM = PM and centre M to intersect the circle drawn in step I at A and B.

Step IV: Join PA and PB.

These PA and PB form the required pair of tangents.



WORKSHEET – 108

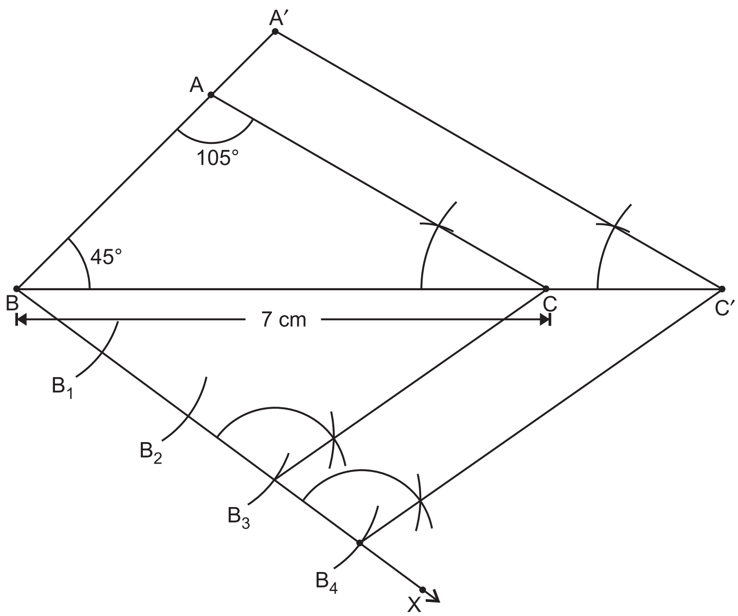
1. The next step should be the line parallel to B_5C should be passed through B_4 as the sides of required triangle are $\frac{4}{5}$ of the corresponding sides of ΔABC.

2. Two distinct tangents to a circle can be constructed from P only when P is situated at a distance more than radius (here $2r$) from the centre.

3. **False.** In the ratio $3 + \sqrt{2} : 3 - \sqrt{2}$, *i.e.*, $11 + 6\sqrt{2} : 7$, $11 + 6\sqrt{2}$ is not a positive integer, while 7 is.

4. **Steps of construction:** In order to construct a ΔABC and its similar triangle with given measurements, follow the following steps:

Step I: Draw a ΔABC in which $BC = 7$ cm, $\angle B = 45°$ and $\angle C = 180° - (45° + 105°) = 30°$.



Step II: Make an acute $\angle CBX$ such that X is on the opposite side of the vertex A and locate points B_1, B_2, B_3 and B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step III: Join B_3C and draw $B_4C' \parallel B_3C$ to intersect BC produced at C′. Also draw $C'A' \parallel CA$ to intersect BA produced at A′.

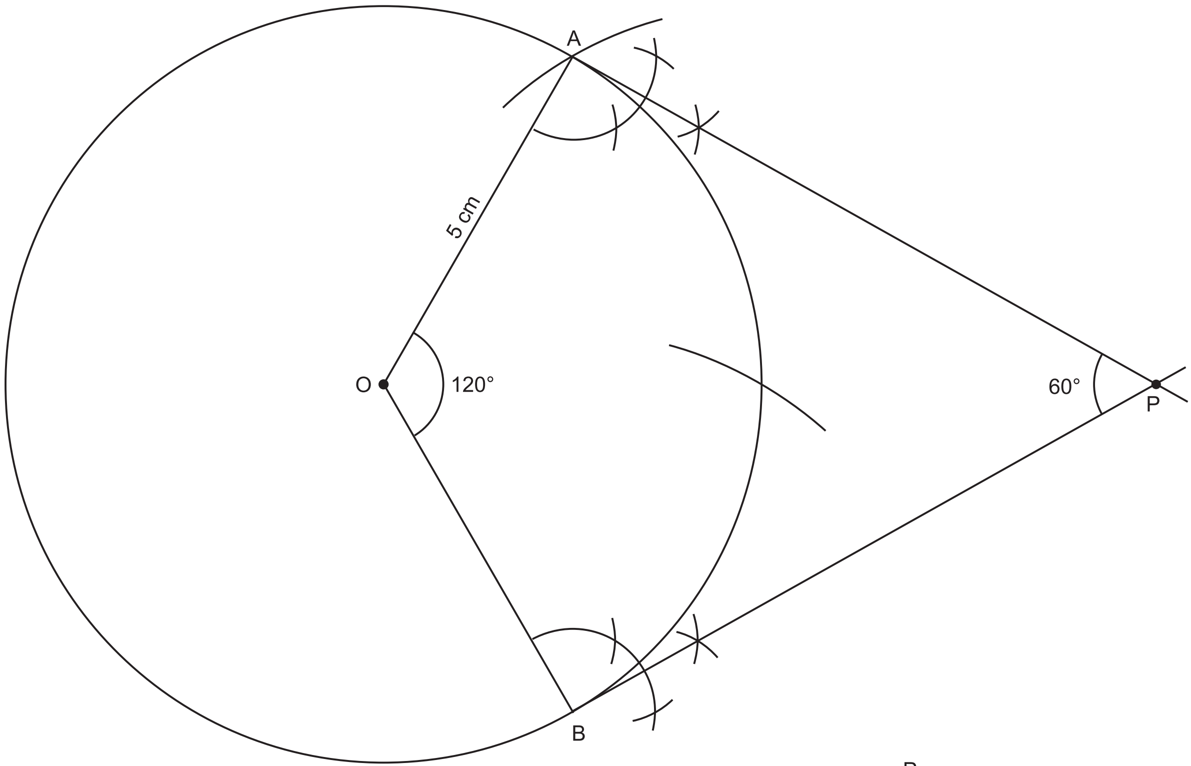
Hence, $\Delta A'BC' \sim \Delta ABC$.

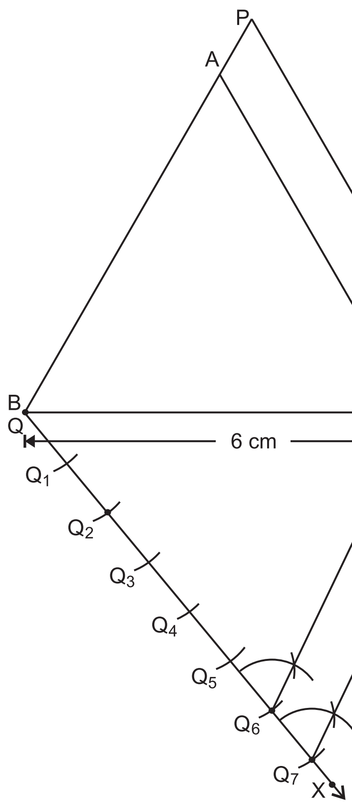
5. **Steps of construction:** In order to draw a pair of tangents to the given circle, follow the following steps:

Step I: Draw a radius AO in the given circle with centre O and draw another radius making an angle AOB of measure $180° - 60° = 120°$.

Step II: Make $\angle OAP = 90°$ and $\angle BOP = 90°$ to intersect each other at P.

Such obtained AP and BP are the required tangents such that $\angle APB = 60°$.





6. We are given a ΔPQR with each side of measure 6 cm.

Steps of construction: In order to construct ΔABC follow the following steps:

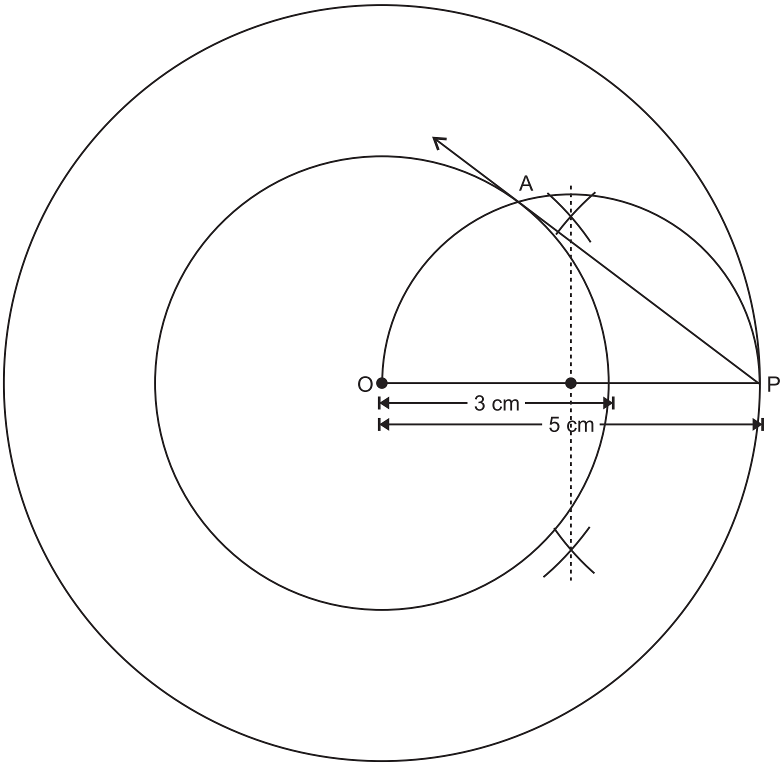
Step I: Make an acute angle RQX and locate seven points Q_1, Q_2, Q_3, Q_4, Q_5, Q_6 and Q_7 on the ray OX such that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_3Q_4 = Q_4Q_5 = Q_5Q_6 = Q_6Q_7$.

Step II: Join Q_7R and draw Q_6C parallel to Q_7R to intersect QR at C.

Step III: Draw CA parallel to RP to intersect BP (B and Q coincide) at A.

Then, ΔABC is the required triangle.

WORKSHEET – 109

1. Line segment A_5B_7 divides the line segment AB in the ratio 5 : 7.
2. Two
3. **True,** because the angle between the tangents must be less than 180°.
4. 

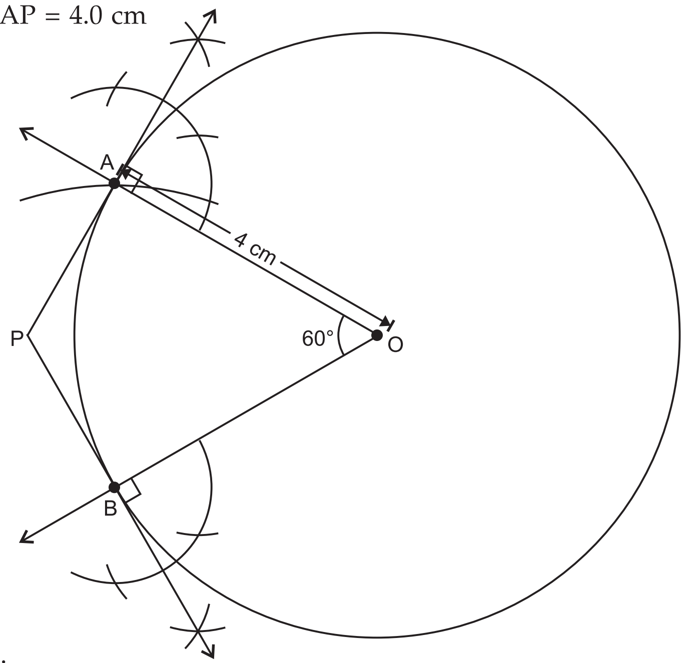
Measuring the tangent AP, we get AP = 4.0 cm

5. We are given a circle of radius 4 cm and centre O.

Steps of construction: In order to draw the required pair of tangents, follow the following steps.

Step I: Draw a pair of radius OA and OB inclined at an angle of 180° – 120° = 60° to intersect the given circle at A and B respectively.

Step II: Draw perpendiculars AP and BP which intersect each other at P. Then AP and BP are the required tangents.



Justification: In quadrilateral, OBPA, applying Angle sum property, we have

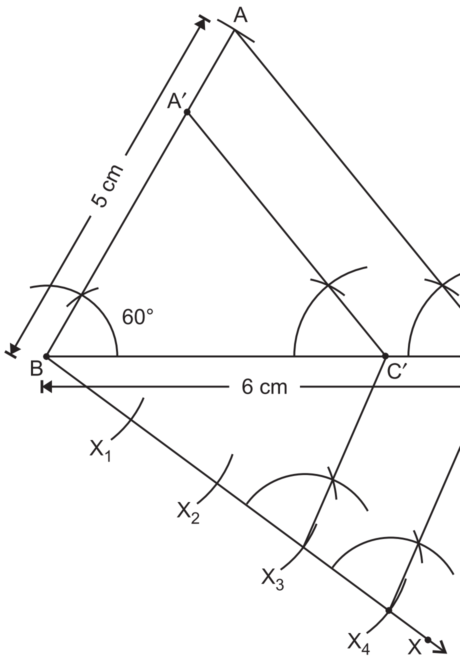
$$\angle O + \angle A + \angle B + \angle P = 360°$$

$$\Rightarrow \quad 60° + 90° + 90° + \angle P = 360°$$

$$\Rightarrow \quad \angle P = 360° - 240° \Rightarrow \quad \angle P = 120°.$$

Angle between the tangents is 120°.

6.



In the figure, $\Delta A'BC' \sim \Delta ABC$.

Steps of construction:

First, we draw ΔABC with the given measurements. Then we draw another triangle $A'BC'$ similar to ΔABC and of scalar factor $\frac{3}{4}$ using the following steps:

Step I: Draw a ray BX such that $\angle CBX$ is an acute angle.

Step II: Mark X_1, X_2, X_3, X_4 on BX such that $BX_1 = X_1X_2 = X_2X_3 = X_3X_4$.

Step III: Join X_4C and draw $X_3C' \parallel X_4C$.

Also, draw $A'C' \parallel AC$.

Thus, $\Delta A'BC' \sim \Delta ABC$.

WORKSHEET – 110

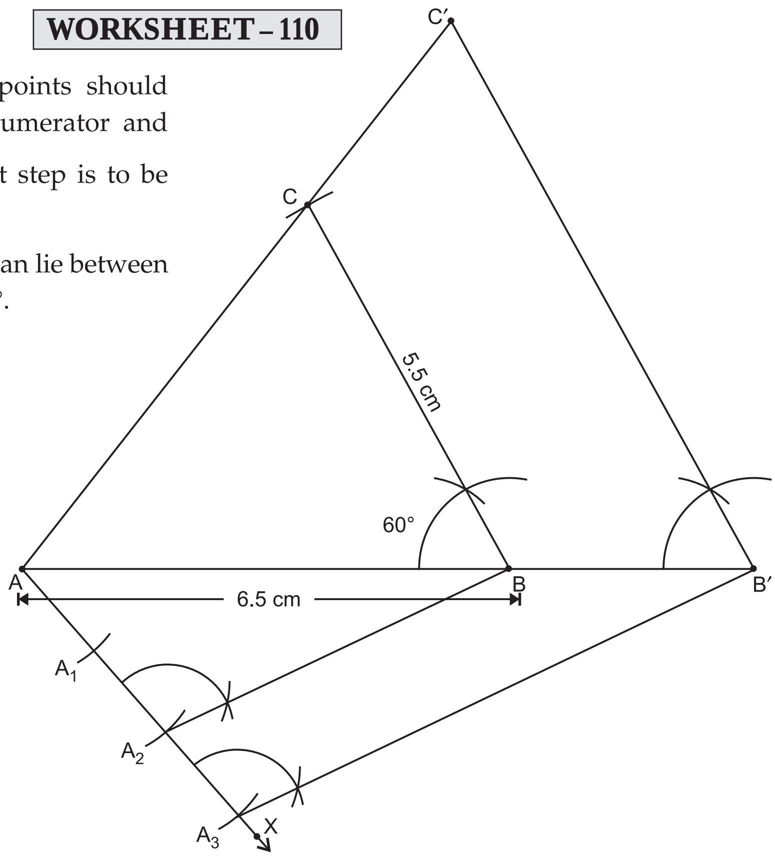
1. The minimum number of points should be 9 as 9 > 5 out of the numerator and denominator of $\frac{9}{5}$. The next step is to be joined B_5 to C.

2. Angle of inclination, here θ, can lie between 0° and 180°. *i.e.*, $0 < \theta < 180°$.

3. In the adjoining figure,

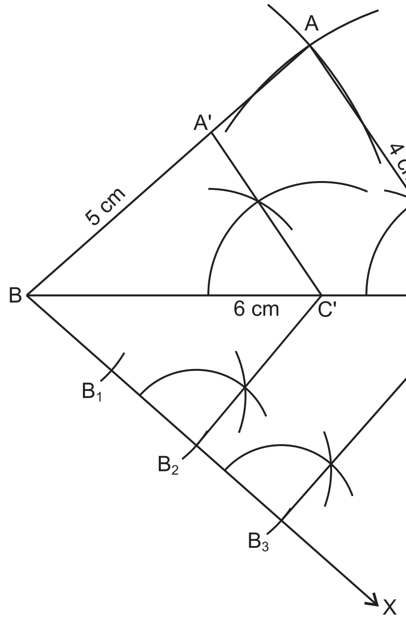
$\Delta AB'C' \sim \Delta ABC$ such that

$\frac{AB'}{AB} = \frac{B'C'}{BC} = \frac{AC'}{AC} = \frac{3}{2}$.



4. Steps of construction:

1. Draw a ΔABC in which AB = 5 cm, BC = 6 cm and AC = 4 cm.
2. Make any acute $\angle CBX$.
3. With suitable distances divide $BB_1 = B_1B_2 = B_2B_3$.
4. Join B_2C.
5. Draw $B_2C' \parallel$ to B_3C.
6. Draw $C'A' \parallel$ to CA.
7. $\Delta A'B'C'$ is the required triangle whose side is $\frac{2}{3}$ of the corresponding sides of given ΔABC.



5. $2\frac{1}{2} = \frac{5}{2}$

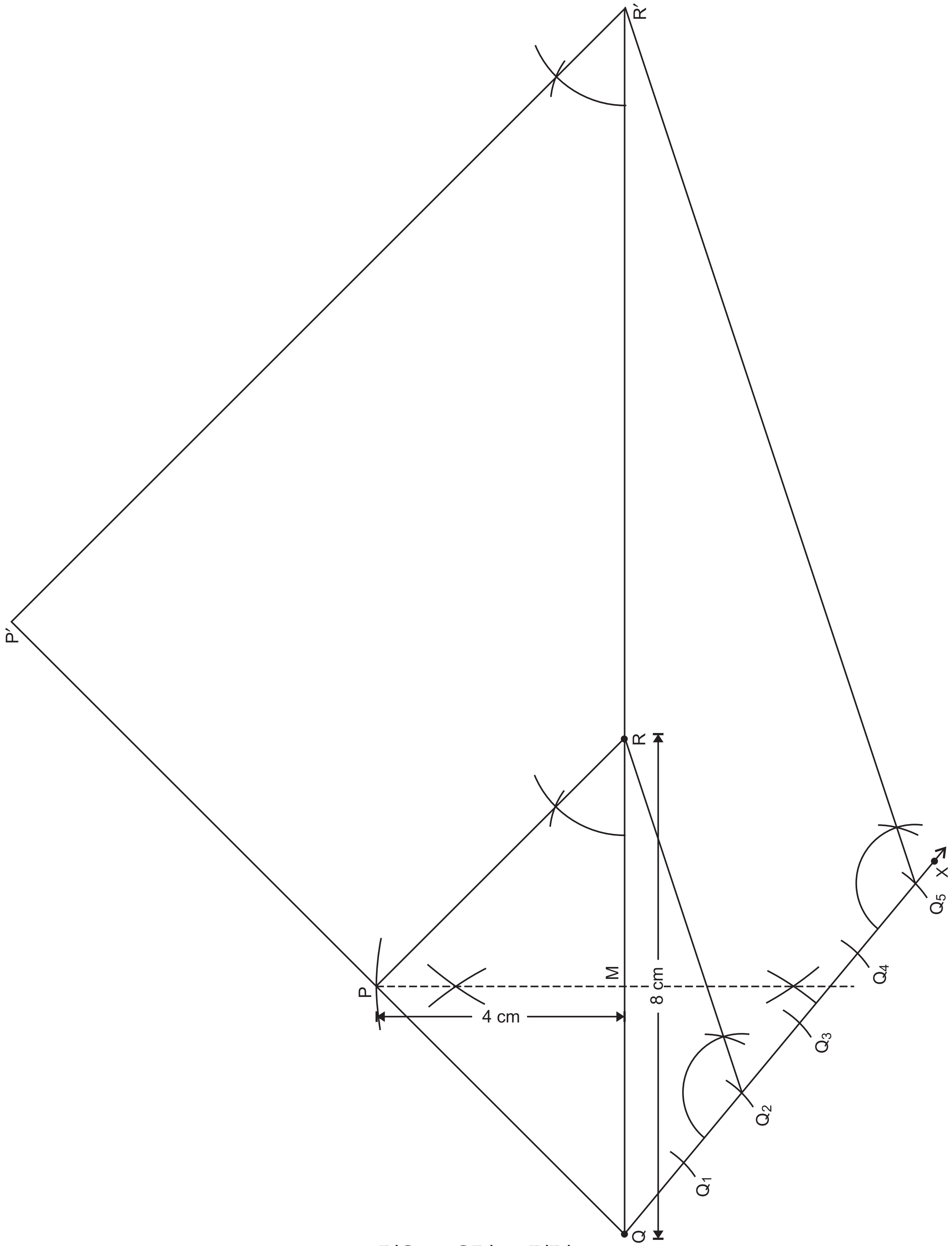
Steps of construction: In order to construct an isosceles triangle and another triangle having $\frac{5}{2}$ of its corresponding sides, follow the steps given below:

Step I: Construct an isosceles triangle having any length of equal sides by drawing base QR = 8 cm and altitude PM = 4 cm passing through the mid-point M of side QR.

Step II: Draw a ray QX such that $\angle RQX$ is and acute angle; and divide the ray in five equal parts, namely QQ_1, Q_1Q_2, Q_2Q_3, Q_3Q_4 and Q_4Q_5.

Step III: Join Q_2R and draw $Q_5R' \parallel QR$ intersecting QR produced at R′.

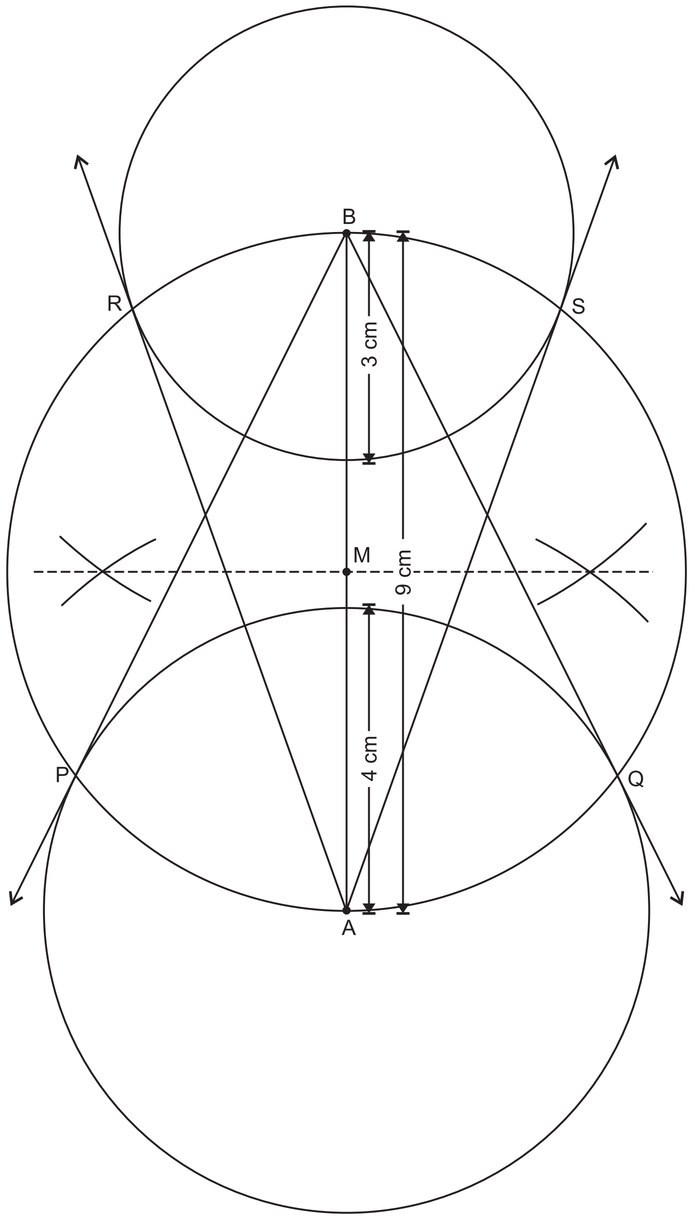
Step IV: Draw $R'P' \parallel RP$ intersecting QP produced at P′.



Hence, $\Delta P'QR'$ is formed so that $\dfrac{P'Q}{PQ} = \dfrac{QR'}{QR} = \dfrac{P'R'}{PR} = \dfrac{5}{2}$.

6. Steps of construction: In order to draw the required pairs of tangents, follow the following steps:

Step I: Draw a line segment AB of length 9 cm. Taking A as centre and radius 4 cm; and B as centre and radius 3 cm, draw circles.



Step II: Find the mid-point M of AB. Then, taking M as centre and radius as AM = MB, draw a circle to intersect circles drawn in step I at P, Q and R, S respectively.

Step III: Join AR, AS, BP and BQ.

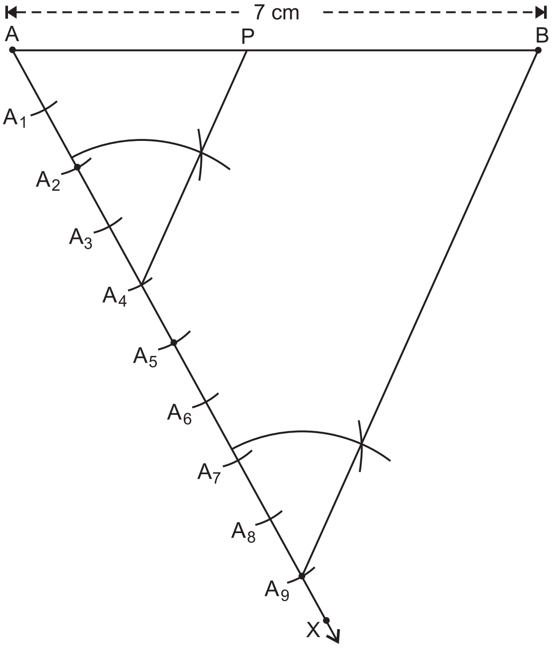
Thus, obtained AR, AS and BP, BQ are the required pairs of tangents.

WORKSHEET – 111

1. Q_7 to R.

2. $5+7 = 12$.

3.

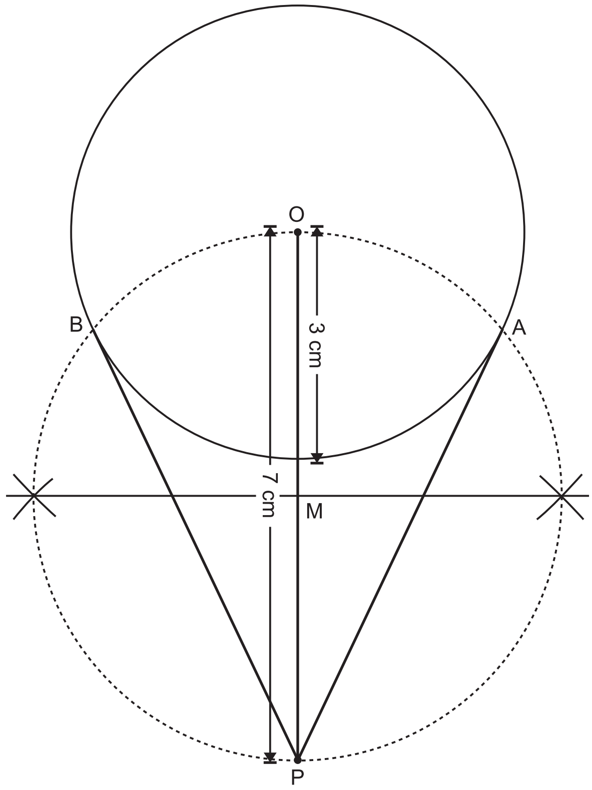


AP : PB = 4 : 5.

4. **False,** because in the ratio $\sqrt{3}-1 : \sqrt{3}+1$, *i.e.*, $2-\sqrt{3} : 1$, $2-\sqrt{3}$ is not a positive integer, while 1 is.

5. To draw a pair of tangents from P to the circle with centre O, we follow the steps as given:

(*a*) Join OP and find its mid-point M.



(*b*) Taking M as centre and radius = MP = MO, draw a circle to intersect the given circle at A and B.

(*c*) Join PA and PB.

PA and PB are the required tangents.

On measuring, PA = 6.35 cm and

PB = 6.35 cm. Clearly, PA and PB are of same length.

6. ΔAPQ is required triangle.

Steps of construction:

Step I: Draw AB = 7cm.

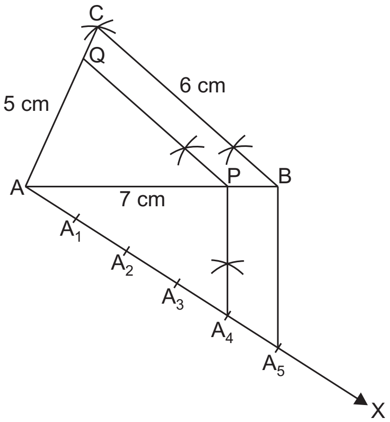
Step II: From A draw an arc of radius 5 cm.

Step III: From B draw an arc of radius 6 cm which cut the arc of step II at point C.

Step IV: Join AC and BC

$\therefore \Delta ABC$ is given triangle with AB = 7cm, AC = 5 cm and BC = 6 cm.

Step V: Draw ray AX.



Step VI: Mark A_1, A_2, A_3, A_4, A_5 at equal distance.

Step VII: Join A_5 to B and draw a line parallel to A_5B from A_4 which cut AB at P.

Step VIII: Draw a line from P parallel to BC which cut AC at Q.

$\therefore$ $\Delta ABQ \sim \Delta ABC$ with scale factor $\frac{4}{5}$.

7. First we construct a ΔABC with the given measurements. Then we construct a ΔA′BC′ similar to ΔABC and scale factor $\frac{5}{7}$.

For it, we follow the steps given below.

(*a*) Draw a ray BX such that $O < \angle CBX < 90°$ and mark points $B_1, B_2, B_3, B_4, B_5, B_6$ and B_7 on it such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

(*b*) Join B_7C and draw $B_5C' \| B_7C$ to intersect BC at C′ and hence draw $A'C' \| AC$ to intersect AB at A′.

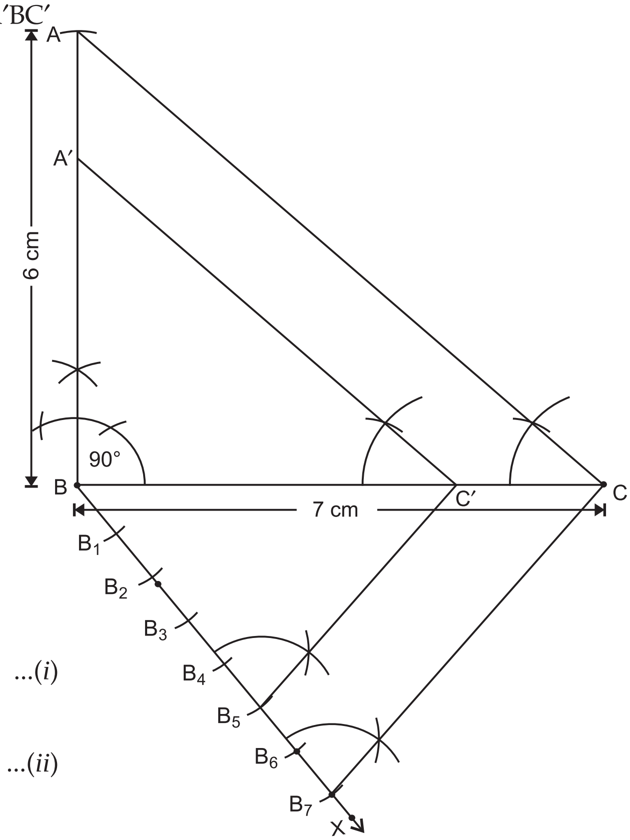
Thus $\Delta A'BC' \sim \Delta ABC$.

Justification: In ΔCBB_7,

$$CB_7 \| C'B_5 \text{ and } \frac{BB_5}{BB_7} = \frac{5}{7}.$$

So, by Thale's theorem, $\frac{BC'}{BC} = \frac{5}{7}$ ...(*i*)

Similarly, in ΔABC, $\frac{A'B}{AB} = \frac{5}{7}$ ...(*ii*)

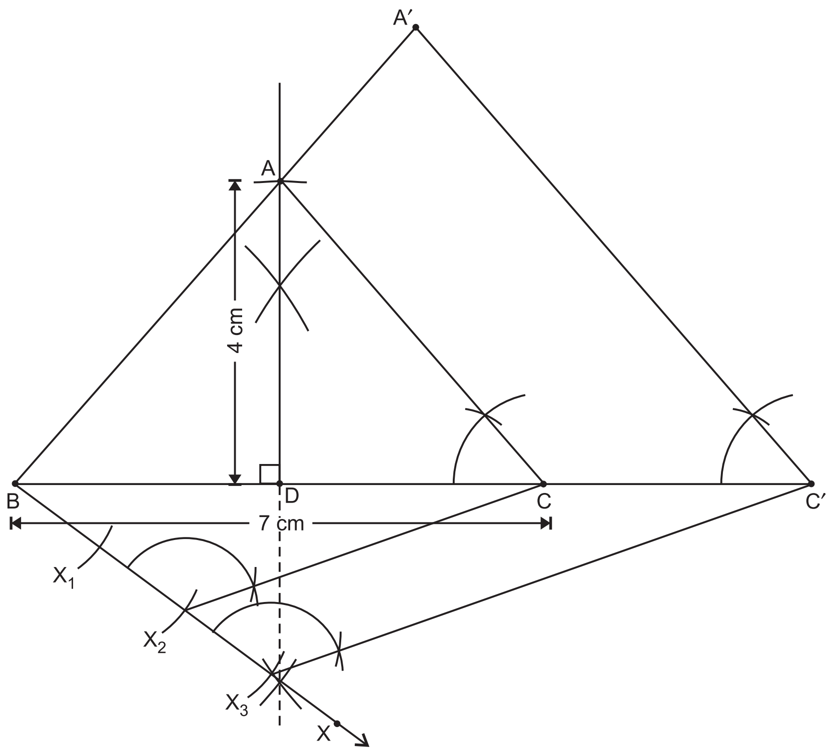


Now, in ΔABC,

$$\frac{BC'}{BC} = \frac{A'B}{AB} = \frac{5}{7}$$ [Using (*i*) and (*ii*); and $\angle B = 90°$ (Given)]

$\Delta A'BC' \sim \Delta ABC$ and $\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{5}{7}$. **Hence justified**.

8. First we draw an isosceles triangle ABC with base BC = 7 cm and altitude AD = 4 cm. Altitude passes through the mid-point D of BC. Hence we construct a $\Delta A'BC'$ similar to ΔABC and of scalar factor $1\frac{1}{2}$, *i.e.*, $\frac{3}{2}$ using following the steps given below:



(*a*) Draw an acute angle CBX opposite to the vertex A with respect to BC.

(*b*) Mark points X_1, X_2, X_3 on ray BX such that $BX_1 = X_1X_2 = X_2X_3$.

(*c*) Join X_2C and draw $X_3C' \| X_2C$ to meet BC produced at C′.

(*d*) Draw $C'A' \| CA$ to meet BA produced at A′.

Thus formed $\Delta A'BC'$ is similar to ΔABC and of scalar factor $\frac{3}{2}$.

CHAPTER TEST

1. Line segment P_3Q_2 divides PQ in 3:2 at M. Therefore, $P_3M:Q_2M = 3:2$ and so

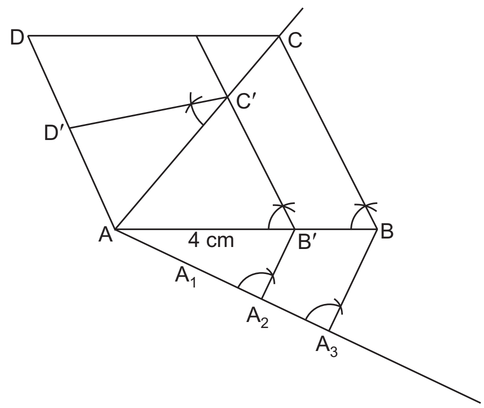
$Q_2M:P_3M = 2:3$.

2. No. A line segment can't be divided in the ratio $\sqrt{6}+1 : \sqrt{6}-1$, *i.e.*, $7+2\sqrt{6} : 5$ as $7+2\sqrt{6}$ is not a positive integer while 5 is.

3. True, because the irrational ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$ can be converted into the rational ratio that is 3:1.

4. First draw the rhombus ABCD in which AB = 4 cm and $\angle ABC = 60°$as given in figure and join AC. Construct the triangle AB′C′ similar to ΔABC with scale factor $\frac{2}{3}$ (see figure).

Finally, draw the line segment C′D′ parallel to CD.



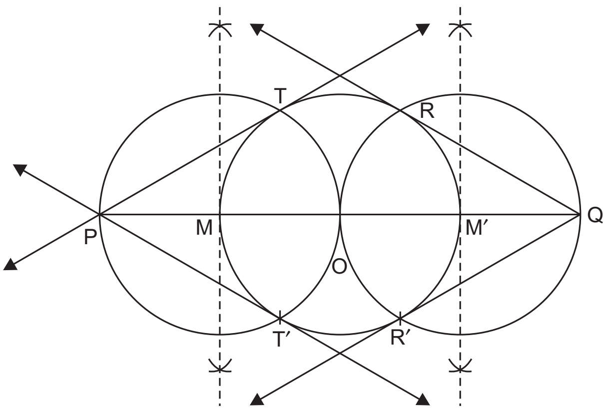
Now, $\frac{AB'}{AB} = \frac{2}{3} = \frac{A'C'}{AC}$

Also, $\frac{AC'}{AC} = \frac{C'D'}{CD} = \frac{AD'}{AD} = \frac{2}{3}$

Therefore, $AB' = B'C' = C'D' = AD' = \frac{2}{3}AB$.

i.e., AB′C′D′ is a rhombus.

5. PT, PT′ and QR, QR′ are required tangents.



6. Steps of construction: In order to construct triangles ABC and AQR, follow the steps given below:

Step I: Draw any line XY and take any point D on it.

Step II: Draw any ray DZ such that $\angle ZDY = 90°$. Locate point C on DZ such that CD = 3 cm.

Step III: Make an $\angle DCB = 30°$ such that CB intersects XY at B.

Step IV: Locate a point A on XB such that AB = 5 cm and by joining AC, we find ΔABC.

Step V: Make an acute angle YAT and locate T_1, T_2 and T_3 on the ray AT such that

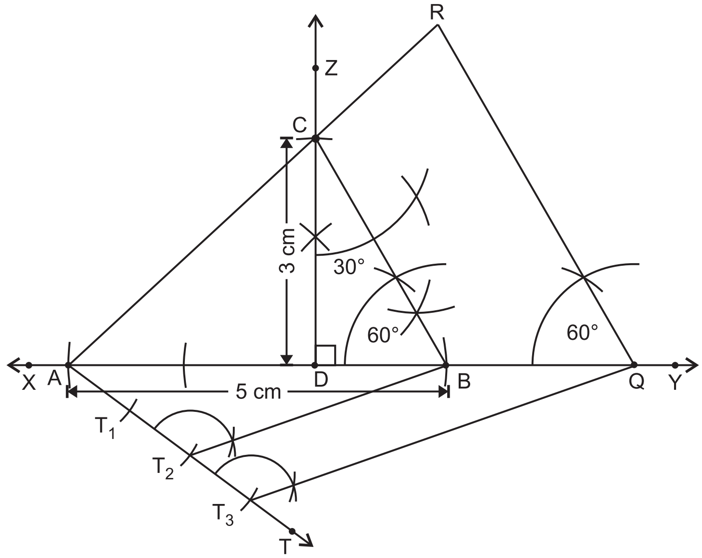
$AT_1 = T_1T_2 = T_2T_3$.

Step VI: Join T_2B and draw $T_3Q \parallel T_2B$ to intersect line AY at Q. Also, draw QR to intersect AC extended at R.

Thus, ΔAQR is obtained such that

$\Delta ABC \sim \Delta AQR$ and

$$\frac{AQ}{AB} = \frac{QR}{BC} = \frac{AR}{AC} = \frac{3}{2}.$$



❑❑

Chapter 12 AREAS RELATED TO CIRCLES

WORKSHEET – 114

1. No.

As AC = diameter = p cm

$\therefore$ If side of square is x cm

then $p = \sqrt{2}.x \Rightarrow x = \frac{p}{\sqrt{2}}$

$\therefore$ Area of square is $x^2 = \frac{p^2}{2}$ cm^2.

2. Perimeter = AB + BC + CD + length of arc $\overset{\frown}{AED}$

$= 20+14+20+\pi \times (7)$

$= 76$ cm.

3. Perimeter = Outer arc length + Inner arc length + 2 × (14).

$= \frac{\pi \times 30°}{180°}(21+7)+28$

$= \frac{22}{7}\times\frac{28}{6}+28 = \frac{44}{3}+28$

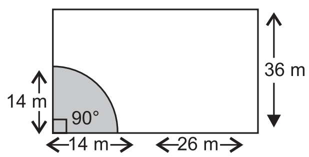
$= \frac{44+84}{3} = \frac{128}{3} = 42\frac{2}{3}$ cm.

4. No.

The correct statement is: "Area of a segment of a circle = Area of the corresponding sector – Area of the corresponding triangle."

5. 154 m^2

Hint: Find area of shaded part.



6. (*i*) $\frac{77}{8}$ cm^2 (*ii*) $\frac{49}{8}$ cm^2

Hint: Area of quadrant $= \frac{\pi r^2}{4}$

7. Length of arc $\overset{\frown}{AP} = 2\pi r \times \frac{\theta}{360°}$

$= \frac{\pi r\theta}{180°}$ …(*i*)

Also $\tan\theta = \frac{AB}{OA} = \frac{AB}{r}$

$\Rightarrow$ $AB = r\tan\theta$ …(*ii*)

Also $\frac{OB}{r} = \sec\theta \Rightarrow OB = r\sec\theta$

$\Rightarrow$ $PB = OB - r = r\sec\theta - r$ …(*iii*)

$\therefore$ Perimeter of shaded part

$= (i) + (ii) + (iii)$

$= r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180°}$

$= r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180°} - 1\right]$.

8. (*i*) Length of wire used

$= 2\pi r + 5\times(2r)$

$= 2\times\frac{22}{7}\times\frac{35}{2}+10\times\frac{35}{2}$

$= 110 + 175 = 285$ mm.

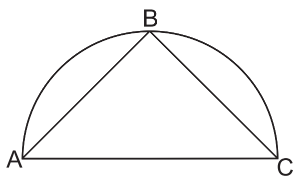
(*ii*) Area of each sector

$= \frac{\pi r^2\theta}{360°} = \frac{\frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}\times 36°}{360°}$

$= \frac{385}{4}$ mm^2.

WORKSHEET – 115

1.



For area of largest triangle: Base of triangle = diameter of circle = $2r$ and height of triangle = radius of circle = r

$\therefore$ Area $= \frac{1}{2}\times(2r)(r) = r^2$ sq. unit.

2. Hint: Area of shaded part

$= \frac{\pi\times 45°}{360°}\left[21^2 - 7^2\right]$

$= 154$ cm^2.

3. 228.57 cm^2

Hint: Shaded portion

= Area of quadrant – Area of square ABCD

Radius OB $= 20 \times \sqrt{2}$

= Diagonal of square.

4. No, because radius of the largest circle must be $\frac{b}{2}$ cm such that the area will be $\frac{\pi b^2}{4}$ cm^2.

5. As BC is diameter

$\Rightarrow \quad \angle BAC = 90°$

$\Rightarrow$ ΔBAC is a right-angled triangle

$$\therefore \quad BC^2 = AC^2 + AB^2 = (24)^2 + (7)^2 = 576 + 49 = 625$$

$\Rightarrow \quad BC = 25$ (diameter of circle)

$$\therefore \quad BO = OC = \frac{1}{2} \times 25 = \frac{25}{2} \text{ cm (Radius)}$$

$\therefore$ Required (shaded Area) = Area of circle – area of ΔBAC – area of quadrant COD.

$$= \pi r^2 - \frac{1}{2} \times AB \times AC - \frac{1}{4}\pi r^2$$

$$= \frac{3\pi r^2}{4} - \frac{1}{2} \times 7 \times 24$$

$$= \frac{3 \times 3.14}{4} \times \frac{25}{2} \times \frac{25}{2} - 84$$

$$= \frac{4.71 \times 625}{8} - 84 = \frac{4.71 \times 625 - 672}{8}$$

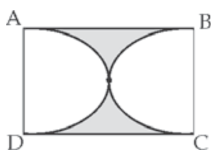
$= 283.96$ cm^2.

OR

Area of shaded part = Area of square ABCD – 2 × (area of semicircles)

$$= 14 \times 14 - 2 \times \left(\frac{1}{2}\pi(7)^2\right)$$

$$= 196 - \pi \times 7 \times 7$$



$$= 196 - \frac{22}{7} \times 7 \times 7$$

$= 196 - 154 = 42$ cm^2

6. Let a = 12 cm = side of equilateral ΔABC

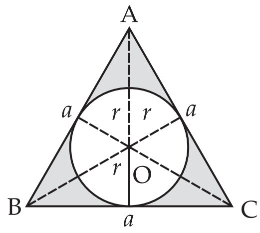
Let r = radius of circle

$\therefore$ $ar(\Delta ABC)$

$= ar(\Delta AOB)$

$+ ar(\Delta BOC)$

$+ ar(\Delta COA)$



$$\Rightarrow \frac{\sqrt{3}}{4} \times (12)^2 = \frac{1}{2} \times AB \times r + \frac{1}{2} \times BC \times r + \frac{1}{2} \times AC \times r$$

$$= \frac{1}{2} r\,(AB + BC + AC)$$

$$= \frac{1}{2} r\,(a + a + a)$$

$$= \frac{1}{2} r\,(12 + 12 + 12)$$

$$= \frac{1}{2} r \times 36$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4} \times 144 = 18r$$

$$\Rightarrow r = \frac{\sqrt{3}}{4} \times \frac{144}{18} = 2\sqrt{3} \text{ cm}$$

Area of shaded part

$= ar(\Delta ABC)$ – *area* of circle

$$= \frac{\sqrt{3}}{4} a^2 - \pi r^2$$

$$= \frac{\sqrt{3}}{4} \times 12 \times 12 - 3.14 \times (2\sqrt{3})^2$$

$= 36\sqrt{3} - 12 \times 3.14$

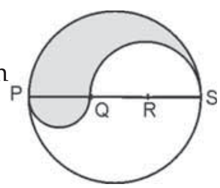
$= 36 \times 1.73 - 12 \times 3.14$

$= 24.638$ cm^2

7. Radius = 6 cm

$\therefore$ Diameter PS

= 2 × 6 = 12 cm



Also, PQ = QR = RS

$$= \frac{12}{3} = 4 \text{ cm.}$$

Perimeter = length of $\widehat{PQ}$ + length of $\widehat{QS}$ + length of $\widehat{PS}$

$$= \pi\,[2 + 4 + 6] = 12 \times \frac{22}{7} = 37.71 \text{ cm.}$$

$$\text{Area} = \frac{\pi}{2}(6)^2 - \frac{\pi}{2}(4)^2 + \frac{\pi}{2}(2)^2$$

$$= \frac{\pi}{2}\,(36 - 16 + 4) = \frac{22}{7\times 2} \times 24$$

$$= \frac{528}{2\times 7} = \frac{264}{7} = 37.71 \text{ cm}^2.$$

8. (*i*) Perimeter of sector = $2r$ + Arc length

$$= 2\times 5.7 + \frac{\pi r\theta}{180°}$$

$$\therefore \quad 27.2 = 11.4 + \frac{\pi r\theta}{180°}$$

$$\Rightarrow \quad \frac{\pi r\theta}{180°} = 27.2 - 11.4 = 15.8 \text{ m}$$

$\therefore$ Arc length = 1580 cm.

(*ii*) 450300 cm^2

Hint: Area of sector = $\frac{1}{2} l r$.

WORKSHEET – 116

1. $2\pi r - r = 37$ cm

$\Rightarrow \quad r(2\pi - 1) = 37$

$$r\left(2\times\frac{22}{7} - 1\right) = 37$$

$$r\left(\frac{44-7}{7}\right) = 37$$

$$r \times \frac{37}{7} = 37$$

$$r = 7 \text{ cm}$$

$\therefore$ Circumference = $2\pi r$

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

2. Hint: $2\pi r = 2\pi r_1 + 2\pi r_2$

$\Rightarrow \quad r = r_1 + r_2$

$= 19 + 9 = 28$ cm.

3. False, because radii are different so their areas will also be different if their corresponding arc lengths are equal.

4. Area of shaded part = area of square – area of quadrant

$$= (7)^2 - \frac{1}{4}\pi(7)^2$$

$$= 49 - \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= 49 - \frac{11}{2} \times 7$$

$$= 49 - \frac{77}{2} = 49 - 38.5 = 10.5 \text{ cm}^2.$$

5. Shaded part = Area of square – 4 × Area of circle

$$= (14)^2 - 4(\pi r^2)$$

$$= 196 - 4 \times \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$$= 196 - 154 = 42 \text{ cm}^2.$$

OR

Length of arc AEB $= \pi \times \frac{2.8}{2} = 1.4\pi$ cm.

Length of arc BFC $= \pi \times \frac{1.4}{2} = 0.7\pi$ cm.

Length of arc ADC $= \pi\,\frac{2.8 + 1.4}{2} = 2.1\pi$ cm

Now, perimeter of shaded region

= Sum of lengths of the arcs AEB, BFC and ADC

$= 1.4\pi + 0.7\pi + 2.1\pi$

$= 4.2\pi = 4.2 \times \frac{22}{7} = 13.2$ cm.

6. Let OP = R = 7 cm

OA = r = 3.5 cm

$\angle$POQ = 30°

$\therefore$ Area of shaded part = Area of sector OPQO – Area of sector OABO

$$= \frac{\pi R^2\theta}{360°} - \frac{\pi r^2\theta}{360°} = \frac{\pi\theta}{360°}\,[R^2 - r^2]$$

$$= \frac{22}{7}\times\frac{30}{360°}\,[7^2 - (3.5)^2]$$

$$= \frac{11}{42}\,[49 - 12.25]$$

$= \frac{11}{42}[36.75] = \frac{11 \times 5.25}{6}$

$= \frac{57.75}{6} = 9.625\text{ cm}^2.$

7. 88.44 cm^2

Hint: Use $\frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin\theta$

$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$.

8. 66.5 cm^2

Hint: ***See solved example 7.***

WORKSHEET – 117

1. $2\pi r = 22$ cm

$\Rightarrow \quad r = \frac{22}{7} = \frac{11}{\pi}$

$\therefore$ Area of quadrant $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \frac{11}{\pi} \times \frac{11}{\pi}$

$= \frac{11 \times 11}{4 \times 22} \times 7 = \frac{77}{8}$.

2. $2\pi r = 22$

$\Rightarrow \quad r = \frac{22}{2\pi} = \frac{11}{\pi}$

$\therefore$ Area of quadrant $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \frac{11}{\pi} \times \frac{11}{\pi}$

$= \frac{121}{4\pi}$ cm^2.

3. Area of shaded part = Area of rectangle + Area of semicircle

$= (8 \times 4) + \frac{1}{2}\pi r^2$

{$\because$ Radius of circle = 6 – 4 = 2 m.}

$= 32 + \frac{\pi(2)^2}{2} = (32 + 2\pi)\text{ cm}^2.$

4. True.

Hint: $\angle AOB = 60°$

$ar(\Delta OAB) = \frac{\sqrt{3}}{4} \times (\text{side})^2.$

5. Let A_1 = area of circle with radius = 14 cm

$= \pi(14)^2 = 196\pi\text{ cm}^2$

A_2 = area of circle with radius = 7 cm

$= \pi(7)^2 = 49\pi\text{ cm}^2$

A_3 = area of sector BOD $= \frac{\pi(7)^2 \times 40}{360}$

and A_4 = area of sector AOC $= \frac{\pi(14)^2 \times 40}{360}$

Let $A_5 = (A_4 - A_3)$ = Area of ABDCA

$= \frac{\pi \times 40}{360}[14^2 - 7^2]$

$= \frac{\pi}{9}[21 \times 7] = \frac{49}{3}\pi\text{ cm}^2$

$\therefore$ Area of shaded part

$= (A_1 - A_2) - A_5$

$= (196\pi - 49\pi) - \frac{49}{3}\pi = 147\pi - \frac{49}{3}\pi$

$= \left(147 - \frac{49}{3}\right) \times \pi = \left(\frac{441 - 49}{3}\right) \times \frac{22}{7}$

$= \frac{392}{3} \times \frac{22}{7} = \frac{56 \times 22}{3}$

$= 410.66\text{ cm}^2.$

6. (*i*) The horse can graze in the shape of a quadrant of a circle with radius 5 m

$\therefore$ Required area $= \frac{1}{4}\pi \times (\text{radius})^2$

$= \frac{1}{4} \times 3.14 \times 5 \times 5$

$= 19.625\text{ m}^2.$

(*ii*) Radius of quadrant of circle

= Length of the rope = 10 m

Area of the sector $= \frac{1}{4} \times \pi \times 10^2$

$$= \frac{1}{4} \times 3.14 \times 100$$

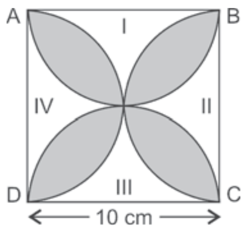
$$= 78.50 \text{ m}^2$$

So, the increase in the grazing area

$$= (78.50 - 19.625)\,\text{m}^2$$

$$= 58.875 \text{ m}^2.$$

7. Let us mark the four unshaded parts as I, II, III, IV in figure.



$\therefore$ Area of I + area of III = area of ABCD – area of two semicircles of radius 5 cm each.

$$= 100 - 3.14 \times 25$$

$$= 21.5 \text{ cm}^2$$

Similarly, area of II + area of IV = 21.5 cm^2

Area of shaded part

$$= ar(\text{ABCD}) - ar(\text{I + II + III + IV})$$

$$= 100 - 2 \times 21.5$$

$$= 57 \text{ cm}^2.$$

8. Radius of the circle having ABC as quadrant

$$= \text{AB} = \text{AC} = 14 \text{ cm}$$

Area of this quadrant ABC $= \frac{1}{4} \times \pi \times 14^2$

$$= 49\pi \text{ cm}^2$$

Area of isosceles ΔABC $= \frac{1}{2} \times \text{AC} \times \text{AB}$

$$= \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$$

$$\text{BC} = \sqrt{\text{AB}^2 + \text{AC}^2}$$

$$= \sqrt{14^2 + 14^2} = 14\sqrt{2} \text{ cm.}$$

Area of semicircle having diameter as BC

$$= \frac{1}{2}\pi \times \left(\frac{14\sqrt{2}}{2}\right)^2$$

$$= \frac{1}{2}\pi \times \left(7\sqrt{2}\right)^2 = 49\pi \text{ cm}^2$$

Now, area of shaded region = Area of ΔABC + Area of the semicircle with BC as diameter – Area of quadrant ABC

$$= 98 + 49\pi - 49\pi = 98 \text{ cm}^2.$$

WORKSHEET – 118

1. Angle $= \frac{360°}{60} \times 35 = 210°$.

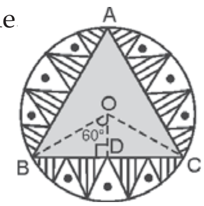
2. Perimeter of square

= Circumference of circle

$$= 2\pi r = 2 \times \frac{22}{7} \times 21 = 132 \text{ cm}$$

$\therefore$ Side of the square $= \frac{132}{4} = 33$ cm.

3. Let O be the centre of circle.



In ΔOBD, $\cos 60° = \frac{\text{OD}}{\text{OB}}$

and $\sin 60° = \frac{\text{BD}}{\text{OB}}$

$\Rightarrow \quad \frac{\text{OD}}{32} = \frac{1}{2}$ and $\frac{\sqrt{3}}{2} = \frac{\text{BD}}{32}$

$\Rightarrow \quad$ OD = 16 and BD = $16\sqrt{3}$

$\Rightarrow \quad$ BC = 2BD = $32\sqrt{3}$

$\therefore \quad$ Area of design

= Area of circle – Area of ΔABC

$$= \left\{\pi \times 32^2 - \frac{\sqrt{3}}{4} \times \left(32\sqrt{3}\right)^2\right\} \text{cm}^2$$

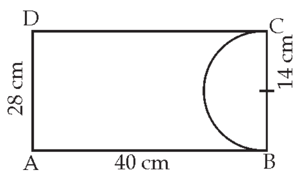
$$= 32^2 \times \left(3.14 - \frac{3 \times 1.73}{4}\right) \text{cm}^2$$

$$= 1024 \times \left(\frac{12.56 - 5.19}{4}\right) \text{cm}^2$$

$$= 256 \times 7.37 = 1886.72 \text{ cm}^2.$$

4. Area of remaining paper

= *ar* of rectangle ABCD – *ar* of semicircle with diameter BC

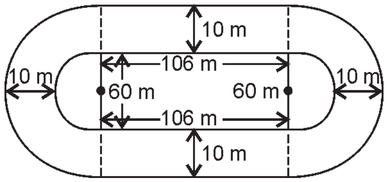


$= 40 \times 28 - \frac{\pi}{2}(14)^2$

$= 1120 - \frac{22}{2 \times 7} \times 14 \times 14$

$= 1120 - 308 = 812 \text{ cm}^2$

5.



(*i*) The distance around the track along the inner edge $= 106 + 106 + (\pi \times 30 + \pi \times 30)$

($\because$ Inner radius = 30 m)

$= 212 + \frac{22}{7} \times 60$

$= 212 + \frac{1320}{7} = \frac{2804}{7} \text{ m} = 400\frac{4}{7} \text{ m}.$

(*ii*) The area of track

$= (106 \times 80 - 106 \times 60) + 2 \times \frac{\pi}{2}\left[40^2 - 30^2\right]$

$= 2120 + 700 \times \frac{22}{7}$

$= 2120 + 2200 = 4320 \text{ m}^2.$

6. We have, area of the shaded region

= Area of the circle with OB (= 7 cm) as diameter + Area of semicircle with CD as diameter – Area of ΔACD

$= \pi \times \left(\frac{7}{2}\right)^2 + \frac{1}{2} \times \pi \times (7)^2 - \frac{1}{2} \times CD \times OA$

$= \left\{\frac{\pi}{4} \times 49 + \frac{\pi}{2} \times 49 - \frac{1}{2} \times 14 \times 7\right\} \text{cm}^2$

$= \left(\frac{3\pi}{4} \times 49 - 49\right) \text{cm}^2$

$= \left(\frac{3}{4} \times \frac{22}{7} \times 49 - 49\right) \text{cm}^2$

$= \frac{231 - 98}{2} \text{ cm}^2 = 66.5 \text{ cm}^2.$

7. With the given information, ΔABC is an equilateral Δ having sides 10 cm.

$\therefore \quad AB = BC = CA = 10 \text{ cm}$

and $\quad \angle A = \angle B = \angle C = 60°$

$\therefore$ E, F and D are the mid-points of AC, AB and BC, respectively and therefore AE = EC = CD = DB = DF = FA = 5 cm = r (Let)

$\therefore$ Area of sector BDF $= \frac{\theta}{360°}\pi r^2$

$= \frac{60°}{360°} \times 3.14 \times (5)^2$

$= \frac{1}{6} \times 3.14 \times 25 = 13.08 \text{ cm}^2$

$\therefore$ Area of total shaded region

= 3(Area of sector BDF)

$= 3 \times 13.08 = 39.24 \text{ cm}^2.$

8. Speed of bus = 40 km/h

$\therefore$ Distance covered in 1 hr = 40 km

$\therefore$ Distance covered in 1 min $= \frac{40 \times 1000}{60}$ m

$= \frac{4000}{6} \text{ m} = \frac{2000}{3} \text{ m}.$

Now radius of wheel $= \frac{126}{2} = 63$ cm

$\therefore$ Distance covered in one revolution

= circumference = $2\pi r$

$= 2 \times \frac{22}{7} \times 63 = 44 \times 9$

$= 396 \text{ cm} = \frac{396}{100} \text{ m} = 3.96 \text{ m}$

$\therefore$ 3.96 m distance makes = 1 revolution

1 m distance makes $= \frac{1}{3.96}$ revolution

$\frac{2000}{3}$ m distance makes $= \frac{1}{3.96} \times \frac{2000}{3}$ revolutions

$= \frac{200000}{1188}$ revolutions

= 168.35 revolutions (approx.)

$\therefore$ Number of complete rotations taken by wheel per min is = 168.

(*ii*) Circumference of circle.

(*iii*) Responsible.

WORKSHEET – 119

1. $\pi r^2 = 1.54 \Rightarrow r^2 = \dfrac{1.54}{22} \times 7 \Rightarrow r = 0.7$ m

Number of revolutions $= \dfrac{176}{2 \times \frac{22}{7} \times 0.7} = 40.$

2. This rhombus must be a square with diagonals as the diameters of the circle.

$\pi r^2 = 1256$

$\Rightarrow \quad r^2 = \dfrac{1256 \times 7}{22} \Rightarrow r = 20$ cm (approx.)

$\therefore$ Diameters $= d_1 = d_2 = 40$ cm

$\therefore$ Area of the rhombus $= \dfrac{1}{2} \times d_1 \times d_2$

$= \dfrac{1}{2} \times 40 \times 40$

$= 800$ cm^2.

3. Minutes elapsed by minute hand

$= 6.40 - 6.05 = 35$

$\because$ Angle covered by minute hand in 60 minutes $= 360°$

$\therefore$ Angle covered by minute hand in

35 minutes $= \dfrac{360°}{60°} \times 35 = 210°$

$\therefore$ Area $= \pi r^2 \times \dfrac{210°}{360°}$

$= \dfrac{22}{7} \times 5^2 \times \dfrac{7}{12} = \dfrac{275}{6}$

$= 45\dfrac{5}{6}$ cm^2.

4. True, because when areas are equal, the radii are equal and so circumferences are equal.

5. In ΔAOB, $AB^2 = 20^2 = 400$

And $AO^2 + BO^2 = (10\sqrt{2})^2 + (10\sqrt{2})^2$

$= 400$

i.e., $AO^2 + BO^2 = AB^2 \Rightarrow \angle AOB = 90°$

$\therefore$ $ar(\Delta AOB) = \dfrac{1}{2} \times AO \times BO$

$= \dfrac{1}{2} \times 10\sqrt{2} \times 10\sqrt{2}$

$= 100$ cm^2

ar(sector AOB) $= \pi \times (AO)^2 \times \dfrac{90°}{360°}$

$= 50\pi$ cm^2

ar(rectangle ABCD) $= AB \times BC$

$= 20 \times 10 = 200$ cm^2

Now, area of the shaded region

$= ar$(rectangle ABCD)

$- ar$(sector AOB) $+ ar(\Delta AOB)$

$= 200 - 50\pi + 100$

$= 300 - 50\pi = 50(6 - \pi)$ cm^2.

6. Area of shaded part

= Area of semicircle – Area ΔABC

$= \dfrac{1}{2}\pi r^2 - \dfrac{1}{2} \times BC \times AC.$

$= \dfrac{1}{2}\left[\pi\left(\dfrac{13}{2}\right)^2 - 5 \times 12\right]$

[$\because$ As $\angle C = 90°$

$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow 13^2 = 12^2 + BC^2$

$\Rightarrow 169 - 144 = BC^2$

$\Rightarrow BC^2 = 25$

$\Rightarrow BC = 5$ cm]

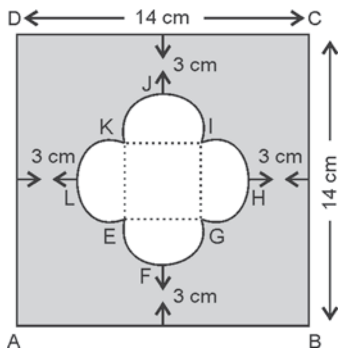
$= \dfrac{1}{2}[3.14 \times (6.5)^2 - 60]$

$= \dfrac{1}{2}[132.665 - 60] = \dfrac{1}{2}[72.665]$

$= 36.33$ cm^2 (approx).

7. Area of square ABCD $= BC^2$

$= 14^2 = 196$ cm^2



EFG, GHI, IJK and KLE are four semicircles of radius $\frac{14-3-3}{4}$ = 2 cm each.

EGIK is a square of side as the diameter of either circle, that is, 4 cm.

Sum of areas of the four semicircles

$= 4 \times$ area of one semicircle

$= 4 \times \frac{1}{2}\pi \times 2^2 = 8\pi \text{ cm}^2$

Area of square EGIK = (Side)^2 = 4^2 = 16 cm^2

Now, area of shaded region

= Area of square ABCD – Sum of areas of four semicircles – Area of square EGIK

$= 196 - 8\pi - 16$

$= (180 - 8\pi) \text{ cm}^2$.

8. In right ΔABC,

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$\Rightarrow \quad AC^2 = 14^2 + 14^2$

$\Rightarrow \quad AC = 14\sqrt{2}$ cm

$ar(\Delta ABC) = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 14 \times 14$

$= 98 \text{ cm}^2$

Radius of quadrant ABCP = AB = BC = 14 cm

ar(quadrant ABCP)

$= \frac{1}{4} \times$ Area of corresponding circle

$= \frac{1}{4} \times \pi(AB)^2 = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154 \text{ cm}^2$

$ar(\text{semicircle ACQ}) = \frac{1}{2} \times \pi \times \left(\frac{AC}{2}\right)^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{14\sqrt{2}}{2} \times \frac{14\sqrt{2}}{2}$ [$\because AC = 14\sqrt{2}$]

$= 154 \text{ cm}^2$

Now, area of shaded portion

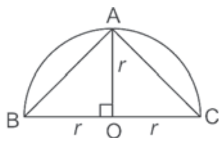
= *ar*(semicircle ACQ) – *ar*(quadrant ABCP) + *ar*(ΔABC)

$= 154 - 154 + 98 = 98 \text{ cm}^2$.

Hence, area of shaded portion is 98 cm^2.

CHAPTER TEST

1. Two vertices of the triangle should coincide with the two extremities of the diameter of the semicircle and the third one lies on the curve.



$ar(\Delta ABC) = \frac{1}{2} \times BC \times AO$

$= \frac{1}{2} \times 2r \times r$

$= r^2$ sq. units.

2. Hint: Diameter of circle = 16 cm

$\therefore$ Diagonal of a square = $\sqrt{2} \times$ side.

$\sqrt{2} \times \text{side} = 16$

$\text{side} = 8\sqrt{2}$

$\therefore$ Area of square = $(8\sqrt{2})^2$

$= 128 \text{ cm}^2$.

3. Let radius of given circle and side of given square be r and a respectively.

Perimeter of the circle = perimeter of the square

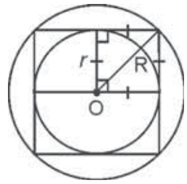
$\Rightarrow \quad 2\pi r = 4a \Rightarrow r = \frac{2a}{\pi}$

Required ratio $= \frac{\pi r^2}{a^2} = \frac{\pi}{a^2} \times \frac{4a^2}{\pi^2}$

$= \frac{4}{\pi} = \frac{4}{\frac{22}{7}} = \frac{14}{11}$

i.e., 14 : 11.

4. True.



Let radius of smaller circle = $r = \frac{5}{2}$

Let radius of larger circle = $R = \sqrt{2}\cdot\left(\frac{5}{2}\right)$

$\therefore$ Ratio of areas = $\frac{\pi R^2}{\pi r^2} = \frac{2\left(\frac{5}{2}\right)^2}{\left(\frac{5}{2}\right)^2} = 2:1.$

5. In 24 hrs, number of complete revolution taken by hour hand = 2

and number of complete revolution taken by minute hand = 24

$\therefore$ Distance travelled by hour hand in 24 hrs

$= 2 \times (2\pi r)$

$= 2 \times \pi(4) \times (2)$

$= 16\pi$ cm

Also distance travelled by minute hand in 24 hrs

$= 2\pi \times (6) \times 24 = 288\pi$ cm

$\therefore$ Sum of distances travelled

$= 16\pi + 288\pi = 304\pi$

$= 304 \times 3.14 = 954.56$ cm

6. Let the angle subtended by arc at the centre be θ.

Area of sector = 54π

$\Rightarrow \frac{\theta}{360^\circ} \times \pi \times (\text{Radius})^2 = 54\pi$

$\Rightarrow \frac{\theta}{360^\circ} \times \pi \times (36)^2 = 54\pi$

$\Rightarrow \theta = 360^\circ \times \frac{54}{36 \times 36} = 15^\circ$

Now, length of the arc = Radius × Angle in radian

$= 36 \times \frac{15^\circ}{180^\circ} \times \pi = 3\pi$ cm.

7. Area of the park = 31.15×4.40 m^2

$= 3115 \times 440$ cm^2

Radius of circle covered by each plant of one type, $r_1 = \frac{56}{2} = 28$ cm

$\therefore$ corresponding area = πr_1^2

$= \frac{22}{7} \times 28 \times 28 = 2464$ cm^2

Radius of circle covered by each plant of another type, $r_2 = 35$ cm

$\therefore$ corresponding area = πr_2^2

$= \frac{22}{7} \times 35 \times 35 = 3850$ cm^2

Let the number of plants of one type and another type be m and n respectively.

Total covered area = $m \times 2464 + n \times 3850$

Uncovered area = $4m \times 2464 + 4n \times 3850$

Total area of the park must be equal to the sum of covered and uncovered area.

$\therefore 2464\,m + 3850n + 4 \times 2464m + 4 \times 3850\,n = 3115 \times 440$

$\Rightarrow 5 \times 2464m + 5 \times 3850n = 3115 \times 440$

Dividing by 10, we get

$1232m + 1925n = 3115 \times 44$

or $1232m + 1925n = 137060$ ...(*i*)

According to the given number of plants, we have

$m + n = 100$ ...(*ii*)

or $1232m + 1232n = 123200$ ...(*iii*)

Subtracting (*iii*) from (*i*), we get

$693n = 13860 \Rightarrow n = \frac{13860}{693} = 20$

So, from (*ii*), we get

$\therefore \quad m = 100 - n = 100 - 20 = 80$

Hence, the required number of plants of each type are 80 and 20 respectively.

(*ii*) Area of circle

(*iii*) Love for plants, keeping environment neat and clean, and become a social worker.

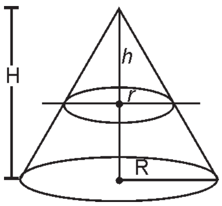
8. ***See Worksheet 119, Sol. 7.***

❑❑

Chapter 13 SURFACE AREAS AND VOLUMES

WORKSHEET – 121

1.



V_1 = Volume of smaller cone = $\frac{1}{3}\pi r^2 h$

V_2 = Volume of bigger cone = $\frac{1}{3}\pi R^2 H$

as $H = 2h$

$\Rightarrow \quad \frac{H}{h} = \frac{2}{1} \Rightarrow \frac{h}{H} = \frac{1}{2}$

Also then $\frac{r}{R} = \frac{1}{2}$

$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \left(\frac{r}{R}\right)^2\left(\frac{h}{H}\right)$

$= \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{1}{8} = 1 : 8.$

2. Volume of big sphere

$V_1 = \frac{4}{3}\pi (3)^3$

Volume of 1 small sphere

$V_2 = \frac{4}{3}\pi (0.3)^3$

$\therefore$ Number of balls obtained

$= \frac{V_1}{V_2} = \frac{27}{0.027} = 1000.$

3. 3 : 1.

Hint: Volume of cylinder = $V_1 = \pi r^2 h$

Volume of cone = $V_2 = \frac{1}{3}\pi r^2 h$

$\therefore \quad V_1 : V_2 = 1 : \frac{1}{3} = 3 : 1.$

4. Number of cones formed

$= \frac{\text{Volume of sphere of radius 10.5 cm}}{\text{Volume of smaller cone}}$

$= \frac{\frac{4}{3}\pi(10.5)^3}{\frac{1}{3}\pi(3.5)^2 \times 3} = \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3}$

$= \frac{4 \times 105 \times 105 \times 3.5 \times 10}{35 \times 10 \times 10}$

$= \frac{12 \times 105}{10} = \frac{1260}{10} = 126.$

5. Let r_1 = radius of conical vessel = 5 cm

h_1 = height of conical vessel = 24 cm

r_2 = radius of cylindrical vessel = 10 cm

h_2 = height of cylindrical vessel = ?

Let Volume of conical vessel = Volume of cylindrical vessel

$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\Rightarrow (5)^2 \times 24 = 3 \times (10)^2 \times h_2$

$\Rightarrow \quad h_2 = \frac{25 \times 24}{3 \times 10 \times 10} = \frac{25 \times 8}{100} = 2 \text{ cm}$

$\therefore$ Water will rise up to a height of 2 cm.

6. Volume of earth dug out = $\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 20 = 154 \times 20 = 3080 \text{ m}^3$

Area of platform = $22 \times 14 - \pi r^2$

$= 22 \times 14 - \frac{22}{7} \times 7^2$

$= 154 \text{ m}^2$

Now, height of the platform

$= \frac{\text{Volume of earth dug out}}{\text{Area of platform}}$

$= \frac{3080}{154} = 20 \text{ m.}$

7. (*i*) Volume of water collected on roof top = Volume of water stored in cylinder

$\therefore$ Volume of water stored in cylinder

$$= l \times b \times h = 22 \times 20 \times \frac{2.5}{100}$$

$$= \frac{22 \times 20 \times 25}{1000} = \frac{11000}{1000} = 11 \text{ m}^3$$

(*ii*) Let h = height of cylinder

$$\therefore \quad \pi r^2 h = 11$$

$$\frac{22}{7} \times 1 \times 1 \times h = 11$$

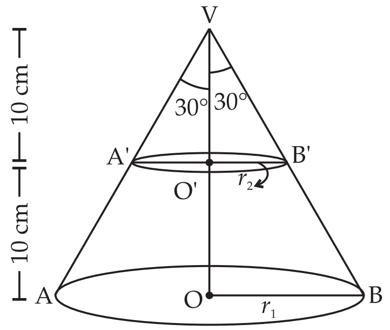
$$\Rightarrow \quad h = \frac{11 \times 7}{22} = 3.5 \text{ m.}$$

(*iii*) Surface area and volume of solids.

(*iv*) Proactive and resourceful.

8. VO = height = 20 cm

$\therefore$ VO′ = O′O = 10 cm



$\therefore$ In triangles VOA and VO′ A′

$$\tan 30° = \frac{OA}{VO} \text{ and } \tan 30° = \frac{O'A'}{VO'}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{r_1}{20} \text{ and } \frac{1}{\sqrt{3}} = \frac{r_2}{10}$$

$$\Rightarrow \quad r_1 = \frac{20}{\sqrt{3}} \text{ cm and } r_2 = \frac{10}{\sqrt{3}} \text{ cm}$$

$$\therefore \quad \text{Volume of frustum} = \frac{1}{3}\pi (r_1^2 + r_2^2 + r_1 r_2)h$$

$$= \frac{\pi}{3}\left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right) \times 10$$

$$= \frac{7000}{9}\pi \text{ cm}^3$$

Now, let length of wire be l cm, diameter is $\frac{1}{12}$ cm

$\therefore$ Volume of metal used in wire

$$= \pi \times \left(\frac{1}{24}\right)^2 \times l$$

$$\Rightarrow \quad \frac{7000}{9}\pi = \frac{\pi l}{24 \times 24}$$

$$\Rightarrow \quad l = \frac{7000 \times 24 \times 24}{9} \text{ cm}$$

$$\Rightarrow \quad l = \frac{70 \times 24 \times 24}{9} \text{ m}$$

$$\Rightarrow \quad l = 70 \times 64$$

$$= 4480 \text{ m.}$$

WORKSHEET – 122

1. Volume of sphere = $\frac{4}{3}\pi(3)^3$

Let length of wire = l.

Radius of wire (r) = $\frac{4}{2} = 2\,\text{mm} = \frac{2}{10}$ cm.

$$\therefore \quad \text{Volume of wire} = \pi r^2 l = \pi\left(\frac{2}{10}\right)^2 \times l$$

$\therefore$ Volume of sphere = Volume of wire

$$\Rightarrow \quad \frac{4}{3} \times 27 = \frac{4}{100} l$$

$$\Rightarrow \quad l = 900 \text{ cm.}$$

2. Hint: $V = \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$

$= 5(20 + 8)$

$= 140 \text{ cm}^3$.

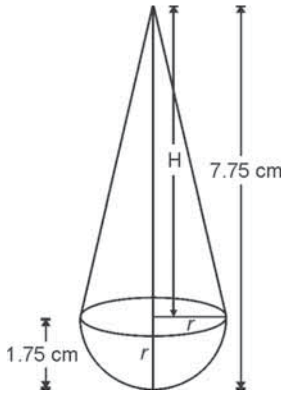
3. Let the base radii be $3x$ and $5x$ respectively and let the same height be h.

$$V_1 = \frac{1}{3}\pi (3x)^2 \times h \,;\; V_2 = \frac{1}{3}\pi (5x)^2 \times h$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{9x^2}{25x^2}, \text{ i.e., } V_1 : V_2 = 9 : 25.$$

4. 53.625 cm^2

Hint:



H = height of cone

$= 7.75 - 1.75$

$= 6$ cm

l = slant height of cone

$= \sqrt{r^2 + H^2}$

∴ T.S.A.

= C.S.A. of cone + C.S.A. of hemisphere

$= \pi rl + 2\pi r^2$

$= \pi r\,[l + 2r]$.

5. T.S.A. = C.S.A. + 2 × (C.S.A. of hemisphere)

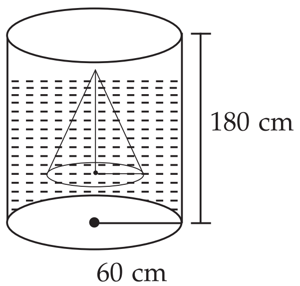
$= 2\pi rh + 2(2\pi r^2) = 2\pi rh + 4\pi r^2$

$= 2\pi r\,[h + 2r] = 2 \times \frac{22}{7} \times 3.5\,[10 + 7]$

$= 2 \times 22 \times 0.5 \times 17$

$= 374$ cm^2.

6.



Let r = radius of cone = 30 cm

h = height of cone = 60 cm

R = radius of cylinder = 60 cm

H = height of cylinder = 180 cm

∴ Volume of water left = Volume of cylinder – Volume of cone

$= \pi R^2 H - \frac{1}{3}\pi r^2 h$

$= \pi[60 \times 60 \times 180 - \frac{1}{3} \times 30 \times 30 \times 60]$

$= \frac{22}{7} \times 60 \times [60 \times 180 - \frac{1}{3} \times 30 \times 30]$

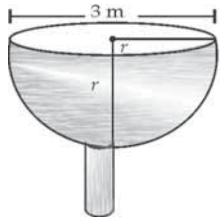
$= 630000 \times \frac{22}{7}$

$= 1980000$ $cm^3 = 1.98$ m^3.

7. Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$ $\quad (\because r = \frac{3}{2})$

$= \left(\frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}\right)$ m^3



∴ Volume to be emptied

$= \frac{1}{2} \times \left(\frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}\right)$

$= \left(\frac{11 \times 3 \times 3}{7 \times 2 \times 2}\right)$ m^3

As 1 m^3 = 1000 l $= \frac{11 \times 3 \times 3}{7 \times 2 \times 2} \times 1000\ l$

∴ According to question, $\frac{25}{7}\,l$ gets emptied in 1 sec.

⇒ 1l gets emptied in $\frac{7}{25}$ sec.

∴ $\frac{11 \times 3 \times 3}{7 \times 2 \times 2} \times 1000\ l$ gets emptied in

$= \frac{7}{25} \times \frac{11 \times 3 \times 3}{7 \times 2 \times 2}$

$= \frac{99}{100} \times 1000 = 990$ sec.

$= \frac{990}{60} = \frac{33}{2} = 16\frac{1}{2}$

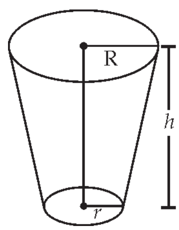
= 16 min. 30 sec.

OR

$r = 2$ cm

R = 4 cm

$h = 14$ cm



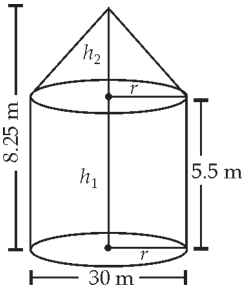
∴ Capacity of glass

$= \frac{1}{3}\pi h[r^2 + R^2 + rR]$

$= \frac{1}{3} \times \frac{22}{7} \times 14[4 + 16 + 8]$

$= \frac{44}{3} \times 28 = \frac{1232}{3} = 410.66$ cm^3.

8. $r = \frac{30}{2} = 15$ m



h_1 = height of cylindrical part = 5.5 m

h_2 = height of conical part = 8.25 – 5.5 = 2.75 m.

Area of canvas used = C.S.A of cone + C.S.A of cylinder

$= \pi rl + 2\pi r h_1$

$A = \pi r(l + h_1)$ ...(*i*)

Now $l = \sqrt{h_2^2 + r^2} = \sqrt{(2.75)^2 + (15)^2}$

$= \sqrt{7.5625 + 225} = \sqrt{232.5625}$

= 15.25 m

$\therefore$ From (*i*), $A = \frac{22}{7} \times 15(15.25 + 5.5)$

$= \frac{22}{7} \times 15 \times 20.75 = 978.214 \text{ m}^2$

$\therefore$ Length of canvas $= \frac{\text{Area of canvas}}{\text{breadth}}$

$= \frac{978.214}{1.5} = 652.14$ m.

WORKSHEET – 123

1. $V_1 = \pi r^2 h$

$V_2 = \pi\left(\frac{r}{2}\right)^2 h$

$\therefore$ $V_1 : V_2 = 1 : \frac{1}{4} = 4 : 1$

2. Let radius of the cone = r

$\frac{1}{3}\pi r^2 x = \frac{4}{3}\pi x^3$

$\Rightarrow$ $r = 2x$ cm.

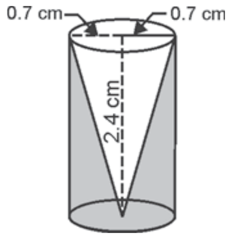
3. Let the required number of cubes be n.

Volume of n cubes = Volume of the ball

$\Rightarrow n \times 1^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

$\Rightarrow$ $n = 4851$.

4. T.S.A. of remaining solid



= C.S.A. of cylinder + Area of circular base + C.S.A. of cone.

$= 2\pi rh + \pi r^2 + \pi rl$

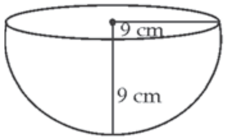
$= \pi r\,[2h + r + l]$;

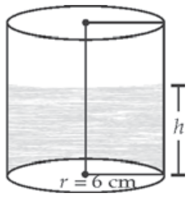
$= \frac{22}{7} \times 0.7[4.8 + 0.7 + 2.5]$

$\left[\therefore\ l = \sqrt{(2.4)^2 + (0.7)^2} = 2.5 \text{ cm}\right]$

$= 17.6 \approx 18 \text{ cm}^2$.

5.





R = 9 cm

r = 6 cm

Let h = height of water in cylindrical vessel

$\therefore$ Volume of water in hemisphere = Volume of water in cylinder upto height h

$\Rightarrow$ $\frac{2}{3}\pi R^3 = \pi r^2 h$

$\Rightarrow$ $\frac{2}{3}(9)^3 = (6)^2 \cdot h$

$\Rightarrow$ $h = \frac{\frac{2}{3} \times 9 \times 9 \times 9}{6 \times 6}$

$= \frac{27}{2} = 13.5$ cm

6. Area of canvas required = C.S.A of conical part + C.S.A of cylindrical part

$= \pi rl + 2\pi RH$

$= \pi\left[\frac{3}{2} \times 2.8 + 3 \times 2.1\right]$

$\left[\begin{array}{l} \because\ r = \text{radius of cone} \\ R = \text{radius of cylinder} \\ l = \text{slant height of cone} \\ H = \text{hight of cylinder} \end{array}\right]$

$= \frac{22}{7}[3 \times 1.4 + 6.3] = \frac{22}{7}[4.2 + 6.3]$

$= \frac{22}{7} \times 10.5 = 22 \times 1.5 = 33.0 = 33\ m^2$

Now, as 1 m^2 costs `500

$\therefore$ Cost of 33 m^2 = 33 × 500 = `16500.

7. 157.5 cm.

Hint: Volume of water flows out of pipe in 1 hour $(V_1) = \pi(1)^2 \times 70 \times 3600\ cm^3$

Let level of water rise in tank = h

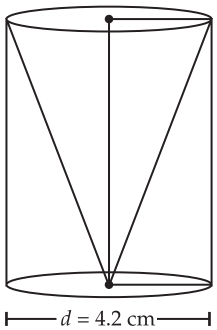
Volume of tank $(V_2) = \pi(40)^2 \times h$

$\therefore \quad V_1 = V_2$

$\Rightarrow \quad h = \frac{70 \times 3600}{40 \times 40}$

$= 157.5$ cm.

8. $r = \frac{4.2}{2} = 2.1$ cm and $h = 2.8$ cm



$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (2.8)^2}$

$= \sqrt{4.41 + 7.84} = \sqrt{12.25}$

$= 3.5$ cm

$\therefore$ T.S.A. of remaining solid

= C.S.A. of cylinder + Area of circular base + C.S.A. of cone

$= 2\pi rh + \pi r^2 + \pi rl$

$= \pi r\,(2h + r + l)$

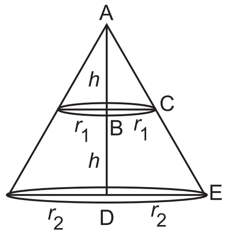
$= \frac{22}{7} \times 2.1\,(2 \times 2.8 + 2.1 + 3.5)$

$= \frac{22}{7} \times \frac{21}{10} \times (5.6 + 2.1 + 3.5)$

$= \frac{66}{10} \times 11.2 = 73.92\ cm^2$

WORKSHEET – 124

1. $\Delta ABC \sim \Delta ADE$



$\frac{r_2}{r_1} = \frac{2h}{h} \Rightarrow r_2 = 2r_1$

Volume of upper part

$V_1 = \frac{1}{3}\pi r_1^2 h$

Volume of lower part

$V_2 = \frac{1}{3}\pi h\,(r_1^2 + 4r_1^2 + 2r_1^2) = \frac{7}{3}\pi r_1^2 h$

$\therefore \quad V_1 : V_2 = 1 : 7.$

2. Let required height = H.

$\pi r^2 H + \frac{1}{3}\pi r^2 h = 3 \times \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad H + \frac{1}{3}h = h \Rightarrow H = \frac{2h}{3}.$

3. Volume of whole solid

= Volume of cone + Volume of cylinder + Volume of hemisphere.

$= \frac{1}{3}\pi r^2 H + \pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2\,[H + 3h + 2r]$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times (2.8 + 19.5 + 7)$

$= \frac{22 \times 7}{3 \times 4} \times 29.3 = 376.016\ cm^3.$

4. Volume of hemisphere $= \frac{2}{3}\pi r^3$

$\Rightarrow \quad \frac{2}{3}\pi r^3 = 2425\frac{1}{2}$

$\Rightarrow \quad \frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{4851}{2}$

$\Rightarrow \quad r^3 = \frac{4851 \times 7 \times 3}{2 \times 22 \times 2}$

$\Rightarrow \quad r^3 = \frac{441 \times 7 \times 3}{2 \times 2 \times 2} = \frac{7 \times 7 \times 3 \times 3 \times 3}{2 \times 2 \times 2}$

$\Rightarrow \quad r = \sqrt[3]{\frac{7 \times 7 \times 7 \times 3 \times 3 \times 3}{2 \times 2 \times 2}} = \frac{7 \times 3}{2}$

$= \frac{21}{2}$ cm.

$\therefore$ C.S.A of hemisphere $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 1386\ cm^2.$

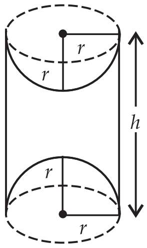
5. $h = 10$ cm

$r = 3.5$ cm

$\therefore$ Volume of wood in toy

= Volume of cylinder

– 2 × (Volume of hemisphere)



$= \pi r^2 h - 2 \times \left(\frac{2}{3}\pi r^3\right)$

$= \pi r^2 h - \frac{4}{3}\pi r^3 = \pi r^2\left[h - \frac{4}{3}r\right]$

$= \frac{22}{7} \times 3.5 \times 3.5 \times \left[10 - \frac{4}{3} \times 3.5\right]$

$= 22 \times 0.5 \times 3.5 \times \left[\frac{30-14}{3}\right]$

$= 11 \times 3.5 \times \frac{16}{3} = 205.33 \text{ cm}^3.$

6. Total surface area of solid cuboidal block

$(S_1) = 2(lb + bh + hl)$

$= 2(15 \times 10 + 10 \times 5 + 15 \times 5)$

$= 550 \text{ cm}^2$

Area of two circular base $= 2\pi r^2$

$(S_2) = 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right) \times \left(\frac{7}{2}\right) = 77 \text{ cm}^2$

Curved surface area of cylinder $= 2\pi rh$

$(S_3) = 2 \times \frac{22}{7} \times \frac{7}{2} \times 5$

$= 110 \text{ cm}^2$

$\therefore$ Required area $= (S_1 + S_3 - S_2)$

$= 550 + 110 - 77 = 583 \text{ cm}^2.$

7. Volume of cylindrical tank $= \pi(5)^2 \times 2$

$= 50\pi \text{ cm}^3$

$\therefore$ Volume of water that flows through pipe in x hours

= Volume of cylinder of radius 10 cm and length (= $4x$ km) = $4000x$ m.

$= \pi \times \left(\frac{1}{10}\text{m}\right)^2 \times 4000x \text{ m} = 40\pi x \text{ m}^3$

$\therefore 40\pi x = 50\pi$

$\Rightarrow \quad x = \frac{5}{4}$ hrs. = 1 hr 15 min.

8. r = radius of pipe = 1 cm

R = radius of tank = 40 cm

Rate of flow = 0.4 m/s. = 40 cm/s.

= 40 × 60 × 60 cm/hr.

Volume of water flow out through the pipe in $\frac{1}{2}$ hr

$V_1 = \pi r^2 \times \left(\frac{40 \times 60 \times 60}{2}\right)$

$= \pi \times 1 \times 1 \times 20 \times 60 \times 60$

Let required height be h.

Volume of water flown in cylinder

$V_2 = \pi R^2 h = \pi(40)^2 h$

According to question,

$V_2 = V_1$

$\Rightarrow \pi \times 20 \times 60 \times 60 = \pi \times 40 \times 40 \times h$

$\Rightarrow \quad h = \frac{20 \times 60 \times 60}{40 \times 40} = \frac{90}{2} = 45$ cm.

WORKSHEET – 125

1. Surface area of cone

= Surface area of hemisphere

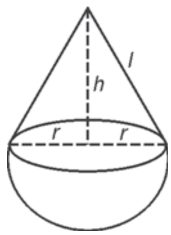
$\Rightarrow \pi r\sqrt{r^2 + h^2} = 2\pi r^2$

$\Rightarrow \quad r^2 + h^2 = 4r^2$

$\Rightarrow \quad 3r^2 = h^2$

$\Rightarrow \quad \frac{r^2}{h^2} = \frac{1}{3}$

$\Rightarrow \quad r : h = 1 : \sqrt{3}$.



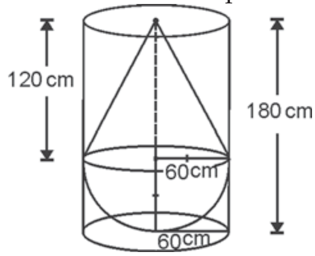
2. $N \times \frac{4}{3}\pi \times \left(\frac{4.2}{2}\right)^3 = 66 \times 42 \times 21$

$\Rightarrow N = \frac{66 \times 42 \times 21 \times 3 \times 7}{4 \times 22 \times 2.1 \times 2.1 \times 2.1} = 1500.$

3. Let H = 120 cm = height of cone

h = 180 cm = height of cylinder

r = 60 cm = radius of cone, cylinder and hemisphere



∴ Volume of water left in cylinder

= Volume of cylinder – Volume of cone – Volume of hemisphere

$$= \pi r^2 h - \frac{1}{3}\pi r^2 H - \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left[h - \frac{H}{3} - \frac{2}{3}r\right]$$

$$= \frac{22}{7} \times 60 \times 60\ [180 - 40 - 40]$$

$$= \frac{22}{7} \times 60 \times 60 \times 100\ cm^3 = 1.131\ m^3.$$

4. False, because $\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{2}$.

5. Let r = radius of cylinder = 3 cm

h = height of cylinder = 5 cm

R = radius of cone = $\frac{3}{2}$ cm

H = height of cone = $\frac{8}{9}$ cm

∴ Volume of cone = $\frac{1}{3}\pi R^2 H$

$$= \frac{1}{3}\pi \left(\frac{3}{2}\right)^2 \times \frac{8}{9} = \frac{2}{3}\pi\ cm^3$$

Now, volume of metal left in cylinder

= Volume of cylinder – Volume of cone

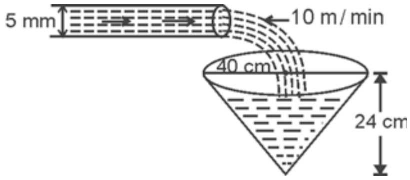
$$= \pi r^2 h - \frac{2}{3}\pi = \pi(3)^2 \times 5 - \frac{2}{3}\pi$$

$$= 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3}\ cm^3$$

$$\therefore \frac{\text{Volume of metal left in cylinder}}{\text{Volume of metal taken out}}$$

$$= \frac{\frac{133\pi}{3}}{\frac{2}{3}\pi} = \frac{133\pi}{3} \times \frac{3}{2\pi} = 133 : 2.$$

6.



r = radius of cone = $\frac{40}{2}$ = 20 cm

h = height of cone = 24 cm

∴ Volume of cone (V_1) = $\frac{1}{3}\pi r^2 h$ cm^3

Also, volume of water flows out of pipe in 1 min

$$V_2 = \pi R^2 H$$

$$= \pi\left(\frac{5}{20}\right)^2 \times 1000\ cm^3$$

Let conical vessel fills in 't' min.

$$\therefore \quad V_1 = t \times V_2$$

$$\Rightarrow \quad t = \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi(20)^2 \times 24}{\pi\left(\frac{5}{20}\right)^2 \times 1000}$$

$$\frac{400 \times 8 \times 400}{25 \times 1000} = 51.2\ \text{min.}$$

= 51 min 12 sec.

7. r_1 = radius of small cylinder = 8 cm

h_1 = height of small cylinder = 60 cm

r_2 = radius of big cylinder = 12 cm

h_2 = height of big cylinder = 220 cm

Volume of pole

= Volume of small cylinder + Volume of big cylinder.

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$

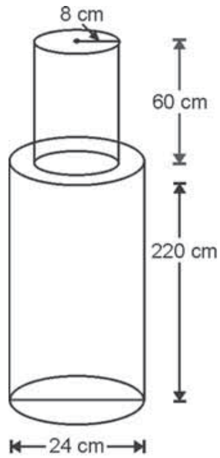
$$= 3.14 \times [64 \times 60 + 144 \times 220]$$

$$= 3.14 \times [3840 + 31680]$$

$$= 111532.8\ cm^3$$

∴ Mass = 111532.8×8 g

= 892.26 kg.



8. r = 2.8 m

h_1 = 3.5 m = height of cylinder

h_2 = 2.1 m = height of cone

l = slant height

$$= \sqrt{h_2^2 + r^2} = \sqrt{(2.1^2) + (2.8)^2}$$

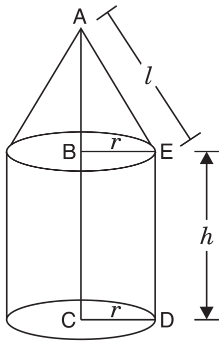
$= \sqrt{4.41+7.84} = \sqrt{12.25} = 3.5$ m

$\therefore$ Area of canvas used for 1 tent = CSA of cone + CSA of cylinder

$= \pi rl + 2\pi rh_1 = \pi r\,(l + 2h_1)$

$= \frac{22}{7} \times 2.8\,(3.5 + 7) = \frac{22\times4}{10}\times\frac{105}{10}$

$= \frac{88\times105}{100} = 92.40\ \text{m}^2$



$\therefore$ Area required for 1500 tents

$= 1500 \times 92.40 = 138600\ \text{m}^2$

Cost = 138600 × 120 = ` 16632000

$\therefore$ Amount shared by each school

$=$ ` $\frac{16632000}{50}$ = ` 332640

WORKSHEET – 126

1. Number of solid spheres

$$= \frac{\text{Volume of cylinder}}{\text{Volume of 1 sphere}}$$

$$= \frac{\pi r^2 h}{\frac{4}{3}\pi R^3} + \frac{3\times(2)^2\times45}{4\times(3)^3} = \frac{45}{3\times3} = 5$$

2. C.S.A.= Inner C.S.A. + Outer C.S.A.

$$= 2\pi r_1^2 + 2\pi r_2^2 = 2\pi\left(r_1^2 + r_2^2\right).$$

3. Number of bags $= \frac{\text{Volume of circular drum}}{\text{Volume of each bag}}$

$$= \frac{3.14\times(4.2)^2\times3.5}{2.1} \approx 92.$$

4. L = 15 cm, B = 10 cm and H = 5 cm

Volume of block $= L\times B\times H = 15\times10\times5$

$= 750\ \text{cm}^3$.

The hole is only possible throughout the surface having area 15 cm × 10 cm.

Volume of circular hole

$$= \pi r^2 h = \frac{22}{7}\times\left(\frac{7}{2}\right)^2\times5$$

$$= \frac{22\times7}{4}\times5$$

$$= \frac{154\times5}{4} = 192.5\ \text{cm}^3$$

$\therefore$ Volume of remaining solid

$= 750 - 192.5$

$= 557.5\ \text{cm}^3$

5. Volume of ice cream $= \frac{5}{6}$ [volume of cone + volume of hemisphere]

$$= \frac{5}{6}\left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right]$$

$$= \frac{5}{6}\times\frac{1}{3}\times\pi r^2\,[h + 2r]$$

Put r = 5 cm, h = 5 cm, $\pi = \frac{22}{7}$

$$\text{Volume} = \frac{5}{6}\times\frac{1}{3}\times\frac{22}{7}\times5\times5\,[5+10]$$

$$= \frac{22\times125}{126}\times15 = \frac{11\times125\times5}{21}$$

$\cong 327.38\ \text{cm}^3$.

6. Let r = radius of sphere $= \frac{12}{2} = 6$ cm

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3$

$$= \frac{4}{3}\pi\times6\times6\times6 = 288\,\pi\ \text{cm}^2$$

Also

Let radius of cylinder = R

Let height up to which water level rises in cylinder $= h = 3\frac{5}{9} = \frac{32}{9}$ cm

$\therefore$ Volume $= \pi R^2 h$

According to question

$\pi r^2 h = 288\,\pi$

$\Rightarrow \quad \pi R^2 \times \frac{32}{9} = 288\,\pi$

$\Rightarrow \quad R^2 = \frac{288 \times 9}{32} = \frac{72 \times 9}{8} = 81$

$\therefore \quad R = \sqrt{81} = 9$ cm

$\therefore$ Required diameter = **18 cm**.

7. h = 24 cm

r_1 = radius of upper circular end = $\frac{30}{2}$ = 15 cm

r_2 = radius of lower circular end = $\frac{10}{2}$ = 5 cm

$\therefore \quad l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{24^2 + 10^2}$

$= \sqrt{576 + 100} = \sqrt{676} = 26$ cm

$\therefore$ T.S.A. of container = $\pi l(r_1 + r_2) + \pi r_2^2$

$= \pi\left[l(r_1 + r_2) + r_2^2\right]$

= 3.14[26 × 20 + 25]

= 3.14 × 545 = 1711.3 cm^2

$\therefore$ Cost of metal sheet used = 1711.3 × $\frac{10}{100}$

= ` 171.13.

8. Diameter = 5 cm, Radius = 2.5 cm,

Height = 10 cm

Volume of glass of type A = $\pi r^2 h$

= 3.14 × 2.5 × 2.5 × 10 = 196.25 cm^3

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$

= 32.71 cm^3

$\therefore$ Volume of glass of type B = 163.54 cm^3

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 2.5 \times 2.5 \times 1.5$

= 3.14 × 2.5 × 2.5 × 0.5

= 9.81 cm^3

Volume of glass of type C = 196.25 – 9.81

= 186.44 cm^3

(*i*) The volume of glass of type A = 196.25 cm^3.

(*ii*) The glass of type B has the minimum capacity of 163.54 cm^3.

(*iii*) Volume of solid figures (Mensuration)

(*iv*) Honesty.

WORKSHEET – 127

1. $n \times \frac{4}{3}\pi \times \left(\frac{6}{2}\right)^3 = \frac{1}{3}\pi \times (12)^2 \times 24$

$\Rightarrow \quad n = \frac{12 \times 12 \times 8 \times 3}{4 \times 3 \times 3 \times 3}$

$n = 32.$

2. C.S.A. = Inner C.S.A. + Outer C.S.A.

$= 2\pi r_1^2 + 2\pi r_2^2 = 2\pi\left(r_1^2 + r_2^2\right).$

3. $\pi r^2 h = 448\,\pi \Rightarrow r^2 \times 7 = 448$

$\Rightarrow r = \sqrt{64} \quad \Rightarrow r = 8$ cm

Curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 8 \times 7$

= 352 cm^2.

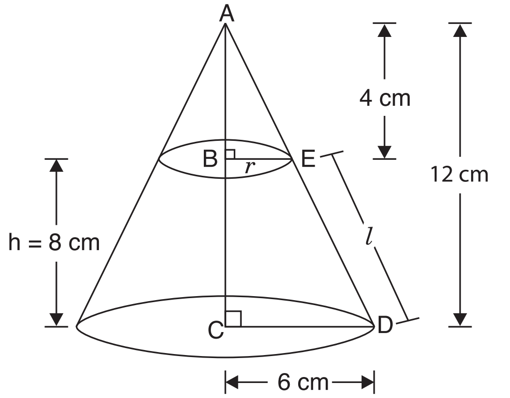
4. False, sides become a, a and $2a$ and so surface area will become

$2(a \times a + a \times 2a + 2a \times a) = 2(a^2 + 2a^2 + 2a^2)$

$= 10a^2.$

5. T.S.A of remaining solid = T.S.A of frustum of cone so obtained

$= \pi r^2 + \pi R^2 + \pi l\,(R + r)$ ...(*i*)



Let $\quad R = 6$ cm

$r = ?$

As $\Delta ABE \sim \Delta ACD$

$\Rightarrow \quad \frac{AB}{AC} = \frac{BE}{CD}$

$\Rightarrow \quad \frac{4}{12} = \frac{r}{R} \quad \Rightarrow \frac{1}{3} = \frac{r}{6} \quad \Rightarrow \quad r = 2$ cm

Also $l = \sqrt{h^2 + (R-r)^2} = \sqrt{8^2 + 2^2} = 4\sqrt{5}$ cm

$\therefore$ From (*i*) $\Rightarrow$ TSA $= \pi[(2)^2 + (6)^2 + 4\sqrt{5}(6+2)]$

$= \frac{22}{7}[32 \times 2.236 + 40] = \frac{22}{7} \times (111.552)$

$= 350.592 \text{ m}^2$

6. $h = 7$ cm; $r = \frac{5}{2}$ mm $= 0.25$ cm

Volume of the barrel $= \pi r^2 h$

$= \frac{22}{7} \times 0.25 \times 0.25 \times 7 = 1.375 \text{ cm}^3$

Volume of ink in the bottle

$= \frac{1}{5}$ litre $= \frac{1}{5} \times 1000 \text{ cm}^3 = 200 \text{ cm}^3$

Number of barrels filled by the ink of bottle

$= \frac{200}{1.375} = \frac{200000}{1375} = \frac{1600}{11}$

$\therefore$ Number of words written by 1 barrel = 330

$\therefore$ Number of words written by $\frac{1600}{11}$ barrels

$= 330 \times \frac{1600}{11} = 48000.$

7. Let r = radius of smaller sphere = 3 cm and R = radius of new sphere formed

$\therefore \quad V_1$ = Volume of smaller sphere

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

V_2 = Volume of bigger sphere

as density of metal $= \frac{\text{mass}}{\text{volume}} = \frac{1}{36\pi}$

$\Rightarrow$ Volume of bigger sphere $\times$ density = Mass

$= \frac{\text{Mass}}{\text{density}} = \frac{7}{\left(\frac{1}{36}\pi\right)}$

$\therefore \quad V_2 = 252\pi \text{ cm}^3$

Now, let V = Volume of new sphere

$\therefore \quad V = V_1 + V_2 = 36\pi + 252\pi$

$\frac{4}{3}\pi R^3 = 288\pi$

$\Rightarrow \quad R^3 = \frac{288 \times 3}{4} = 72 \times 3 = 216$

$\Rightarrow \quad R = 6$ cm

$\therefore \quad$ Diameter = 12 cm.

8. (*i*) $A_1 = 40 \text{ cm}^2$, $A_2 = 160 \text{ cm}^2$, $h = 45$ cm

$\therefore$ Volume of bucket

$= \frac{h}{3}[A_1 + A_2 + \sqrt{A_1A_2}]$

$= \frac{45}{3}[40 + 160 + \sqrt{6400}]$

$= 15[200 + 80]$

$= 15 \times 280 = 4200 \text{ cm}^3$

(*ii*) Volume of each glass $= \pi r^2 h$

$= \frac{22}{7} \times 2 \times 2 \times 7 \left(\because r = \frac{\text{diameter}}{2}\right)$

$= 88 \text{ cm}^3$

Number of glasses needed

$= \frac{\text{Volume of the bucket}}{\text{Volume of one glass}} = \frac{4200}{88}$

$= 47.72$ (approx.)

$= 48$ glasses. (rounded)

(*iii*) Kindheartedness and Cooperation.

CHAPTER TEST

1. $49 \times 33 \times 24 = \frac{4}{3} \times \frac{22}{7} \times r^3$

$\Rightarrow \quad r = \sqrt[3]{9261} \quad \Rightarrow \quad r = 21$ cm.

2. $\frac{V_1}{V_2} = \frac{64}{27} \Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{4^3}{3^3} \Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$

$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

$\therefore \quad S_1 : S_2 = 16 : 9.$

3. $\quad r_1 = \frac{44}{2} = 22$ cm,

$r_2 = \frac{24}{2} = 12$ cm, $h = 35$ cm

$$\text{Capacity} = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{22}{7} \times \frac{35}{3} \times (484 + 144 + 264)$$

$$= 32706.67\text{cm}^3 = \frac{32706.67}{1000} l$$

$= 32.7\ l.$

4. True, because capacity

$$= \pi r^2 h - \frac{2}{3}\pi r^3 = \frac{\pi r^2}{3}(3h - 2r).$$

5. Radius of each cone = $r = \frac{6}{2} = 3$ cm

Let the heights of the cone be h_1 and h_2 respectively.

$\therefore \quad h_1 + h_2 = 21$ cm ...(*i*)

Given: $\frac{V_1}{V_2} = \frac{2}{1}$

$$\therefore \quad \frac{\frac{1}{3}\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{2}{1} \Rightarrow h_1 = 2h_2$$

Substitute $h_1 = 2h_2$ in equation (*i*) to get $h_2 = 7$ cm $\quad \therefore h_1 = 14$ cm

$$\text{Now, } V_1 = \frac{1}{3}\pi r^2 h_1 = \frac{1}{3} \times \frac{22}{7} \times 3^2 \times 14 = 132 \text{ cm}^3$$

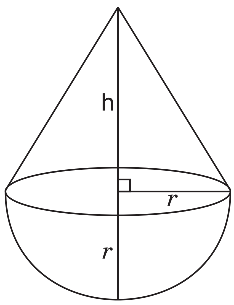
$$\text{and } V_2 = \frac{1}{3}\pi r^2 h_2 = \frac{1}{3} \times \frac{22}{7} \times 3^2 \times 7 = 66 \text{ cm}^3$$

Volume of remaining portion
= Volume of the cylinder – ($V_1 + V_2$)
$= \pi r^2 \times 21 - (132 + 66)$

$$= \frac{22}{7} \times 3^2 \times 21 - (132 + 66)$$

$= 594 - 198 = 396$ cm^3.

6.



$r = 3.5$ cm
H = height of toy
h = height of cone

Volume of toy = $166\frac{5}{6}$ cm^3

$\therefore \quad 166\frac{5}{6}$ = Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 [h + 2r]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 [h + 3.5 \times 2]$$

$$\Rightarrow \quad \frac{1001}{6} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}[7 + h]$$

$$\Rightarrow \quad 7 + h = \frac{1001 \times 7}{22 \times 7} = 13$$

$\Rightarrow \quad h = 6$ cm

Also area hemispherical part of toy = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 3.5 \times 3.5 = 77 \text{ cm}^2$$

$\therefore$ Cost of painting = 77 × 10 = ` 770.

7. Let internal radius of pipe = x m.
and radius of base of tank

$$= 40 \text{ cm} = \frac{2}{5} \text{ m}$$

Level of water raised in tank

$$= 3.15 \text{ or } \frac{315}{100}$$

Volume of water delivered in $\frac{1}{2}$ hr

$= \pi r^2 h = \pi(x)^2 \times 1260$ m

[$\because$ 2.52 km = 1 hr]

2520 m = 1 hr

$\therefore$ In $\frac{1}{2}$ hr height = $\frac{1}{2} \times 2520 = 1260$ m

$\therefore$ According to question,

$$\Rightarrow \quad \pi(x^2)(1260) = \pi \times \frac{2}{5} \times \frac{2}{5} \times \frac{315}{100}$$

$$\Rightarrow \quad x^2 = \frac{4}{25} \times \frac{315}{100} \times \frac{1}{1260} = \frac{1}{2500}$$

$$\Rightarrow \quad x = \frac{1}{50} \text{ m} = 2 \text{ cm}$$

$\therefore$ Internal diameter of pipe = 4 cm.

8. (*i*) V_1 = Volume of juice in cubical container

$= (5 \times 6 \times 22) \text{ cm}^3$

V_1 = Volume of juice in cubical container

$$= \frac{22}{7} \times (7)^2 \times 22 \text{ cm}^3$$

V_3 = Volume of juice in each small cone

$$= \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times 3.5 \text{ cm}^3$$

$\therefore$ **Case I.** If cubical packing is purchased, then number of small cones needed

$$= \frac{V_1}{V_3} = \frac{5 \times 6 \times 22}{\frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 3.5}$$

$$= \frac{6 \times 10 \times 3}{2 \times 2} = 3 \times 5 \times 3 = 45$$

Case II. If cylindrical packing is purchased

$\therefore$ Number of small cones needed

$$= \frac{V_2}{V_3} = \frac{\frac{22}{7} \times 7 \times 7 \times 22}{\frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 3.5}$$

$$= \frac{7 \times 7 \times 22 \times 3 \times 10}{2 \times 2 \times 35} = 7 \times 11 \times 3 = 231.$$

(*ii*) Mr Sharma must purchase cylindrical packing to serve maximum children.

(*iii*) Surface area and volume of solids.

(*iv*) Kindheartedness and helpful.

❑❑

Chapter 14

STATISTICS

WORKSHEET – 129

1. 21.1

Hint: 3 Median = mode + 2 mean.

2. 3 Median = 2 mean + mode

$$\Rightarrow \quad \text{Mean} = \frac{3\text{ median } - \text{mode}}{2}$$

$$= \frac{3 \times 45.5 - 50.5}{2} = \frac{136.5 - 50.5}{2}$$

$$= \frac{86.0}{2} = 43.$$

3. In such case, mean will increase by 3.

$\therefore$ New mean = 18 + 3 = 21.

4. $x = 26$

Hint: Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i}$.

5. As

Cost of living index	No. of weeks f_i	Cumulative frequency c.f.
1400–1550	8	8
1550–1700	15	23
1700–1850	21	44
1850–2000	8	N = 52
	N = 52	

$$\therefore \quad \frac{N}{2} = \frac{52}{2} = 26$$

$\therefore$ c.f. just greater than 26 is 44

$\therefore$ Median class is 1700–1850.

6. 36.25

Hint: Here maximum class frequency is 32. So, the modal class is 30-40.

Now, $l = 30$, $f_1 = 32$, $f_0 = 12$, $f_2 = 20$, $h = 10$

Use the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h.$$

7. For less than ogive:

Age	No. of participants
Less than or equal to 15	37
Less than or equal to 30	82
Less than or equal to 45	109
Less than or equal to 60	118
Less than or equal to 75	125
Less than or equal to 90	128

So we'll plot following points on graph

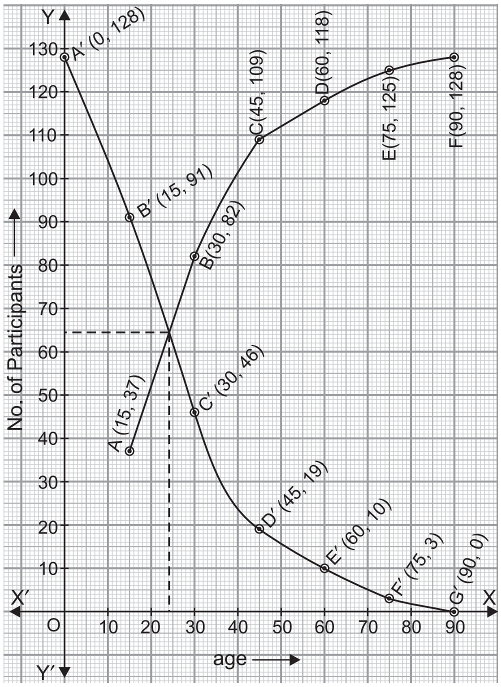
A(15, 37), B(30, 82), C(45, 109)

D (60, 118), E(75, 125), F(90, 128)

More than ogive:

Age	No. of participants
More than or equal to 0	128
More than or equal to 15	91
More than or equal to 30	46
More than or equal to 45	19
More than or equal to 60	10
More than or equal to 75	3
More than or equal to 90	0

So we will plot following points on graph.
A′(0, 128), B′(15, 91), C′(30, 46), D′(45, 19), E′(60, 10), F′(75, 3), G′(90, 0).



From Graph: The two ogive are intersecting at P drop a line from P, perpendicular to x-axis it meets x-axis at 24

$\therefore$ Median = 24.

WORKSHEET – 130

1. Here, $a = 25$, $h = 10$.

$$\therefore \quad \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 25 + 10\left(\frac{20}{100}\right) = 27.$$

2. 4

3. The given distribution can be represented as:

Marks obtained	No. of students
0-10	5
10-20	3
20-30	4
30-40	3
40-50	6
More than 50	42

Clearly, the frequency of the class 30-40 is 3.

4. Let us rewrite the given table with cumulative frequencies.

Class interval	f	cf
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66
	N = 66	

$\because \quad N = 66$

$\therefore \quad \frac{N}{2} = 33$

$\therefore$ Median class = 10-15

Modal class = 15-20

Required sum = 10 + 15 = 25.

5. In the given distribution, maximum class frequency is 20, so the modal class is 40-50.

Here, lower limit of modal class: $l = 40$

Frequency of the modal class: $f_1 = 20$

Frequency of the class preceding the modal class: $f_0 = 12$

Frequency of the class succeeding the modal class: $f_2 = 11$

Size of class: $h = 10$

Using the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{20 - 12}{2 \times 20 - 12 - 11}\right) \times 10$$

$$= 40 + 4.70 = 44.70.$$

Hence, mode of the given data is about 45 cars.

6. Let us rewritten the table with class intervals.

Class interval	f	cf
36-38	0	0
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35
	N = 35	

We mark the upper class limits on x-axis and cumulative frequencies on y-axis with a suitable scale.

We plot the points (38, 0); (40, 3); (42, 5); (44, 9); (46, 14); (48, 28); (50, 32) and (52, 35). These points are joined by a free hand smooth curve to obtain a less than type ogive as shown in the figure.

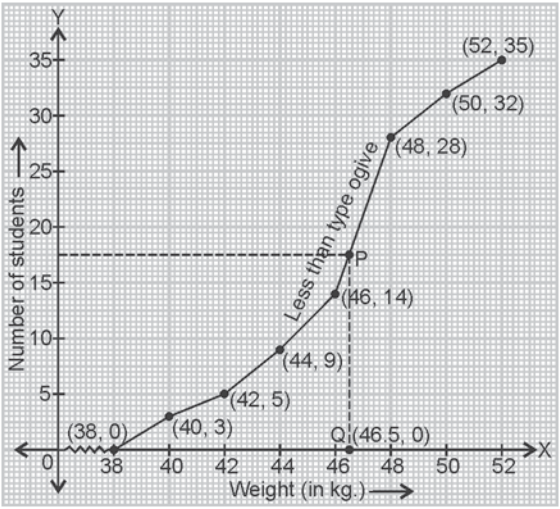


Figure: Less than type ogive

To obtain median from the graph:

We first locate the point corresponding to $\frac{N}{2} = \frac{35}{2} = 17.5$ students on the y-axis. From this point, draw a line parallel to the x-axis to cut the curve at P. From the point P, draw a perpendicular PQ on the x-axis to meet it at Q. The x-coordinate of Q is 46.5. Hence, the median is 46.5 kg.

Let us verify this median using the formula.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$$

$$= 46 + \frac{7}{14} = 46 + 0.5$$

$$= 46.5 \text{ kg.}$$

Thus, the median is the same in both methods.

7. (*i*) By making the given data continuous, we get: $a = 57$, $h = 3$.

No. of mangoes	No. of boxes (f_i)	Mid-points (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
49.5-52.5	15	51	– 2	– 30
52.5-55.5	110	54	– 1	– 110
55.5-58.5	135	$a = 57$	0	0
58.5-61.5	115	60	1	115
61.5-64.5	25	63	2	50
	$\sum f_i = 400$			$\sum f_i u_i = 25$

$$\therefore \quad \text{Mean} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right) = 57 + 3 \times \left(\frac{25}{400}\right) = 57 + \frac{75}{400} \cong 57.19.$$

(*ii*) Step devitation method

(*iii*) Vikram Singh **believes in quality serving, fruits will remian fresh and free from germs and flies.**

WORKSHEET – 131

1. Sum of 11 numbers = 11 × 35 = 385

Sum of first 6 numbers = 6 × 32 = 192

Sum of last 6 numbers = 6 × 37 = 222

$\therefore$ 6^{th} number = 192 + 222 – 385 = 29.

2. We have

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

$$\Rightarrow \quad 45 = 3\ \text{Median} - 2 \times 27$$

$$\Rightarrow \quad \text{Median} = 33.$$

3. 25-30

Hint: $\frac{N}{2} = \frac{5+8+3+2}{2} = 9$.

4. Required number of athletes

$= 2 + 4 + 5 + 71 = 82.$

5.

Class interval	Frequency f_i	x_i	$d_i = x_i - a$	$u_i = \frac{d_i}{20}$	$f_i u_i$
0-20	17	10	– 42	– 2.1	– 35.7
20-40	f_1	30	– 22	– 1.1	$-1.1 f_1$
40-60	32	50	– 2	– 0.1	– 3.2
60-80	f_2	70	18	0.9	$0.9 f_2$
80-100	19	90	38	1.9	36.1
	$\Sigma f_i = 120$		$\Sigma f_i u_i = -2.8 - 1.1 f_1 + 0.9 f_2$		

Let us assumed mean be $a = 52$

Here, $h = 20$

Using the formula:

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\Rightarrow \quad 50 = 52 + \frac{-2.8 - 1.1 f_1 + 0.9 f_2}{120} \times 20$$

$$\Rightarrow \quad 1.1 f_1 - 0.9 f_2 = 9.2 \quad ...(i)$$

But $68 + f_1 + f_2 = 120$

$$\Rightarrow \quad f_1 + f_2 = 52 \quad ...(ii)$$

Solving (*i*) and (*ii*), we obtain

$f_1 = 28$

and $f_2 = 24$.

6. Let us convert the given data into less than type distribution.

Class interval	f	Lifetimes (in hrs.)	cf
0-20	10	less than 20	10
20-40	35	less than 40	45
40-60	52	less than 60	97
60-80	61	less than 80	158
80-100	38	less than 100	196
100-120	29	less than 120	225

We mark the upper class limits along the *x*-axis with a suitable scale and the cumulative frequencies along the *y*-axis with a suitable scale. For this, we plot the points A(20, 10), B(40, 45), C(60, 97), D(80, 158), E(100, 196) and F(120, 225) on a graph paper. These points are joined by a free hand smooth curve to obtain a less than type ogive as shown in the given figure.

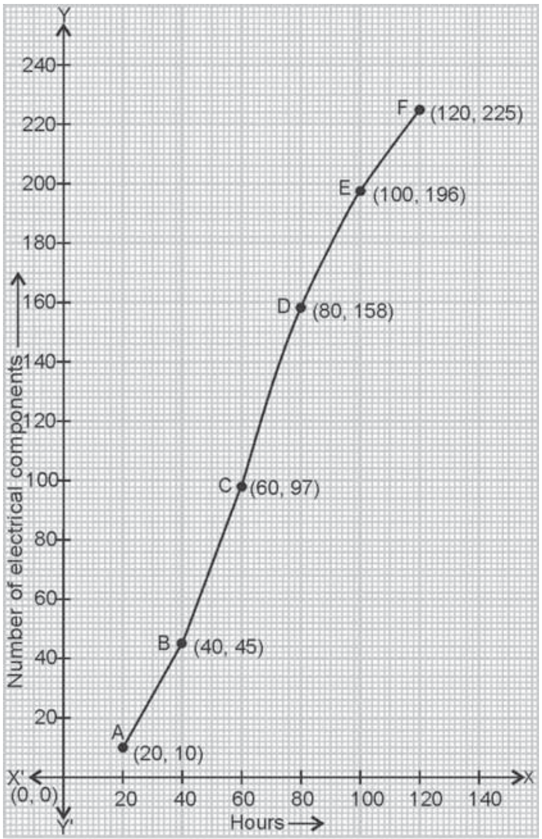


Figure: Less than type ogive

7. The given distribution can be again represented with the cumulative frequencies as given below:

Class interval	f_i	x_i	cf	$f_i x_i$
100-120	12	110	12	1320
120-140	14	130	26	1820
140-160	8	150	34	1200
160-180	6	170	40	1020
180-200	10	190	50	1900
	50			7260

Mean: $\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\because \quad \Sigma f_i = 50$ and $\Sigma f_i x_i = 7260$

$\therefore \quad \text{Mean} = \frac{7260}{50} = 145.20.$

Hence, the mean is ` 145.20

Median:

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$\because$ N = 50, $\frac{N}{2}$ = 25, f = 14, cf = 12,

l = 120 and h = 20

$$\therefore \quad \text{Median} = 120 + \left(\frac{25-12}{14}\right) \times 20$$

$$= 120 + 18.57 = 138.57$$

Hence, the median is ` 138.57.

Mode:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$\because$ l = 120, f_1 = 14, f_0 = 12

f_2 = 8 and h = 20

$$\therefore \quad \text{Mode} = 120 + \left(\frac{14-12}{2\times 14 - 12 - 8}\right) \times 20$$

$$= 120 + \frac{40}{8} = 125$$

Hence, the mode is ` 125.

8.

C.I.	No. of consumers (f_i)	(*c.f.*)
65-85	4	4
85-105	5	9
105-125	13	22 = *c.f.*
125-145	20 = *f*	42
145-165	14	56
165-185	8	64
185-205	4	68
	N = 68	

$\therefore \frac{N}{2} = \frac{68}{2} = 34$ $\therefore$ *c.f.* just greater than 34 is 42.

$\therefore$ Median class is 125-145.

$$\therefore \text{Median} = l + \frac{\frac{N}{2} - c.f.}{f} \times h$$

$$= 125 + \frac{34-22}{20} \times 20$$

$$= 125 + 12 = 137.$$

(*ii*) 20 + 14 + 8 + 4 = 46 families.

(*iii*) Since, Mr Sharma is saving electricity so his consumption is less, which means his monthly bill will also be less. So, **he believes in saving and hence is responsible also.**

WORKSHEET – 132

1. 30-40

Hint:

Class interval (C.I.)	Frequency (f)	Cumulative Frequency
0-10	4	4
10-20	4	8
20-30	8	16
30-40	10	26
40-50	12	38
50-60	8	46
60-70	4	50

2. 45

Hint:

Draw a line parallel to the x-axis at the point $y = \frac{40}{2} = 20$. This line cuts the curve at a point. From this point, draw a perpendicular to the x-axis. The abscissa of the point of intersection of this perpendicular with the x-axis determines the median of the data.

3. The given distribution can also be represented as follows:

Class interval	Frequency
0-10	3
10-20	9
20-30	15
30-40	30
40-50	18
50-60	5

As the maximum frequency is 30, the modal class is 30-40.

4.

C.I.	f_i	x_i	$f_i x_i$
1-3	9	2	18
3-5	22	4	88
5-7	27	6	162
7-10	17	8.5	144.5
	$\Sigma f_i = 75$		$\Sigma f_i x_i = 412.5$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{412.5}{75} = 5.5 .$$

5. In the given distribution, the classes are in the inclusive form. Let us convert them into exclusive form by subtracting $\frac{163-162}{2}$, *i.e.*, 0.5 from lower limit and adding the same to upper limit of each class.

Class interval	f
159.5-162.5	15
162.5-165.5	118
165.5-168.5	142
168.5-171.5	127
171.5-174.5	18

Here, the maximum frequency is 142.

$\therefore$ $l = 165.5$, $f_1 = 142$, $f_0 = 118$, $f_2 = 127$, $h = 3$

Now,

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 165.5 + \left(\frac{142-118}{284-118-127}\right) \times 3$$

$$= 165.5 + 1.85 = 167.35$$

Hence, the modal height of the students is 167.35 cm.

6. The given data may be re-tabulated by the following manner with corresponding cumulative frequencies.

Heights (in cm.) C.I.	No. of girls (f)	Cumulative frequency (cf)
Below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51
	N = 51	

Now, N = 51. So, $\frac{N}{2} = 25.5$.

This observation lies in the class 145-150.

Then $l = 145$, $cf = 11$, $f = 18$, $h = 5$

Now,
$$\text{median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 145 + \left(\frac{25.5-11}{18}\right) \times 5$$

$$= 149.03.$$

Hence, the median height of the girls is 149.03 cm.

7.

C.I.	f_i	x_i	$f_i x_i$
10-12	7	11	77
12-14	12	13	156
14-16	18	15	270
16-18	13	17	221
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 724$

$$\therefore \text{ Mean mileage} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{724}{50}$$

$$= 14.48 \text{ km}/l.$$

(*ii*) No, the manufacturer is claiming mileage 1.52 km/l more than average mileage.

(*iii*) **The manufacturer should be honest with his customer.**

8. 69.5.

Hint: Change the given distribution into less than type and more than type distributions. For drawing the 'less than type' ogive, take upper class limits and corresponding cumulative frequencies; and for drawing the 'more than type' ogive take lower class limits and corresponding cumulative frequencies.

WORKSHEET – 133

1. $x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots + x_n = n \times \bar{x}$

$$\Rightarrow \frac{x_1}{k} + \frac{x_2}{k} + \ldots\ldots\ldots + \frac{x_n}{k} = \frac{n}{k}\bar{x}$$

(Dividing throughout by k)

$$\Rightarrow \frac{\frac{x_1}{k} + \frac{x_2}{k} + \ldots\ldots + \frac{xn}{k}}{n} = \frac{\bar{x}}{k}$$

(Dividing throughout by n)

$\Rightarrow$ Required mean $= \dfrac{\bar{x}}{k}$.

2. The first ten prime numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Median $= \dfrac{11+13}{2} = \dfrac{24}{2} = 12.$

3. Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$\Rightarrow 15 = \frac{5 \times 6 + 10 \times k + 15 \times 6 + 20 \times 10 + 25 \times 5}{6 + k + 6 + 10 + 5}$$

$$\Rightarrow \frac{445 + 10k}{27 + k} = 15$$

$\Rightarrow k = 8.$

4. False, because the values of these three measures depend upon the type of data, so it can be the same.

5. Let us use the assumed mean method to find the mean of the given data.

Marks (C.I.)	No. of students (f_i)	Class mark (x_i)	$d_i = x_i - 35$	$f_i d_i$
0-10	4	5	−30	−120
10-20	6	15	−20	−120
20-30	8	25	−10	−80
30-40	10	35	0	0
40-50	12	45	10	120
50-60	30	55	20	600
	$\Sigma f_i = 70$			$\Sigma f_i d_i = 400$

Here, assumed mean, $a = 35$

Now, required mean $= a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

$= 35 + \dfrac{400}{70} = 35 + 5.71 = 40.71.$

6. Since mode = 36, which lies in the class interval 30-40, so the modal class is 30-40.

$\therefore f_1 = 16, f_0 = f, f_2 = 12, l = 30$ and $h = 10$.

Now, mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$\Rightarrow 36 = 30 + \left(\frac{16 - f}{32 - f - 12}\right) \times 10$$

$$\Rightarrow \frac{6}{10} = \frac{16 - f}{20 - f}$$

$\Rightarrow 120 - 6f = 160 - 10f$

$\Rightarrow 4f = 40 \Rightarrow f = 10.$

7. 31.5 marks.

Hint:

Classes	No. of students	Cumulative frequency
0-10	5	5
10-20	8	13
20-30	6	19
30-40	10	29
40-50	6	35
50-60	6	41

Draw the ogive by plotting the points: (10, 5), (20, 13), (30, 19), (40, 29), (50, 35) and (60, 41). Here $\dfrac{N}{2} = 20.5$. Locate the point on the ogive whose ordinate is 20.5. The x-coordinate of this point will be the median.

8. We prepare the cumulative frequency table by less than method as given below:

Scores	Frequency (f)	Score less than	Cumulative frequency (f)	Point
200-250	30	250	30	(250, 30)
250-300	15	300	45	(300, 45)
300-350	45	350	90	(350, 90)
350-400	20	400	110	(400, 110)
400-450	25	450	135	(450, 135)
450-500	40	500	175	(500, 175)
500-550	10	550	185	(550, 185)
550-600	15	600	200	(600, 200)

We plot the points given in above table on a graph paper and then join them by free hand smooth curve to draw the cumulative frequency curve by less than method.

Similarly for the cumulative frequency curve by more than method, we prepare the corresponding frequency table.

Scores	Frequency (f)	Score more than	Cumulative frequency (cf)	Point
200-250	30	200	200	(200, 200)
250-300	15	250	170	(250, 170)
300-350	45	300	155	(300, 155)
350-400	20	350	110	(350, 110)
400-450	25	400	90	(400, 90)
450-500	40	450	65	(450, 65)
500-550	10	500	25	(500, 25)
550-600	15	550	15	(550, 15)

We plot the points given in this last table on the same graph and join them by free hand smooth curve to draw the cumulative frequency curve by more than method (see figure).

Median: The two curves intersect each other at a point. From this point, we draw a perpendicular on the x-axis. The foot of this perpendicular is P(375, 0). The abscissa of the point P, *i.e.*, 375 is the required median.

Hence, the median is 375.

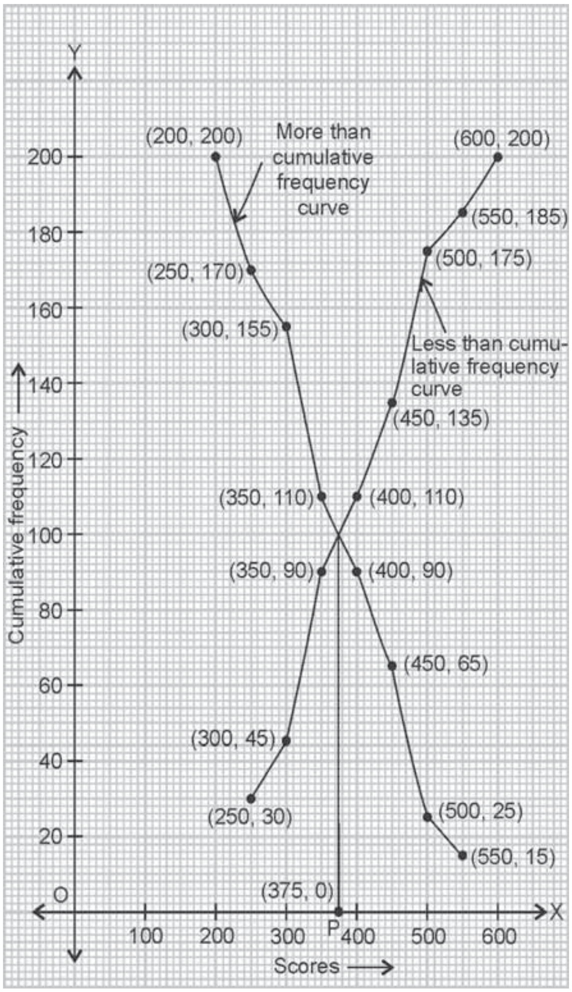


Figure: Less than and more than type cumulative frequency curves

CHAPTER TEST

1. Mode = 3 Median – 2 Mean

80 = 3 Median – 2(110)

80 = 3 Median – 220

3 Median = 220 + 80

$$\text{Median} = \frac{300}{3} = 100.$$

2. It will be 2 + 4 + 5 + 71 = 82.

3. 17.5

Hint: First, transform the given class-intervals into exclusive form and then find the cumulative frequency table.

Here, N = 13 + 10 + 15 + 8 + 11 = 57

$$\therefore \quad \frac{N}{2} = 28.5.$$

4.

Monthly income (in `)	No. of families
10000-13000	15
13000-16000	16
16000-19000	19
19000-22000	17
22000-25000	18
25000 or more	15

Hence, required number of families is 19.

5. No, because an ogive is a graphical representation of a cumulative frequency distribution.

6. Yes; as we know

mode = 3 median – 2 mean

⇒ 3 median = mode + 2 mean

$$\Rightarrow \text{Median} = \frac{1}{3}\text{ mode} + \frac{2}{3}\text{ mean}$$

$$= \text{mode} - \frac{2}{3}\text{ mode} + \frac{2}{3}\text{ mean}$$

$$= \text{mode} + \frac{2}{3}(\text{mean} - \text{mode}).$$

7.

C.I.	x_i	f_i	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
800-820	810	7	$-\frac{40}{20} = -2$	– 14
820-840	830	14	$-\frac{20}{20} = -1$	– 14
840-860	850	19	$\frac{0}{20} = 0$	0
860-880	870	15	$\frac{20}{20} = 1$	15
880-900	890	9	$\frac{40}{20} = 2$	18
		$\Sigma f_i = 64$		$\Sigma f_i u_i = 5$

Let assumed mean be

A = 850

h = 20

$$\text{Mean} = A + \left(\frac{\Sigma u_i f_i}{\Sigma f_i}\right) \times h$$

$$= 850 + \left(\frac{5}{64}\right) \times 20$$

$$= 850 + 1.5625 = 851.5625.$$

Hence, the required mean is 851.5625.

8.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Here, $l = 30$, $f_1 = 45$, $f_0 = 30$, $f_2 = 12$, $h = 10$

$$\therefore \quad \text{Mode} = 30 + \left(\frac{45 - 30}{90 - 30 - 12}\right) \times 10$$

$$= 30 + 3.125 = 33.125 \text{ marks.}$$

9. (*i*)

Class intervals (in daily pocket allowances) (in `)	Frequency (No. of children) (f_i)	Mid-points of C.I. (x_i)	$f_i x_i$
11-13	7	12	84
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	x	20	20
21-23	5	22	110
23-25	4	24	96
	$\Sigma f_i = 44 + x$		$\Sigma f_i x_i = 752 + 20x$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{752 + 20x}{44 + x}$$

As Mean = ` 18 (given) $\therefore \; 18 = \dfrac{752 + 20x}{44 + x}$

$\Rightarrow \quad 792 + 18x = 752 + 20x \Rightarrow 40 = 2x \quad \Rightarrow \quad x = 20.$

(*ii*) Arithmetic mean of grouped data.

(*iii*) **One shouldn't be spend thrift, but should save his money for future use.**

❑❑

Chapter 15 PROBABILITY

WORKSHEET – 135

1. Prime number less than 23 are 2, 3, 5, 7, 11, 13, 17

$\therefore$ Probability = $\frac{7}{90}$.

2. Favourable number of cases = 9

Total number of cases = 36

$\therefore$ Required probability = $\frac{9}{36} = \frac{1}{4}$.

3. Factors of 8 are: 1, 2, 4, 8

$\therefore$ Total numbers = 8

$\therefore$ Required probability = $\frac{4}{8} = \frac{1}{2}$.

4. **No**, because the number of favourable outcomes of getting '6' and 'not 6' are respectively 1 and 5; and so their probabilities are $\frac{1}{6}$ and $\frac{5}{6}$.

5. Sample space is:

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$\therefore$ $n(s) = 8$

(*i*) Let E = getting at least 2 heads

= {THH, HTH, HHT, HHH}

$\therefore$ $n(E) = 4$

$\therefore$ $P(E) = \frac{4}{8} = \frac{1}{2}$.

(*ii*) Let F = getting at most 2 heads

$\therefore$ F = {TTT, TTH, THT, HTT, HHT, HTH, THH}

$\therefore$ $n(F) = 7$

$\therefore$ $P(F) = \frac{7}{8}$.

6. (*i*) $\frac{1}{23}$ (*ii*) $\frac{5}{46}$

Hints:

(*i*) Prime numbers are 5 and 7.

(*ii*) Perfect square numbers are 9, 16, 25, 36, 49.

7. Let A = The event that 5 will not come up either time.

Now sample space is given by

S ={(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total number of outcomes in sample space

$n(S) = 36$

$\therefore$ $\bar{A}$ = {(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 5)}

$\therefore$ $n(\bar{A}) = 11$

$\therefore$ $n(A) = n(S) - n(\bar{A}) = 36 - 11 = 25$

(*i*) $P(A) = \frac{n(A)}{n(S)} = \frac{25}{36}$

(*ii*) $P(\bar{A}) = \frac{n(\bar{A})}{n(S)} = \frac{11}{36}$.

8. Total number of pens = 144

It is the number of all possible outcomes.

Number of defective pens = 20

Number of good pens = 144 – 20 = 124

(*i*) Required probability

$= \frac{\text{No. of good pens}}{\text{Total no. of pens}} = \frac{124}{144} = \frac{31}{36}$

(*ii*) Required probability

$= \frac{\text{No. of defective pens}}{\text{Total no. of pens}} = \frac{20}{144} = \frac{5}{36}$

(*iii*) Rationality.

9. The sample space is

S = {1, 2, 3, 4, 5, 6, 7, 8}

$\therefore$ $n(S) = 8$

(*i*) Let E_1 be the event that the arrow will point at 8, then

$n(E_1) = 1$

$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{8}$.

(*ii*) Let E_2 be the event that the arrow will point at 1, 3, 5 or 7; then

$n(E_2) = 4$

$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{8} = \frac{1}{2}$.

(*iii*) Let E_3 be the event that the arrow will point at 3, 4, 5, 6, 7, or 8; then

$n(E_3) = 6$

$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{8} = \frac{3}{4}$.

(*iv*) Let E_4 be the event that the arrow will point at 1, 2, 3, 4, 5, 6, 7 or 8; then

$n(E_4) = 8$

$\therefore \quad P(E_4) = \frac{n(E_4)}{n(S)} = \frac{8}{8} = 1$.

WORKSHEET – 136

1. Hint: Outcomes in favourable event of getting the sum as a perfect square are (1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (5, 4), (6, 3).

2. Hint: $|x| \leq 4 \Rightarrow -4 \leq x \leq 4$

$\Rightarrow x = -4, -3, -2, -1, 0, 1, 2, 3, 4$.

3. Total number of outcomes = 52

Since, the drawn card should not be red or queen

Total number of red cards (including a red queen) = 13

Total number of queens (excluding red queen) = 3

$\therefore$ Total favourable outcomes = 13 + 3 = 16

$\therefore$ Required probability = $\frac{16}{52} = \frac{4}{13}$.

4. No, because the theoretical probability of getting a head on tossing a coin is $\frac{1}{2}$ and the experimental probability tends to $\frac{1}{2}$ when the number of tosses increases.

OR

$n(S) = 100$

Let E be the event of getting a prime.

The primes from 1 to 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

$\therefore \quad n(E) = 25$

Now, $\quad P(E) = \frac{n(E)}{n(S)} = \frac{25}{100} = \frac{1}{4}$.

5. Total cards = 52

Since card drawn is neither a king nor queen

$\therefore$ favourable cards are = 52 – (4 + 4)

= 52 – 8 = 44

$\therefore$ Required probability = $\frac{44}{52} = \frac{11}{13}$.

6. False; because the outcomes are not equally likely. As for

No girl: Cases are {BBB} (*i.e.*, Three boys)

One girl: Cases are {BGB, BBG, GBB}

Two girls: Cases are {BGG, GBG, GGB}

All girls: Cases are {GGG}

7. The sample space is

S = {HH, HT, TH, TT}

$\therefore \quad n(S) = 4$

(*i*) The outcomes for atleast one head:

{HH, HT, TH}

$\therefore$ Probability (at least one head) = $\frac{3}{4}$.

(*ii*) The outcomes for at most one head:

{HT, TH, TT}

$\therefore$ Probability (at most one head) = $\frac{3}{4}$.

(*iii*) The outcomes for one head: {HT, TH}

$\therefore$ Probability (one head) = $\frac{2}{4} = \frac{1}{2}$.

OR

Total number of students = 23

$\therefore \quad n(S) = 23$

Let E be the event that the selected student is not from A, B and C.

$\therefore \quad n(E) = 23 - 4 - 8 - 5 = 6$

Now, $\quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{23}$.

8. Total face cards = 12

Number of black face cards = 6

Number of cards left after removing 6 black face cards = 52 – 6 = 46

$\therefore$ (*i*) Probability (a face card) = $\frac{6}{46} = \frac{3}{23}$

(*ii*) Probability (a red card) = $\frac{26}{46} = \frac{13}{23}$

(*iii*) Probability (a black card) = $\frac{20}{46} = \frac{10}{23}$

(*iv*) Probability (a king) = $\frac{2}{46} = \frac{1}{23}$

9. Total cards = 65

(*i*) P(one digit number *i.e.*, 6, 7, 8, 9) = $\frac{4}{65}$.

(*ii*) P(Number divisible by 5, *i.e.*, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70) = $\frac{13}{65} = \frac{1}{5}$.

(*iii*) P(odd number < 30) = $\frac{11}{65}$

i.e., P(7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29).

(*iv*) Non composite number between 50 and 70 are : 53, 59, 61, 67, 69

$\therefore$ Probability of getting a non-composite number between 50 and 70 is = $\frac{5}{19}$

$\therefore$ P(composite number) = $1 - \frac{5}{19} = \frac{14}{19}$.

WORKSHEET – 137

1. Required prime numbers are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

$\therefore$ Required probability = $\frac{10}{30} = \frac{1}{3}$.

2. Hint: The sum of probabilities of having a particular event and not having the same event is one.

3. Total balls = 5 + 8 + 4 + 7 = 24

Let G = getting a green ball

Total green ball = 4.

$\therefore$ $P(G) = \frac{4}{24} = \frac{1}{6}$

$\therefore$ $P(\text{not } G) = 1 - \frac{1}{6} = \frac{5}{6}$.

4. False, because the probability of each outcome will be $\frac{1}{2}$ only when the two outcomes are equally likely otherwise not.

5. No, because areas of regions 3, 5 and 7 are not equal.

6. All possible outcomes are given by
{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(*i*) $\frac{5}{36}$

Hint: Favourable outcomes:
(2, 6), (6, 2), (3, 5), (5, 3), (4, 4).

(*ii*) 0

Hint: No, favourable outcome is possible.

(*iii*) 1

Hint: Favourable outcomes are the same as the outcomes in sample space.

7. (*i*) Total cards left
= 52 – (4 King) – (4 Queen) – (4 Aces)
= 52 – 12 = 40

Now probability (black face card)

$= \frac{2}{40} = \frac{1}{20}$

(*ii*) A red card = $\frac{20}{40} = \frac{1}{2}$.

8. Number of all cards = 50 – 5 + 1 = 46

i.e., $n(S) = 46$

(*i*) Let E_1 be the event that the number on the card taken out is a prime less than 10.

Prime numbers from 5 to 9 are 5 and 7.

$\therefore$ $n(E_1) = 2$

$\therefore$ $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{46} = \frac{1}{23}$.

(*ii*) Let E_2 be the event that the number on the card taken out is a perfect square.

The perfect square numbers from 5 to 50 are 9, 16, 25, 36 and 49.

$\therefore \quad n(E_2) = 5$

$\therefore \quad P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{5}{46}$.

9. Total = 12

A : extremely patient = 3

B : extremely honest = 6

∴ C : extremely kind = 12 – 9 = 3

(*i*) $P(A) = \dfrac{3}{12} = \dfrac{1}{4}$

(*ii*) $P(B \text{ or } C) = \dfrac{6+3}{12} = \dfrac{9}{12} = \dfrac{3}{4}$.

Extremely patient.

WORKSHEET – 138

1. Prime numbers are: 2, 3, 5, 7, 11

$\therefore$ Probability $= \dfrac{5}{10} = \dfrac{1}{2}$.

2. Given: $P(E) = 3\,P(E')$ ...(*i*)

We have $P(E) + P(E') = 1$ ...(*ii*)

(*i*) and (*ii*) gives $P(E) = 3\{1 - P(E)\}$

i.e., $4\,P(E) = 3$, *i.e.*, $P(E) = \dfrac{3}{4}$.

3. The number of outcomes when a pair of dice is rolled $= 6^2 = 36$.

The outcomes such that the sum is divisible by 3 are:

(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6).

These are 12 outcomes.

The outcomes such that the sum is divisible by 2 are:

(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6).

These are 18 outcomes.

The outcomes such that the sum is divisible by 6 are:

(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6).

These are 6 outcomes.

Now, the number of outcomes which are divisible by 3 or 2 is 12 + 18 – 6 = 24.

Hence, the required probability $= \dfrac{24}{36} = \dfrac{2}{3}$.

4. Number of cards = 50

Prime numbers from 51 to 100 are: 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Therefore, number of all possible outcomes = 50.

And number of favourable outcomes = 10.

$\therefore$ Required probability $= \dfrac{10}{50} = \dfrac{1}{5}$.

5. All possible outcomes are given by

S = {1, 2, 3, ..., 1000}

$\therefore n(S) = 1000$

(*i*) Let E_1 be the event that the first player wins a prize. Then,

E_1 = Perfect square numbers greater than 500 and less than 1001.

= 529, 576, 625, 676, 729, 784, 841, 900, 961.

$\therefore n(E_1) = 9$

Now, $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{9}{1000}$.

(*ii*) Let E_2 be the event that the second player wins a prize, if the first has won.

$\therefore \quad n(E_2) = n(E_1) - 1 = 9 - 1 = 8$

And number of all possible outcomes

$= n(S) - 1 = 1000 - 1 = 999$

Now, $P(E_2) = \dfrac{n(E_2)}{999} = \dfrac{8}{999}$.

6. Let E_1 be the event 'the mobile phone is acceptable to Varnika' and E_2 be the event 'the mobile phone is acceptable to the trader'.

$\therefore \quad n(E_1)$ = Number of good mobile phones

= 42

And $n(E_2)$ = Number of good mobile phones + Number of mobile phones having only minor defects

= 42 + 3 = 45

Number of all mobile phones is given by

$n(S) = 48$

(*i*) $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{42}{48} = \frac{7}{8}$

(*ii*) $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{45}{48} = \frac{15}{16}$.

7. Consider

y \ x	1	4	9	16
xy:				
1	1	4	9	16
2	2	8	18	32
3	3	12	27	48
4	4	16	36	64

∴ Total possible values of distinct (xy) = 15.

Number of cases in which $xy > 16 = 7$

∴ Required probability $= \frac{7}{15}$.

8. (*i*) P (odd number) $= \frac{25}{50} = \frac{1}{2}$

(*ii*) P (a perfect square) $= \frac{4}{50} = \frac{2}{25}$

(*iii*) P (divisible by 5) $= \frac{10}{50} = \frac{1}{5}$

(*iv*) P (prime number less than 20) $= \frac{4}{50} = \frac{2}{25}$.

CHAPTER TEST

1. Number of faces having B or C

$= 2 + 1 = 3$

Number of all faces $= 6$

P(getting B or C) $= \frac{3}{6} = \frac{1}{2}$.

2. P(drawing a green ball)

= 3 × P(drawing a red ball)

$\Rightarrow \frac{n}{5+n} = 3 \times \frac{5}{5+n} \Rightarrow n = 15.$

3. Total number of outcomes = 36

Let E = getting sum more than 10

∴ Sum of digits on dice = 11 or 12

For 11: (5, 6), (6, 5)

For 12: (6, 6)

∴ Favourable outcomes for events E are (5, 6), (6, 5), (6, 6)

∴ $P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$.

4. Case I: 2 dice are thrown.

Number of all outcomes in the sample space, $n(S) = 6^2 = 36$

Favourable numbers, $n(E_1) = 1$

∴ $P(E_1) = \frac{1}{36}$

Case II: 1 die is thrown.

Number of all outcomes, $n(S) = 6$

Favourable numbers, $n(E_2) = 1$

∴ $P(E_2) = \frac{1}{6}$

So, the student throwing one die has the better chance because he has more probability.

OR

The sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

∴ $n(S) = 8$

The outcomes having at least two heads are

E = {HHH, HHT, HTH, THH}

∴ $n(E) = 4$

∴ $P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$.

5. False, because there are equal probabilities of getting the head or tail, that is $\frac{1}{2}$.

OR

Total number of outcomes, $n(S) = 36$

Favourable outcomes,

E = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

∴ $n(E) = 5$

∴ $P(E) = \frac{5}{36}$.

6. There are 52 cards in the pack. Therefore, the number of outcomes in sample space is given by

$n(S) = 52$

Number of hearts cards $= 13$

Number of queens $= 4$

Number of queens of hearts $= 1$

So, number of favourable outcomes is given by

$$n(E) = 52 - (13 + 4 - 1) = 36$$

Now, the required probability will be given by

$$P(E) = \frac{n(E)}{n(S)} = \frac{36}{52} = \frac{9}{13}.$$

7. (*i*) Probability that selected student is a car driver $= \frac{25}{100} = \frac{1}{4}$

(*ii*) Total students who drive bicycle

$$= 100 - (25 + 20) = 55$$

$\therefore$ Probability that selected student rides on bicycle $= \frac{55}{100} = \frac{11}{20}$.

(*iii*) Use of bicycle should be encouraged in campus as it saves fuel and helps reducing the pollution in environment.

8. (*i*) Probability of getting an odd number

$$= \frac{25}{49}$$

(*ii*) Probability of getting a multiple of 5

$$= \frac{9}{49}$$

(*iii*) Probability of getting a perfect square

$$= \frac{7}{49}$$

(*iv*) Probability of getting an even prime number $= \frac{1}{49}$.

9. Total balls in bag = 18

No. of red ball $= x$

No. of non-red balls $= 18 - x$

(*i*) $\therefore$ P(not red) $= \frac{18 - x}{18}$

(*ii*) Total balls = 18 + 2 = 20

Number of red balls $= (x + 2)$

$$P(\text{red ball}) = \frac{x+2}{20}$$

Also, P(red ball) in first case $= \frac{x}{18}$

According to question

$$\frac{x+2}{20} = \frac{9}{8}\left(\frac{x}{18}\right) \Rightarrow \frac{x+2}{20} = \frac{x}{16}$$

$$\Rightarrow \quad 4x + 8 = 5x \quad \Rightarrow \quad x = 8.$$

❑❑

PRACTICE PAPERS

Practice Paper-1

Section-A

1. Sum of zeroes (S) $= -\dfrac{2}{\sqrt{3}} + \dfrac{\sqrt{3}}{4}$

$$= \frac{3-8}{4\sqrt{3}} = -\frac{5}{4\sqrt{3}}$$

Product of zeroes (P) $= -\dfrac{2}{\sqrt{3}} \times \dfrac{\sqrt{3}}{4} = -\dfrac{1}{2}$

Now, required polynomial will be $p(x) = x^2 - 5x + P$, *i.e.*, $p(x) = x^2 + \dfrac{5}{4\sqrt{3}}x - \dfrac{1}{2}$ or $p(x) = 4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$.

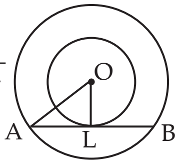
2. $$\frac{147}{120} = \frac{49}{40} = \frac{49}{4\times 10}$$

$$= \frac{12.25}{10} = 1.225$$

So, decimal expansion of $\dfrac{147}{120}$ terminates after three places of decimal.

3. OA = 13 cm,

OL = 5 cm



Therefore, AL $= \sqrt{(13)^2 - (5)^2}$

AL $= \sqrt{169 - 25}$

AL = 12 cm

AB = 2AL

AB = 2 × 12 = 24 cm.

4. Mid-point of AB is:

$$\left(\frac{3+k}{2}, \frac{4+6}{2}\right) = \left(\frac{3+k}{2}, 5\right)$$

Then, $\left(\dfrac{3+k}{2}, 5\right) = (x, y)$

$\Rightarrow \quad x = \dfrac{3+k}{2}$; $y = 5$

Since, $x + y - 10 = 0$

$\Rightarrow \quad \dfrac{3+k}{2} + 5 - 10 = 0$

$\Rightarrow \quad 3 + k = 10$

$\Rightarrow \quad k = 7.$

5. Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow 15 = \dfrac{5\times 6 + 10\times k + 15\times 6 + 20\times 10 + 25\times 5}{6 + k + 6 + 10 + 5}$

$\Rightarrow \quad \dfrac{445 + 10k}{27 + k} = 15$

$\Rightarrow \quad k = 8.$

6. As, the sum of an event and its complementary event is unity

$\therefore \quad p + p$ (complementary event) = 1

$\Rightarrow \quad p$(complementary event) $= 1 - p$.

Section-B

7. No. Prime factors of 6^n will be of type $2^n \times 3^n$. As it doesn't have 5 as a prime factor, so 6^n can't end with the digit 5.

8. $a = 1;\ d = -3.$

$a_n = -236$

$\Rightarrow \quad a + (n-1)\,d = -236$

$\Rightarrow \quad 1 + (n-1)(-3) = -236$

$\Rightarrow \quad -3n + 4 = -236$

$\Rightarrow \quad -3n = -240 \quad \Rightarrow \quad n = 80$

$\Rightarrow \quad S_n = \dfrac{n}{2}\{a + a_n\}$

$\Rightarrow \quad S_{80} = \dfrac{80}{2}\{1 - 236\}$

$= 40 \times (-235) = -9400.$

9. $PQ = \sqrt{58} \Rightarrow PQ^2 = 58$

$\Rightarrow \quad (k-3)^2 + (2+5)^2 = 58$

$\Rightarrow \quad (k-3)^2 = 58 - 49$

$\Rightarrow \quad (k-3)^2 = 9$

$\Rightarrow \quad k - 3 = \pm 3 \Rightarrow k = 0$ or 6.

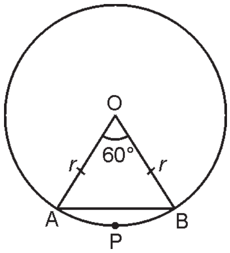
10. $$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$

$$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

11. $r = 10$

Let AB is the chord which subtend an angle of 60° at centre of circle.



∴ Area of corresponding segment

= Area of APB

$$A = \frac{\pi r^2 \theta}{360°} - ar(\Delta AOB)$$

As ΔAOB is equilateral

$$\therefore \quad A = \frac{\pi(10)^2 \times 60}{360°} - \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= \left(\frac{157}{3} - 25\sqrt{3}\right) \text{cm}^2.$$

12. Sample space: {1, 2, 3, ... , 99}

$\therefore \quad n(S) = 99.$

The numbers divisible by 3 and 5 both are numbers divisible by 15.

So, favourable outcomes are: {15, 30, 45, 60, 75, 90}

Let E be the event getting a number divisible by 3 and 5.

$\therefore \quad n(E) = 6$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{6}{99} = \frac{2}{33}.$$

Section-C

13. Let x be any positive integer.

Then it is either of the form $3q$ or $3q + 1$ or $3q + 2$.

Case I. $x = 3q$

Cubing both sides, we get

$$x^3 = (3q)^3 = 27q^3 = 9m \qquad \ldots(i)$$
$$m = 3q^3$$

Case II. $x = 3q + 1$

Cubing both sides, we get

$$x^3 = (3q + 1)^3$$
$$= 27q^3 + 1 + 3(3q + 1) \times 3q$$
$$= 27q^3 + 27q^2 + 9q + 1$$
$$= 9q(3q^2 + 3q + 1) + 1$$
$$= 9m + 1; \qquad \ldots(ii)$$
$$m = q(3q^2 + 3q + 1).$$

Case III. $x = 3q + 2$

Cubing both sides, we get

$$x^3 = (3q + 2)^3$$
$$= 27q^3 + 8 + 3(3q + 2)\,6q$$
$$= 27q^3 + 54q^2 + 36q + 8$$
$$= 9q(3q^2 + 6q + 4) + 8$$
$$= 9m + 8; \qquad \ldots(iii)$$
$$m = q(3q^2 + 6q + 4).$$

Thus, from equations (*i*), (*ii*) and (*iii*), it is clear that cube of any positive integer is either of the form $9m$, or $9m + 1$ or $9m + 8$.

14. Since α, β are the zeroes of $x^2 + px + q$, then

$$\alpha + \beta = -p; \ \alpha\beta = q$$

Now, $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -\frac{p}{q}$$

and $$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{q}$$

So the polynomial having zeroes $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ will be

$$p(x) = x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha} \times \frac{1}{\beta}\right)$$

$$= x^2 + \frac{p}{q}x + \frac{1}{q}$$

or $$p(x) = qx^2 + px + 1.$$

OR

Since $x = \sqrt{\frac{5}{3}}$ and $x = -\sqrt{\frac{5}{3}}$ are zeroes of $p(x)$ $= 3x^4 + 6x^3 - 2x^2 - 10x - 5$, so $p(x)$ is divisible by $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$, *i.e.*, $x^2 - \frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2 + 6x + 3 \\
x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & \underline{\overset{-}{3x^4} \qquad \overset{+}{-\,5x^2}} \\
 & 6x^3 + 3x^2 - 10x - 5 \\
 & \underline{\overset{-}{6x^3} \qquad \overset{+}{-\,10x}} \\
 & 3x^2 - 5 \\
 & \underline{\overset{-}{3x^2} \qquad \overset{+}{-\,5}} \\
 & 0
\end{array}
$$

Here, other two zeroes of $p(x)$ are the two zeroes of quotient $3x^2 + 6x + 3$

Put $3x^2 + 6x + 3 = 0$

$\Rightarrow$ $3(x + 1)^2 = 0$

$\Rightarrow$ $x = -1$ and $x = -1$

Hence, all the zeroes of $p(x)$ are , $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.

15. Let the first term and common difference of the A.P. be A and D respectively.

Now, pth term $= a$

$\Rightarrow$ $A + (p - 1)D = a$ ...(*i*)

qth term $= b$

$\Rightarrow$ $A + (q - 1)D = b$ ...(*ii*)

rth term $= c$

$\Rightarrow$ $A + (r - 1)D = c$ ...(*iii*)

Subtracting equation (*ii*) from (*i*), (*iii*) from (*ii*), (*i*) from (*iii*), we get respectively:

$a - b = (p - q)D$ ...(*iv*)

$b - c = (q - r)D$ ...(*v*)

$c - a = (r - p)D$ ...(*vi*)

Multiplying (*iv*), (*v*) and (*vi*) by respectively r, p and q; and then adding the results to get

$(a - b)r + (b - c)p + (c - a)q$

$= (pr - qr)D + (pq - pr)D + (qr - pq)D$

$= (pr - qr + pq - pr + qr - pq)D$

$= 0 \times D = 0.$ **Hence the result.**

16. Let the two digits number be $10x + y$.

Since ten's digit exceeds twice the unit's digit by 2

$\therefore$ $x = 2y + 2$

$\Rightarrow$ $x - 2y - 2 = 0$ ...(*i*)

Since the number obtained by inter-changing the digits, *i.e.*, $10y + x$ is 5 more than three times the sum of the digits.

$\therefore$ $10y + x = 3(x + y) + 5$

$\Rightarrow$ $2x - 7y + 5 = 0$ ...(*ii*)

On solving equations (*i*) and (*ii*), we obtain

$x = 8$ and $y = 3$

$\therefore$ $10x + y = 83$

Hence, the required two-digit number is 83.

17. Steps of construction:

Step I: First, draw a circle with radius as 3 cm and centre at O. Then take a point P so that OP = 7 cm.

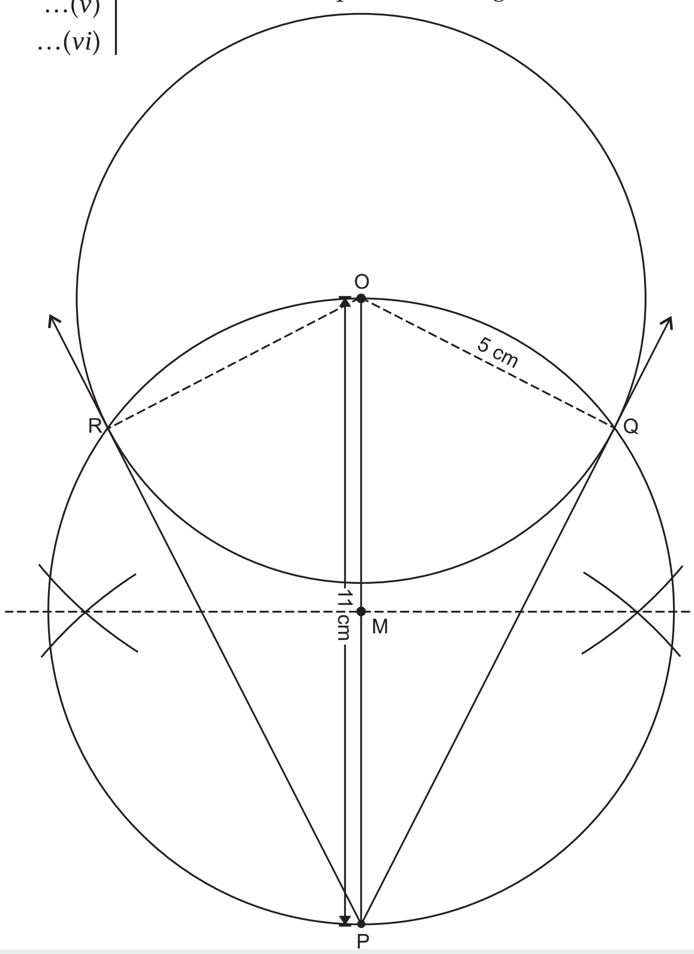
Step II: Bisect OP to find mid-point M of OP. Then take M as centre and MP = MO as radius, draw a circle to intersect the previous circle at Q and R.

Step III: Join PQ and PR which are the required tangents.

After measuring PQ and PR, we find PQ = PR = 6.32 cm (approximately).

So, tangents are perpendicular to radii passing through their respective points of contact.

i.e., $PQ \perp OQ$ and $PR \perp OR$.



18. BC is trisected at D and E.

$\therefore \quad BD = DE = EC = \frac{1}{3}BC$

Let us use Pythagoras theorem.

In ΔABD; $\quad AD^2 = AB^2 + BD^2$

$= AB^2 + \frac{1}{9}BC^2$

$[\because BD = \frac{1}{3}BC]$

$\Rightarrow \quad 5AD^2 = 5AB^2 + \frac{5}{9}BC^2 \quad \ldots(i)$

Also in ΔABE; $AE^2 = AB^2 + BE^2$

$= AB^2 + \left(\frac{2}{3}BC\right)^2$

$= AB^2 + \frac{4}{9}BC^2$

$\Rightarrow \quad 8AE^2 = 8AB^2 + \frac{32}{9}BC^2 \quad \ldots(ii)$

In ΔABC; $\quad AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 3AC^2 = 3AB^2 + 3BC^2 \quad \ldots(iii)$

Adding equations (*i*) and (*iii*), and then subtracting the result from equation (*ii*), we get

$8AE^2 - (5AD^2 + 3AC^2)$

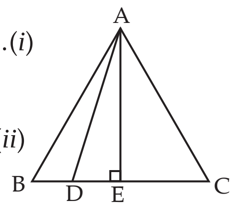
$= 8AB^2 + \frac{32}{9}BC^2 - (5AB^2 + \frac{5}{9}BC^2$

$+ 3AB^2 + 3BC^2)$

$= 8AB^2 - 8AB^2 + \frac{32}{9}BC^2 - \frac{32}{9}BC^2 = 0$

$\Rightarrow \quad 8AE^2 = 3AC^2 + 5AD^2.$

OR



Let $AB = BC = AC = a \quad \ldots(i)$

Draw $AE \perp BC$

$\Rightarrow \quad BE = EC = \frac{1}{2}a \quad \ldots(ii)$

and $\quad BD = \frac{1}{3}BC = \frac{1}{3}a \ldots(iii)$

Using Pythagoras theorem in ΔACE, we have $AC^2 = AE^2 + EC^2 \quad \ldots(iv)$

Similarly, in ΔADE, we have $AD^2 = AE^2 + DE^2 = AE^2 + (BE - BD)^2$

$AD^2 = AE^2 + BE^2 + BD^2 - 2BE.BD$

$\Rightarrow \quad AD^2 = AE^2 + EC^2 + \left(\frac{1}{3}a\right)^2 - 2\left(\frac{1}{2}a\right)\left(\frac{1}{3}a\right)$

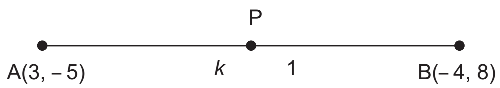
[By (*ii*) and (*iii*)]

$= AC^2 + \frac{a^2}{9} - \frac{a^2}{3}$ [By (*iv*)]

$= a^2 + \frac{a^2}{9} - \frac{a^2}{3} = \frac{7a^2}{9}$ [By (*i*)]

$9AD^2 = 7AB^2.$

19.



Let coordinates of P are (x, y)

$\therefore$ Using section formula:

$x = \frac{-4k+3}{k+1}; \; y = \frac{8k-5}{k+1}$

As P lies on $x + y = 0$

$\Rightarrow \quad \frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0$

$\Rightarrow \quad 4k - 2 = 0 \Rightarrow k = \frac{2}{4} = \frac{1}{2}.$

OR

Let the height of parallelogram taking AB as base be *h*.

Now, $\quad AB = \sqrt{(7-4)^2 + (2+2)^2}$

$= \sqrt{3^2 + 4^2} = 5$ units

Area of $\Delta ABC = \frac{1}{2}[4(2-9) + 7(9+2)$

$+ 0(-2-2)]$

$= \frac{49}{2}$ sq. units

Now, $\frac{1}{2} \times AB \times h = \frac{49}{2}$

$\Rightarrow \quad \frac{1}{2} \times 5 \times h = \frac{49}{2}$

$\Rightarrow \quad h = \frac{49}{5} = 9.8$ units.

20. In the given distribution, the classes are in the inclusive form. Let us convert them into exclusive form by subtracting $\frac{163-162}{2}$, *i.e.*, 0.5 from lower limit and adding the same to upper limit of each class.

Class interval	f
159.5-162.5	15
162.5-165.5	118
165.5-168.5	142
168.5-171.5	127
171.5-174.5	18

Here, the maximum frequency is 142.

$\therefore\ l = 165.5, f_1 = 142, f_0 = 118, f_2 = 127, h = 3$

Now, mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 165.5 + \left(\dfrac{142-118}{284-118-127}\right) \times 3$

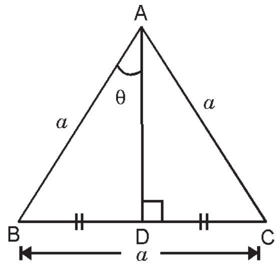
$= 165.5 + 1.85 = 167.35$

Hence, the modal height of the students is 167.35 cm.

21. Draw Δ ABC with

AB = BC = AC = a (say) [*i.e.*, equilateral Δ]

Draw AD ⊥ BC



$\therefore\ \angle BAD = \angle DAC = \theta = 30°\quad [\because \angle A = 60°]$

and BD = DC = $a/2$

$\therefore\quad \sin\theta = \dfrac{BD}{AB} = \dfrac{a/2}{a} = \dfrac{1}{2} \Rightarrow \sin 30° = \dfrac{1}{2}$.

OR

$\sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides,

$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos = 2$

$\Rightarrow 1 + 2\sin\theta.\cos\theta = 2\ (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow 2\sin\theta.\cos\theta = 1$

$\Rightarrow \sin\theta.\cos\theta = \dfrac{1}{2}$ ...(*i*)

We know that,

$\sin^2\theta + \cos^2\theta = 1$ ...(*ii*)

Dividing (*ii*) by (*i*), we get

$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta} = \dfrac{1}{\frac{1}{2}} \Rightarrow \tan\theta + \cot\theta = 2.$

22. $\dfrac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta}$

$= \underset{\text{I}}{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = \underset{\text{II}}{\dfrac{1+\cos\theta}{1-\cos\theta}} = \underset{\text{III}}{\dfrac{\tan^2\theta}{(\sec\theta - 1)^2}}$

LHS $= \dfrac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \dfrac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$

$= \dfrac{\sin\theta + \sin\theta\cos\theta}{\sin\theta - \sin\theta\cos\theta}$

Dividing numerator and denominator by sin θ.cos θ, we get

LHS $= \dfrac{\frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}}{\frac{\sin\theta}{\sin\theta\cos\theta} - \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}}$

$= \dfrac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1} = \dfrac{\sec\theta + 1}{\sec\theta - 1}$ = (1st Result)

Again, $\dfrac{\sec\theta + 1}{\sec\theta - 1}$

$= \dfrac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1} = \dfrac{1+\cos\theta}{1-\cos\theta}$ = (IInd Result)

For IIIrd Result consider 1st Result, *i.e.*, LHS

$= \dfrac{\sec\theta + 1}{\sec\theta - 1}$

Multiplying numerator and denominator by (sec θ – 1), we get

$= \dfrac{(\sec\theta + 1)(\sec\theta - 1)}{(\sec\theta - 1)(\sec\theta - 1)} = \dfrac{\sec^2\theta - 1}{(\sec\theta - 1)^2}$

$= \dfrac{\tan^2\theta}{(\sec\theta - 1)^2}$. $(\because \sec^2\theta - 1 = \tan^2\theta)$

Section-D

23. Table for values of x and y corresponding to equation $4x - 5y - 20 = 0$ is

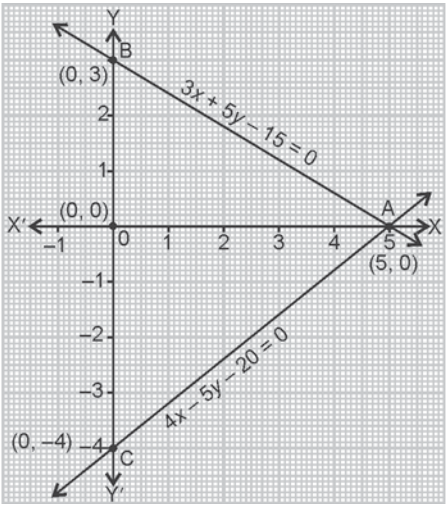
x	5	0
y	0	-4

Similarly for the equation $3x + 5y - 15 = 0$

x	5	0
y	0	3

Let us draw the graphs for the two equations.

As the graphs of the two lines intersect each other at the point A(5, 0), the required solution is $x = 5$, $y = 0$.



The graphs intersect the y-axis at B (0, 3) and C(0, – 4). Therefore, the coordinates of vertices of the triangle ABC are A(5, 0), B(0, 3) and C(0, – 4).

Hence the answer: $x = 5$, $y = 0$ and (5, 0), (0, 3), (0, – 4).

24. Let present Nisha's age = x years

$\therefore$ Present age of Asha's age $x^2 + 2$

When Nisha's age = $x^2 + 2$

then Asha's age = $10x - 1$

According to question, we have

$\therefore \quad x^2 + 2 + [x^2 + 2 - x] = 10x - 1$

$\Rightarrow \quad 2x^2 - x + 4 = 10x - 1$

$\Rightarrow \quad 2x^2 - 11x + 5 = 0$

$\Rightarrow \quad 2x^2 - 10x - x + 5 = 0$

$\Rightarrow \quad 2x(x - 5) - 1(x - 5) = 0$

$\Rightarrow \quad (2x - 1)(x - 5) = 0$

$\Rightarrow \quad x = \frac{1}{2}, 5 \therefore x = 5$

Hence, Nisha's present age = 5 years

and Asha's present age $= (5)^2 + 2$

$= 27$ years.

25. $\because$ PQ and PR are two tangents of the same circle at point Q and R.

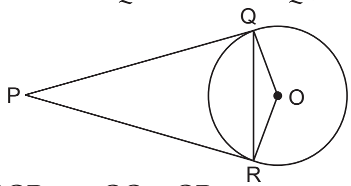
$\therefore$ PQOR is a cyclic quadrilateral.

$\angle QPR + \angle QOR = 180°$

[Opposite of cyclic quadrilateral]

$\angle QOR = 180° - \angle QPR$

$\angle QPR = 80° - \angle QOR \quad \ldots(i)$



In ΔOQR, $OQ = OR$

[Radii of the same circle]

$\angle OQR = \angle ORQ$

[Opposite angles of equal side]

$\angle OQR + \angle ORQ + \angle QOR = 180°$

[Angle sum property of triangle]

$\angle OQR + \angle OQR + \angle QOR = 180°$

$2\angle OQR = 180° - \angle QOR$

$\angle OQR = \frac{1}{2}(180° - \angle QOR)$

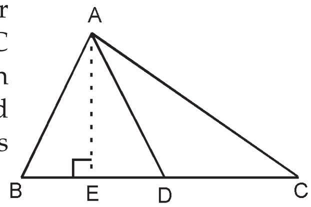
$= \frac{1}{2}\angle QPR$

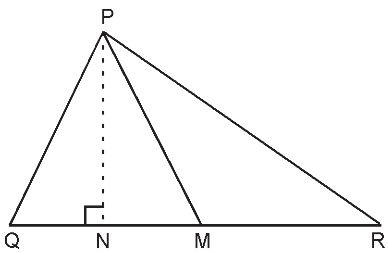
[From (i), $\angle QPR = 180° - \angle QOR$]

$\therefore \quad \angle QPR = 2\angle OQR.$

Hence proved.

26. Let the given similar triangles be ABC and PQR with medians AD and PM respectively as shown in the figure.





$\therefore \quad \Delta ABC \sim \Delta PQR$

$\Rightarrow \quad \dfrac{AB}{PQ} = \dfrac{BC}{QR} \quad \ldots(i)$

and $\quad \angle B = \angle Q \quad \ldots(ii)$

We need to prove

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$$

Draw AE ⊥ BC and PN ⊥ QR, $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ [By (*i*)]

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad \ldots(iii)$$

In ΔABD and ΔPQM, ∠B = ∠Q [By (*ii*)]

$$\frac{AB}{PQ} = \frac{BD}{QM} \quad \text{[By } (iii)\text{]}$$

$\therefore \quad \Delta ABD \sim \Delta PQM$ (SAS similarity)

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} \Rightarrow \frac{BC}{QR} = \frac{AD}{PM} \quad \ldots(iv)$$

[By (*i*)]

In ΔABE and ΔPQN,

$\angle B = \angle Q$ [By (*ii*)]

$\angle E = \angle N$ [Each 90°]

$\therefore \quad \Delta ABE \sim \Delta PQN$ (AA similarity)

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AE}{PN} \Rightarrow \frac{BC}{QR} = \frac{AE}{PN} \quad \text{[By } (i)\text{]}$$

$$\Rightarrow \quad \frac{AE}{PN} = \frac{AD}{PM} \quad \ldots(v) \text{ [By } (iv)\text{]}$$

$$\text{Now, } \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times QR \times PN}$$

$$= \left(\frac{BC}{QR}\right) \times \left(\frac{AE}{PN}\right)$$

$$= \frac{AD}{PM} \times \frac{AD}{PM}$$

[Using (*iv*) and (*v*)]

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PM^2}$$

OR

Given: A triangle ABC, right angled at B. We have to prove that $AC^2 = AB^2 + BC^2$.

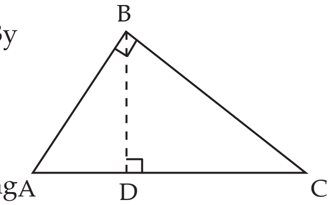
Construction: Draw BD ⊥ AC.

Proof. In triangles ADB and ABC,

$\angle A = \angle A$ (Common)

$\angle ADB = \angle ABC$ (Each 90°)

$\therefore$ ΔADB ~ ΔABC (By AA corollary)



$\therefore \quad \dfrac{AD}{AB} = \dfrac{AB}{AC}$ (Corresponding sides)

$\Rightarrow \quad AB^2 = AD \times AC \quad \ldots(i)$

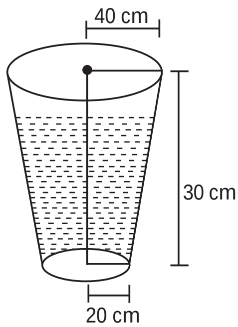
Similarly, $\quad BC^2 = DC \times AC \quad \ldots(ii)$

Adding (*i*) and (*ii*), we get

$$AB^2 + BC^2 = AD \times AC + CD \times AC$$
$$= AC(AD + CD)$$
$$= AC \times AC$$

$\Rightarrow \quad AB^2 + BC^2 = AC^2$ **Hence Proved.**

27.



Let $\quad R = 40$ cm

$r = 20$ cm

$h = 30$ cm

$\therefore$ Volume of frustum $= \dfrac{1}{3}\pi h\,(R^2 + r^2 + Rr)$

$$= \frac{1}{3} \times \frac{22}{7} \times 30\,(40^2 + 20^2 + 40 \times 20)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 30\,[1600 + 400 + 800]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 30 \times 2800$$

$= 88000 \text{ cm}^3 = 88$ litres

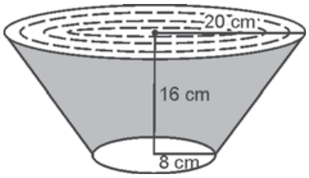
$\therefore$ Number of containers needed

$$= \frac{\text{Total volume of milk}}{\text{Volume of container}} = \frac{880}{88} = 10.$$

Cost of milk = 35 × 88 × 10 = ` 30800

Value: Helping the needy.

28. $h = 16$ cm, $r_1 = 8$ cm, $r_2 = 20$ cm



Capacity of container (frustum)

$$= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{3.14 \times 16}{3}\ (8^2 + 20^2 + 8 \times 20)$$

$$= \frac{50.24}{3}\ (64 + 400 + 160)$$

$$= \frac{50.24}{3} \times 624 = 50.24 \times 208$$

$$= 10449.92 \text{ cm}^3 = \frac{10449.92}{1000}\ l = 10.44992\ l$$

Cost of milk = Capacity in litres × Rate per litre

= 10.44992 × 20

= ` 208.9984 ≃ ` 209.

If l be the slant height of the frustum, then

$$l = \sqrt{h^2 + (r_2 - r_1)^2} = \sqrt{16^2 + 12^2}$$

$$= \sqrt{400} = 20 \text{ cm}$$

Total surface area of the frustum

$= \pi (r_1 + r_2)\, l + \pi r_1^2$

$= 3.14\ [(8 + 20) \times 20 + 8^2]$

$= 3.14 \times 624 = 1959.36 \text{ cm}^2$

Cost of metal sheet used

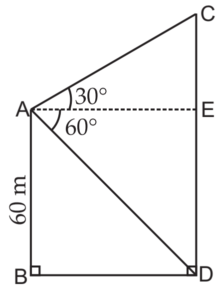
$$= \frac{1959.36}{100} \times 8$$

= 156.7488 ≃ ` 156.75

Thus, cost of milk is ` 209 and cost of metal sheet is ` 156.75.

29. Let AB = height of building

and CD = height of tower



$\therefore$ To find: (*i*) Difference between heights

= CD – DE [$\because$ AB = DE]

(*ii*) BD = Distance between bottoms

In right-angled ΔABD,

$$\angle ADB = \angle EAD = 60°$$

$$\therefore \quad \tan 60° = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{60}{BD}$$

$$\Rightarrow \quad BD = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \text{ m}$$

$$\therefore \quad BD = 20\sqrt{3} \text{ m}$$

Also as ABDE is a rectangle

$$\therefore \quad AB = DE = 60 \text{ m}$$

and $\quad BD = AE = 20\sqrt{3}$ m

$\therefore$ In right-angled ΔAEC,

$$\tan 30° = \frac{CE}{AE}$$

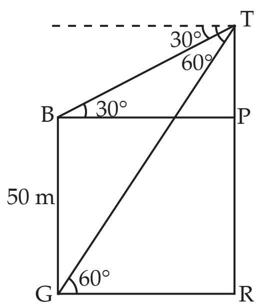
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{CE}{20\sqrt{3}}$$

$$\Rightarrow \quad CE = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}$$

$\therefore$ Difference between heights = CE = 20 m.

OR

In ΔBTP $\Rightarrow \tan 30° = \frac{TP}{BP}$



$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{TP}{BP}$$

$$BP = TP\sqrt{3} \quad \ldots(i)$$

In ΔGTR,

$$\tan 60° = \frac{TR}{GR} \Rightarrow \sqrt{3} = \frac{TR}{GR}$$

$$\Rightarrow \quad GR = \frac{TR}{\sqrt{3}} \quad \ldots(ii)$$

Now, $$TP\sqrt{3} = \frac{TR}{\sqrt{3}} \text{ (as BP = GR)}$$

$$\Rightarrow \quad 3TP = TP + PR$$

$$\Rightarrow \quad 2TP = BG \Rightarrow TP = \frac{50}{2} \text{ m} = 25 \text{ m}$$

Now, $$TR = TP + PR = (25 + 50) \text{ m}.$$

Height of tower = TR = 75m.

Distance between building and tower

$$= GR = \frac{TR}{\sqrt{3}}$$

$$\Rightarrow \quad GR = \frac{75}{\sqrt{3}} \text{ m} = 25\sqrt{3} \text{ m}.$$

30. We note that the classes are continuous, so, we proceed for making cumulative frequency table.

Marks (*C.I.*)	Number of Students (f)	*c.f.*
0–10	5	5
10–20	f_1	$5 + f_1$
20–30	15	$20 + f_1$ ←
30–40	f_2	$20 + f_1 + f_2$
40–50	6	$26 + f_1 + f_2$
	50	

As median is given to be 28, therefore, median class is (20-30), marked with arrow (←).

Here, $l = 20$, $c = 5 + f_1$, $h = 10$, $N = 50$, $f = 15$

Using the formula for median, we get

$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h$$

$$\Rightarrow \quad 28 = 20 + \frac{25 - (5 + f_1)}{15} \times 10$$

$$\Rightarrow \quad 8 = \frac{(25 - 5 - f_1)2}{3}$$

$$\Rightarrow \quad 12 = 20 - f_1 \Rightarrow f_1 = 8.$$

Also, $26 + f_1 + f_2 = 50 \Rightarrow f_1 + f_2 = 24$.

Substituting for $f_1 = 8$, we get

$$f_2 = 24 - 8 = 16$$

Therefore, $f_1 = 8,\ f_2 = 16.$

OR

Daily pocket allowance (in `)	Number of children (f_1)	Mid-point (x_i)	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11–13	3	12	–3	–9
13–15	6	14	–2	–12
15–17	9	16	–1	–9
17–19	13	18	0	0
19–21	k	20	1	k
21–23	5	22	2	10
23–25	4	24	3	12
	$\Sigma f_i = 40 + k$			$\Sigma f_i u_i = k - 8$

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \Rightarrow 18 = 18 + 2\left(\frac{k-8}{40+k}\right) \Rightarrow k = 8.$$

Practice Paper-2

Section-A

1. Rational number = 0.27
Irrational number = 0.26010010001… .

2. Sum of zeroes $= -\dfrac{-3\sqrt{2}}{3} = \sqrt{2}$

and product of zeroes $= \dfrac{1}{3}$.

3. Any point on y-axis be $(0, y)$

$\therefore \quad \sqrt{(6)^2+(5-y)^2} = \sqrt{(0+4)^2+(3-y)^2}$

$\Rightarrow \quad y = 9 \therefore$ Point is (0, 9).

4. $\angle BAT = \angle ACB = 55°$.

5. The modal class is 125-145 as the largest frequency is 20.
Required difference = 145 – 125 = 20.

6. $n(s) = 52$
Let E be the event (a red face card)

$\therefore \quad n(E) = 6 \;\therefore\; p(E) = \dfrac{6}{52} = \dfrac{3}{26}$.

Section-B

7. (*i*) $\dfrac{12027}{2^2\times5^3} = \dfrac{12027\times2}{2^3\times5^3} = \dfrac{24054}{1000} = 24.054$.

(*ii*) $\dfrac{37}{2^2\times5} = \dfrac{37\times2\times25}{2^3\times5^3} = \dfrac{1850}{1000} = 1.850$.

8. $\because$ The numbers $x-2$, $4x-1$ and $5x+2$ are in AP.

$\therefore \quad 4x-1-(x-2) = 5x+2-(4x-1)$
[$\because$ Common difference in AP is same]

$\Rightarrow \quad 4x-1-x+2 = 5x+2-4x+1$

$\Rightarrow \quad 3x+1 = x+3$

$\Rightarrow \quad 3x-x = 3-1 \quad\Rightarrow\quad 2x = 2$

$\Rightarrow \quad x = 1$.

9. Let A(8, 1), B(3, –2k) and C(k, –5) are collinear.

$\Rightarrow \quad$ Area $\Delta ABC = 0$

$\Rightarrow \quad |x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)| = 0$

$\Rightarrow \quad |8(-2k+5) + 3(-5-1) + k(1+2k)| = 0$

$\Rightarrow \quad |-16k + 40 - 18 + k + 2k^2| = 0$

$\Rightarrow \quad |2k^2 - 15k + 22| = 0$

$\Rightarrow \quad 2k^2 - 15k + 22 = 0$

$\Rightarrow \quad 2k^2 - 11k - 4k + 22 = 0$

$\Rightarrow \quad k(2k-11) - 2(2k-11) = 0$

$\Rightarrow \quad (k-2)(2k-11) = 0$

$\Rightarrow \quad k-2 = 0$ or $2k-11 = 0$

$\Rightarrow \quad k = 2$ or $k = \dfrac{11}{2}$.

10. $\text{Sec}\,\theta + \tan\theta = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{(1+\sin\theta)}{\cos\theta}$

$$= \frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{1+\frac{a}{b}}{\sqrt{1-\frac{a^2}{b^2}}}$$

$$= \frac{b+a}{\sqrt{b^2-a^2}} = \frac{b+a}{\sqrt{b+a}\sqrt{b-a}}$$

$$= \sqrt{\frac{b+a}{b-a}}.$$

11. Length of arc = 20 cm (Given)
and angle (θ) = 60°
Let the radius = r cm
We know that,

$$\text{Length of arc} = \frac{\theta}{360°}\times 2\pi r$$

$$20 = \frac{60}{360°}\times 2\pi r$$

$$r = \frac{60}{\pi}\text{ cm}$$

$$\text{Area of corresponding sector} = \pi r^2 \times \frac{\theta}{360°}$$

$$= \pi \times \frac{60\times60}{\pi^2} \times \frac{60}{360°}$$

$$= \frac{600}{\pi}\text{ cm}^2.$$

12. Sample space: {1, 2, 3, ... , 99}

$\therefore \quad n(S) = 99$.

The numbers divisible by 3 and 5 both are numbers divisible by 15.

So, favourable outcomes are: {15, 30, 45, 60, 75, 90}

Let E be the event getting a number divisible by 3 and 5.

$\therefore \qquad n(\text{E}) = 6$

$\therefore \qquad \text{P(E)} = \dfrac{n(\text{E})}{n(\text{S})} = \dfrac{6}{99} = \dfrac{2}{33}.$

Section-C

13. Let us assume, to the contrary that $\sqrt{3}$ is rational. We can take integers a and b such that

$\sqrt{3} = \dfrac{a}{b}$, where a and b are coprime.

$\Rightarrow \qquad 3b^2 = a^2$

$\Rightarrow \qquad a^2$ is divisible by 3

$\Rightarrow \qquad a$ is divisible by 3 ...(*i*)

We can write $a = 3c$ for some integer c

$\Rightarrow \qquad a^2 = 9c^2$

$\Rightarrow \qquad 3b^2 = 9c^2 \quad (\because\ a^2 = 3b^2)$

$\Rightarrow \qquad b^2 = 3c^2$

$\Rightarrow \qquad b^2$ is divisible by 3

$\Rightarrow \qquad b$ is divisible by 3 ...(*ii*)

From (*i*) and (*ii*), we observe that a and b have atleast 3 as a common factor. But this contradicts the fact that a and b are co-prime. This means that our assumption is not correct.

Hence, $\sqrt{3}$ is an irrational number.

OR

Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is rational.

So we can find coprimes a and b such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$

Rearranging, $\qquad \sqrt{5} = \dfrac{a - 3b}{2b}$

a and b are integers $\Rightarrow$ $a - 3b$ is an integer

$\Rightarrow \dfrac{a-3b}{2b}$ is rational number

$\therefore$ $\sqrt{5}$ should be rational. But we know that $\sqrt{5}$ is irrational. So our assumption that $3 + 2\sqrt{5}$ is rational is wrong.

Hence $3 + 2\sqrt{2}$ is irrational.

14. Since $x = \sqrt{\dfrac{5}{3}}$ and $x = -\sqrt{\dfrac{5}{3}}$ are zeroes of $p(x)$ $= 3x^4 + 6x^3 - 2x^2 - 10x - 5$, so $p(x)$ is divisible by $\left(x - \sqrt{\dfrac{5}{3}}\right)\left(x + \sqrt{\dfrac{5}{3}}\right)$, *i.e.*, $x^2 - \dfrac{5}{3}$.

$$\begin{array}{r|l} & 3x^2 + 6x + 3 \\ \hline x^2 - \frac{5}{3} & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ & \underset{-}{3x^4} \qquad\quad \underset{+}{-5x^2} \\ \hline & 6x^3 + 3x^2 - 10x - 5 \\ & \underset{-}{6x^3} \qquad\quad \underset{+}{-10x} \\ \hline & 3x^2 \qquad -5 \\ & \underset{-}{3x^2} \qquad \underset{+}{-5} \\ \hline & 0 \end{array}$$

Here, other two zeroes of $p(x)$ are the two zeroes of quotient $3x^2 + 6x + 3$

Put $\qquad 3x^2 + 6x + 3 = 0$

$\Rightarrow \qquad 3(x + 1)^2 = 0$

$\Rightarrow \qquad x = -1$ and $x = -1$

Hence, all the zeroes of $p(x)$ are $\sqrt{\dfrac{5}{3}}$, $-\sqrt{\dfrac{5}{3}}$, -1 and -1.

15. Let $\qquad a =$ first term

$d =$ common difference

$\therefore \qquad a_n = a + (n - 1)d$

$\therefore \qquad a_{17} = a + 16d$

$a_8 = a + 7d$

$a_{11} = a + 10d$

Now $\qquad a_{17} = 2 \cdot a_8 + 5$

$\Rightarrow \qquad a + 16d = 2(a + 7d) + 5$

$= 2a + 14d + 5$

$\Rightarrow \qquad 2d = a + 5$

$\Rightarrow \qquad a = 2d - 5$ ...(*i*)

Also as $\qquad a_{11} = 43$

$\Rightarrow \qquad a + 10d = 43$

Using (*i*), $2d - 5 + 10d = 43$

$\Rightarrow \qquad 12d = 48$

$d = 4$

$\therefore$ from (*i*) $\Rightarrow \qquad a = 2 \times 4 - 5 = 3$

$\therefore \qquad a_n = a + (n - 1) \cdot d$

$\Rightarrow \qquad a_n = 3 + (n - 1) \cdot 4$

$= 3 + 4n - 4$

$a_n = 4n - 1.$

OR

Integers between 100 and 200 divisible by 9 are 108, 117, 126, …, 198 which forms an AP with,

$$a = 108, d = 117 - 108 = 9$$

nth term in AP,

$$a_n = a + (n-1)d$$
$$198 = 108 + (n-1)d$$
$$198 - 108 = (n-1) \times 9$$
$$\frac{90}{9} = n - 1 \quad \Rightarrow \quad n = 10 + 1 = 11$$

Sum of nth term in AP,

$$S_n = \frac{n}{2}(a + l) = \frac{11}{2}(108 + 198)$$
$$= \frac{11}{2} \times 306 = 11 \times 153 = 1683.$$

16. For infinite number of solutions, we have

$$\frac{2}{p+q} = \frac{-3}{-(p+q-3)} = \frac{-7}{-(4p+q)}$$

Consider, $\frac{2}{p+q} = \frac{-3}{-(p+q-3)}$

$\Rightarrow \quad p + q = 6 \quad \ldots(i)$

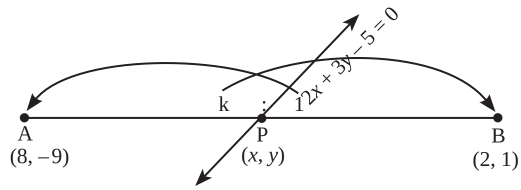
and $\frac{-3}{-(p+q-3)} = \frac{-7}{-(4p+q)}$,

$\Rightarrow \quad 5p - 4q = -21 \quad \ldots(ii)$

Solving (i) and (ii) we get;

$$p = -5, q = -1.$$

17. Let the line $2x + 3y - 5 = 0$ divides the line segment joining the points (8, – 9) and (2, 1) in the ratio $k : 1$.



By using formula

$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$, then

$$\therefore \quad P\left(\frac{2k+8}{k+1}, \frac{k-9}{k+1}\right) = P(x, y)$$

Thus, $x = \frac{2k+8}{k+1}$ and $y = \frac{k-9}{k+1}$

Point P(x, y) lies on the given line.

$$\therefore \quad 2\left(\frac{2k+8}{k+1}\right) + 3\left(\frac{k-9}{k+1}\right) - 5 = 0$$
$$\Rightarrow \quad 4k + 16 + 3k - 27 - 5k - 5 = 0$$
$$\Rightarrow \quad 2k = 16 \Rightarrow k = 8$$

Hence, the required ratio is 8 : 1.

Point of division is given as

$$P\left(\frac{2(8)+8}{8+1}, \frac{8-9}{8+1}\right) \text{ i.e., } P\left(\frac{8}{3}, \frac{-1}{9}\right).$$

OR

Let x-axis divides the line segment joining (–4, –6) and (–1, 7) at the point P in the ratio $1 : k$.

Now, coordinates of point of division

$$= P\left(\frac{-1-4k}{k+1}, \frac{7-6k}{k+1}\right)$$

Since P lies on x-axis, therefore $\frac{7-6k}{k+1} = 0$

$$\Rightarrow \quad 7 - 6k = 0 \Rightarrow k = \frac{7}{6}$$

Hence the ratio is $1 : \frac{7}{6} = 6 : 7$

Now, the coordinate of p are $\left(\frac{-34}{13}, 0\right)$.

18. Given: ΔABC, in which AD $\perp$ BC and BD $= \frac{1}{3}$ CD.

To Prove: $2CA^2 = 2AB^2 + BC^2$.

Proof: $\because$ BD $= \frac{1}{3}$ CD $\therefore \frac{BD}{CD} = \frac{1}{3}$

Let BD $= x$ then CD $= 3x$

$\Rightarrow$ BC $= x + 3x = 4x$.

In right-angled ΔABD, by Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$
$$AD^2 = AB^2 - BD^2 \quad \ldots(i)$$

Similarly, in right-angled ΔACD,

$$AC^2 = AD^2 + DC^2$$
$$AD^2 = AC^2 - DC^2 \quad \ldots(ii)$$

Equating the value of AD^2 from equations (*i*) and (*ii*), we get

$$AB^2 - BD^2 = AC^2 - DC^2$$

$$AB^2 - BD^2 + DC^2 = AC^2$$

$$AC^2 = AB^2 - \left(\frac{1}{4}BC\right)^2 + \left(\frac{3}{4}BC\right)^2$$

$$\left[\because BD = \frac{1}{4} BC \text{ and } DC = \frac{3}{4} BC\right]$$

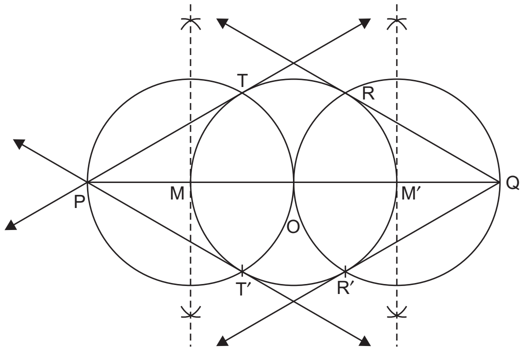
$$= AB^2 - \frac{BC^2}{16} + \frac{9BC^2}{16}$$

$$= \frac{16AB^2 - BC^2 + 9BC^2}{16} = \frac{16AB^2 + 8BC^2}{16}$$

$$\therefore AC^2 = \frac{8(2AB^2 + BC^2)}{16} \Rightarrow AC^2 = \frac{2AB^2 + BC^2}{2}$$

$2AC^2 = 2AB^2 + BC^2$. **Hence proved.**

19. PT, PT′ and QR, QR′ are required tangents.



20.

C.I.	Frequency (f_i)	Mid value (x_i)	$u_i = \frac{x-a}{h}$	$f_i u_i$
0-10	7	5	–2	–14
10-20	12	15	–1	–12
20-30	13	25	0	0
30-40	10	35	1	10
40-50	8	45	2	16
	$\Sigma f_i = 50$			$\Sigma f_i u_i = -26 + 26 = 0$

Let assumed mean $a = 25$; $h = 10$

$$\text{Mean } \bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 25 + \frac{0}{50} \times 10$$

$$\Rightarrow \quad \text{Mean } \bar{x} = 25.$$

21. Given expression $= \dfrac{2\sin 68°}{\cos 22°} - \dfrac{2\cot 15°}{5\tan 75°}$

$$- \frac{3\tan 45° \tan 20° \tan 40° \tan 50° \tan 70°}{5}$$

$$= \frac{2\sin(90° - 22°)}{\cos 22°} - \frac{2\cot(90° - 75°)}{5\tan 75°}$$

$$- \frac{\frac{3 \times 1 \times \tan(90° - 70°)\tan(90° - 50°)}{\tan 50° \tan 70°}}{5}$$

$$= \frac{2\cos 22°}{\cos 22°} - \frac{2\tan 75°}{5\tan 75°} - \frac{\frac{3\cot 70° \cot 50°}{\tan 50° \tan 70°}}{5}$$

$$= 2 - \frac{2}{5} - \frac{3 \times \frac{1}{\tan 70°} \times \frac{1}{\tan 50°} \tan 50° \tan 70°}{5}$$

$$= 2 - \frac{2}{5} - \frac{3}{5} = \frac{10 - 2 - 3}{5}$$

$$= \frac{5}{5} = 1.$$

OR

$$\frac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°}$$

$$+ \frac{\sin^2 63° + \cos 63° \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}$$

$$= \frac{\text{cosec}^2 63° + \tan^2 24°}{\tan^2(90° - 66°) + \text{cosec}^2(90° - 27°)}$$

$$+ \frac{\sin^2 63° + \cos 63° \cos(90° - 27°) + \sin 27° \text{cosec}(90° - 63°)}{2[\text{cosec}^2 65° - \cot^2(90° - 25°)]}$$

$$= \frac{\text{cosec}^2 63° + \tan^2 24°}{\tan^2 24° + \text{cosec}^2 63°}$$

$$+ \frac{\sin^2 63° + \cos^2 63° + \sin 27° \text{cosec } 27°}{2(\text{cosec}^2 65° - \cot^2 65°)} = 1 + \frac{1+1}{2(1)}$$

= 2.

22. LHS $= \dfrac{1}{\dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A}} - \dfrac{1}{\sin A}$

$$= \frac{\sin A}{1 - \cos A} - \frac{1}{\sin A} = \frac{\sin^2 A - 1 + \cos A}{\sin A (1 - \cos A)}$$

$$= \frac{1 - \cos^2 A - 1 + \cos A}{\sin A (1 - \cos A)} = \frac{\cos A (1 - \cos A)}{\sin A\ (1 - \cos A)}$$

= cot A.

Section-D

23. Given equations of lines are:

$$3x + y + 4 = 0 \quad ...(i)$$

and $$6x - 2y + 4 = 0 \quad ...(ii)$$

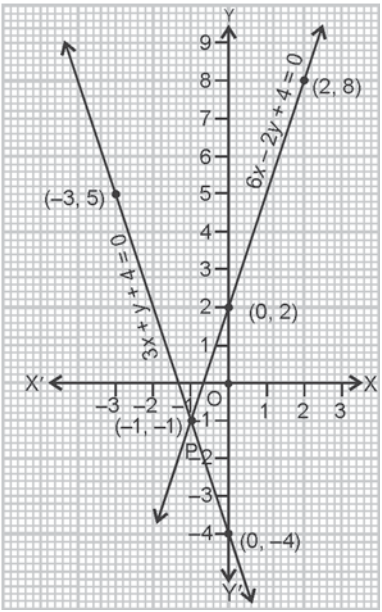
To draw the graphs of lines (*i*) and (*ii*), we need atleast two solutions of each equation.

For equation (*i*), two solutions are:

x	0	– 3
y	– 4	5

For equation (*ii*), two solutions are:

x	0	2
y	2	8



Let us draw the graphs of the lines (*i*) and (*ii*). From the graph it is clear that the two lines intersect each other at a point, P(– 1, – 1), therefore, the pair of equations consistent.

The solution is $x = -1$, $y = -1$.

24. (*i*) Let width of grass paths = x m

∴ Length of rectangular pond

$= (50 - 2x)$ m

breadth of rectangular pond

$= (40 - 2x)$ m

∴ Area of grass path = Area of lawn – Area of rectangular pond

$= 50 \times 40 - (50 - 2x)(40 - 2x)$

$= 2000 - 2000 + 100x + 80x - 4x^2$

$= 180x - 4x^2$

According to question

$180x - 4x^2 = 1184$

$\Rightarrow \quad 4x^2 - 180x + 1184 = 0$

$\Rightarrow \quad x^2 - 45x + 296 = 0$

$\Rightarrow \quad x^2 - 8x - 37x + 296 = 0$

$\Rightarrow \quad x(x - 8) - 37(x - 8) = 0$

$\Rightarrow \quad (x - 8)(x - 37) = 0$

$\Rightarrow \quad x = 8$ or $x = 37$

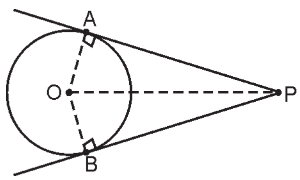
Reject $x = 37$ as it is not possible

∴ $x = 8$ m.

(*ii*) ∴ Length of pond = 34 m; breadth of pond = 24 m.

(*iii*) Love for environment.

25. Given: Let PA and PB be two tangents drawn from an external point P to a circle C(O, *r*).



To prove: PA = PB.

Construction: Join OA, OB and OP.

Proof: ∠OAP = 90° ...(*i*)

[Tangent is perpendicular to radius at the point of contact]

Similarly, ∠OBP = 90° ...(*ii*)

From (*i*) and (*ii*), we get

∠OAP = ∠OBP = 90° ...(*iii*)

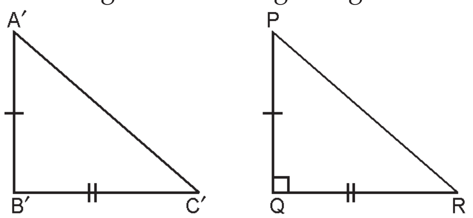
Now, in right ∆s OAP and OBP,

$$OP = OP \quad \text{[Common]}$$
$$OA = OB \quad \text{[Radii]}$$
$$\angle OAP = \angle OBP = 90° \quad \text{[From } (iii)\text{]}$$
$$\therefore \quad \Delta OAP \cong \Delta OBP \quad \text{[SAS]}$$
$$\Rightarrow \quad PA = PB. \quad \text{[CPCT]}$$

AE = AH (Length of tangents from external points are equal)

$$\Rightarrow \quad x = 4 - x \quad \Rightarrow \quad 2x = 4 \Rightarrow x = 2$$
$$\Rightarrow \quad DH = (5 - 2) = 3 \text{ cm}$$
$$\Rightarrow \quad DH = DG = 3 \text{ cm}$$
$$\Rightarrow \quad CF = CG \Rightarrow 2y - 3 = y \Rightarrow y = 3$$
$$DC = DG + GC$$
$$= 3 + 3 = 6 \text{ cm}.$$

26. Statement: In a triangle, if square of the largest side is equal to the sum of the squares of the other two sides, then the angle opposite to the largest side is a right angle.



Proof: We are given a triangle A' B' C' with

$$A'C'^2 = A'B'^2 + B'C'^2 \quad ...(i)$$

We have to prove that ∠B' = 90°

Let us construct a ∆PQR with ∠Q = 90° such that

$$PQ = A'B' \text{ and } QR = B'C' \quad ...(ii)$$

In ∆PQR,

$$PR^2 = PQ^2 + QR^2$$

(Pythagoras Theorem)

$$= A'B'^2 + B'C'^2 \quad ...(iii)\text{[From } (ii)\text{]}$$

But $A'C'^2 = A'B'^2 + B'C'^2$ ...(*iv*) [From (*i*)]

From equations (*iii*) and (*iv*), we have

$$PR^2 = A'C'^2$$
$$\Rightarrow \quad PR = A'C' \quad ...(v)$$

Now, in ∆A'B'C' and ∆PQR,

$$A'B' = PQ \quad \text{[From } (ii)\text{]}$$
$$B'C' = QR \quad \text{[From } (ii)\text{]}$$
$$A'C' = PR \quad \text{[From } (v)\text{]}$$

Therefore, ∆A'B'C' ≅ ∆PQR

(SSS congruence rule)

$$\Rightarrow \quad \angle B' = \angle Q \quad \text{(CPCT)}$$
$$\text{But} \quad \angle Q = 90°$$
$$\therefore \quad \angle B' = 90°.$$

Hence proved.

2nd part

In ∆ABD, ∠D = 90°

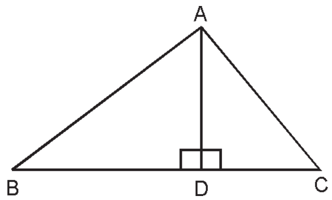
$$\therefore \quad AB^2 = AD^2 + BD^2$$

In ∆ACD, ∠D = 90°

$$\therefore \quad AC^2 = AD^2 + CD^2$$

So, $AB^2 + AC^2 = 2AD^2 + BD^2 + CD^2$

$$= 2.\, BD \times CD + BD^2 + CD^2$$
$$(\because \quad AD^2 = BD \times CD)$$
$$= (BD + CD)^2$$
$$\Rightarrow \quad AB^2 + AC^2 = BC^2$$



Now, by above theorem, we have in ∆ABC,

$$\angle BAC = 90°$$

⇒ ∆ABC is a right-angled triangle.

OR

Try yourself

27. (*i*) V_1 = Volume of juice in cubical container

$$= (5 \times 6 \times 22) \text{ cm}^3$$

V_2 = Volume of juice in cubical container

$$= \frac{22}{7} \times (7)^2 \times 22 \text{ cm}^3$$

V_3 = Volume of juice in each small cone

$$= \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times 3.5 \text{ cm}^3$$

∴ **Case I.** If cubical packing is purchased, then number of small cones needed

$$= \frac{V_1}{V_3} = \frac{5 \times 6 \times 22}{\frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 3.5}$$

$$= \frac{6 \times 10 \times 3}{2 \times 2} = 3 \times 5 \times 3 = 45$$

Case II. If cylindrical packing is purchased.

$\therefore$ Number of small cones needed = $\dfrac{V_2}{V_3}$

$$= \frac{\frac{22}{7} \times 7 \times 7 \times 22}{\frac{1}{3} \times \frac{22}{7} \times 2 \times 2 \times 3.5} = \frac{7 \times 7 \times 22 \times 3 \times 10}{2 \times 2 \times 35}$$

= 7 × 11 × 3 = 231.

(*ii*) Mr Sharma must purchase cylindrical packing to serve maximum children.

(*iii*) Surface area and volume of solids.

(*iv*) Kindheartedness and helpful.

28. Let internal radius of pipe = x m.

and radius of base of tank

$$= 40 \text{ cm} = \frac{2}{5} \text{ m}$$

Level of water raised in tank

$$= 3.15 \text{ or } \frac{315}{100}$$

Volume of water delivered in $\frac{1}{2}$ hr

$$= \pi r^2 h = \pi(x)^2 \times 1260 \text{ m}$$

[∵ 2.52 km = 1 hr]

2520 m = 1 hr

$\therefore$ in $\frac{1}{2}$ hr height $= \frac{1}{2} \times 2520 = 1260$ m

$\therefore$ According to question,

$$\Rightarrow \pi(x^2)(1260) = \pi \times \frac{2}{5} \times \frac{2}{5} \times \frac{315}{100}$$

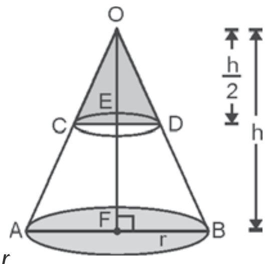
$$\Rightarrow \quad x^2 = \frac{4}{25} \times \frac{315}{100} \times \frac{1}{1260} = \frac{1}{2500}$$

$$\Rightarrow \quad x = \frac{1}{50} \text{ m} = 2 \text{ cm}$$

$\therefore$ Internal diameter of pipe = 4 cm.

OR

Let r and h be the radius of the base and height of a cone OAB.



Let OE = $\dfrac{h}{2}$

Since, triangles OED and OFB are similar.

$$\therefore \frac{OE}{OF} = \frac{ED}{FB}$$

$$\Rightarrow \frac{\frac{h}{2}}{h} = \frac{ED}{r} \Rightarrow ED = \frac{r}{2}$$

$$\text{Volume of cone OCD} = \frac{1}{3}\pi \times \frac{r}{2} \times \frac{r}{2} \times \frac{h}{2} = \frac{\pi r^2 h}{24}$$

$$\text{Volume of cone OAB} = \frac{1}{3}\pi r^2 h$$

$$\therefore \frac{\text{Volume of part OCD}}{\text{Volume of part CDBA}} = \frac{\frac{\pi r^2 h}{24}}{\frac{\pi r^2 h}{3} - \frac{\pi r^2 h}{24}}$$

$$= \frac{\frac{1}{24}}{\frac{1}{3} - \frac{1}{24}} = \frac{\frac{1}{24}}{\frac{8-1}{24}} = \frac{1}{7}.$$

29. Let the tower be PQ and the objects be A and B.

∵ ∠XQA = 45° and ∠XQB = 60°

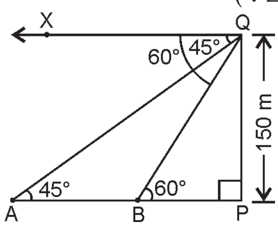
∴ ∠QAP = 45° and ∠QBP = 60° (Alternate angles)

In right ΔAPQ,

∠PAQ + ∠PQA = 90°

⇒ ∠PQA = 90° – 45° = 45°

(∵ ∠PAQ = 45°)



∴ AP = PQ = 150 (∵ PQ = 150 m)

⇒ AB + BP = 150 ...(*i*)

In right ΔBPQ,

$$\tan 60° = \frac{PQ}{BP} \Rightarrow \sqrt{3} = \frac{150}{BP}$$

$$\Rightarrow \quad BP = \frac{150}{\sqrt{3}} \quad \text{...(ii)}$$

Putting BP = $\dfrac{150}{\sqrt{3}}$ in equation (*i*), we get

$$AB + \frac{150}{\sqrt{3}} = 150$$

$$\Rightarrow \quad AB = 150\left(1 - \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad AB = 150 \times \frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 50 \times (3-\sqrt{3}) = 50\ (3 - 1.73)$$

$$= 50 \times 1.27 \Rightarrow AB = 63.50 \text{ m}$$

Thus, distance between the two objects is 63.50 m.

30. Let us convert the given data into less than type distribution.

Class interval	*f*	Lifetimes (in hrs.)	*cf*
0–20	10	less than 20	10
20–40	35	less than 40	45
40–60	52	less than 60	97
60–80	61	less than 80	158
80–100	38	less than 100	196
100–120	29	less than 120	225

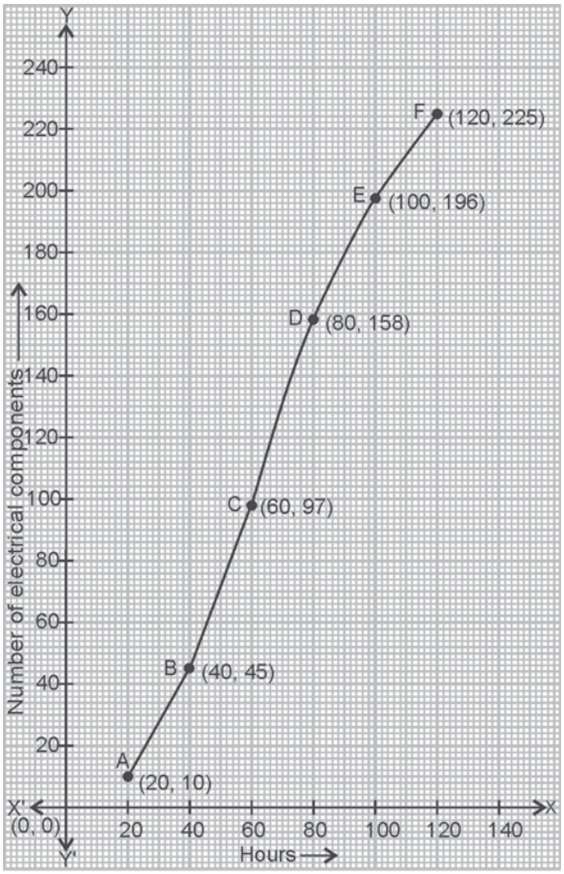


Figure: Less than type ogive

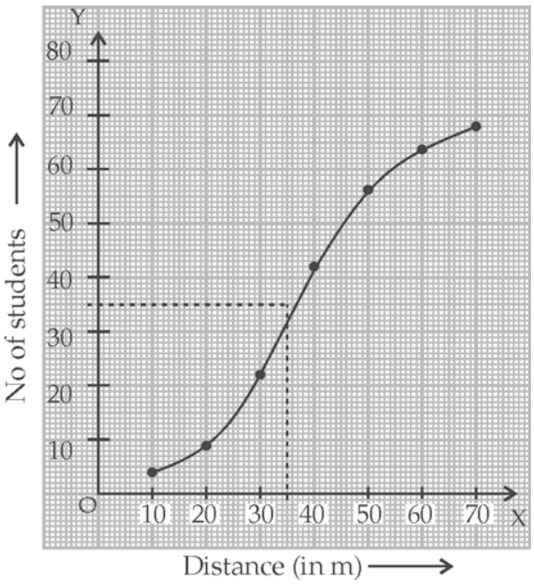
We mark the upper class limits along the *x*-axis with a suitable scale and the cumulative frequencies along the *y*-axis with a suitable scale. For this, we plot the points A(20, 10), B(40, 45), C(60, 97), D(80, 158), E(100, 196) and F(120, 225) on a graph paper. These points are joined by a free hand smooth curve to obtain a less than type ogive as shown in the given graph.

OR

Less than	Number of Students
10	4
20	9
30	22
40	42
50	56
60	64
70	68

Median distance is value of x that corresponds to cumulative frequency $\frac{N}{2} = \frac{68}{2} = 34$

Therefore, median distance = 36.



Less than O'give

Practice Paper-3

Section-A

1. $\frac{125}{2^4 \cdot 5^3} = \frac{5^3}{16 \times 5^3} = \frac{1}{16} = 0.0625$

Clearly, the decimal form of $\frac{125}{2^4 \cdot 5^3}$ terminates after four places.

2. $f(x) = 3x^2 - 3 + 2x - 5$
$= 3x^2 + 2x - 8$

$\therefore$ Sum of zeroes $= -\frac{b}{a} = -\frac{2}{3}$

Product of zeroes $= \frac{c}{a} = -\frac{8}{3}$.

3. Condition of collinearity is:

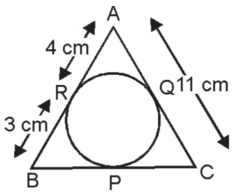
$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\therefore$ $1(k - 4) + 3(4 - 1) + (-1)(1 - k) = 0$

$\Rightarrow$ $k - 4 + 9 - 1 + k = 0$

$\Rightarrow$ $k = -2.$

4. BC = BP + PC
= BR + CQ
= 3 + [AC−AQ]
= 3 + [11 − 4]
= 10 cm.



5. $\because$ 3 Median = 2 Mean + Mode

$\because$ Median $= \frac{2 \times 27 + 45}{3}$

= 33.

6. Required prime numbers are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

$\therefore$ Required probability $= \frac{10}{30} = \frac{1}{3}$.

Section-B

7. Let us represent each of the numbers 30, 72 and 432 as a product of primes.

$30 = 2 \times 3 \times 5$

$72 = 2^3 \times 3^2$

$432 = 2^4 \times 3^3$

Now, HCF $= 2 \times 3 = 6$

and LCM $= 2^4 \times 3^3 \times 5 = 2160.$

8. 15th term from end of −10, −20, −30, ..., −980, −990, −1000

= 15th term of −1000, −990, −980, ..., −20, −10

$= -1000 + (15 - 1) \times (-990 + 1000)$

$= -1000 + 140 = -860.$

9. Let A(3, 0), B(6, 4) and C(−1, 3) are the vertices.

$\therefore$ Consider $AB = \sqrt{(6-3)^2 + (4-0)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$BC = \sqrt{(-1-6)^2 + (3-4)^2}$

$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

$AC = \sqrt{(-1-3)^2 + (3-0)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

Clearly AB = AC

$\Rightarrow$ Triangle is isosceles

also $AB^2 + AC^2 = 5^2 + 5^2 = 50$

and $BC^2 = \left(5\sqrt{2}\right)^2 = 50$

$\Rightarrow$ $AB^2 + AC^2 = BC^2$

$\Rightarrow$ By converse of Pythagoras theorem

$\angle A = 90°$.

$\Rightarrow$ ΔABC is right-angled isosceles triangle.

Hence proved

10. $\frac{4}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cos^2 45°$

$= \frac{4}{\left(\sqrt{3}\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2} = \frac{8+8-3}{6} = \frac{13}{6}$.

11. Area of shaded part = area of square − area of quadrant

$= (7)^2 - \frac{1}{4}\pi(7)^2$

$= 49 - \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$

$= 49 - \frac{11}{2} \times 7$

$= 49 - \frac{77}{2} = 49 - 38.5 = 10.5 \text{ cm}^2.$

12. Total number of outcomes = 52

Since, the drawn card should not be red or queen

Total number of red cards (including a red queen) = 13

Total number of queens (excluding red queen) = 3

$\therefore$ Total favourable outcomes

$= 13 + 3 = 16$

$\therefore$ Required probability $= \frac{16}{52} = \frac{4}{13}$.

Section-C

13. Hint: Let a be any positive integer

$\therefore \quad a = 3q$ or $3q + 1$ or $3q + 2$

$\therefore \quad a^2 = 9q^2 = 3m; m = 3q^2$

or $\quad a^2 = (3q + 1)^2 = 3m + 1, m = q(3q + 2)$

or $\quad a^2 = (3q + 2)^2 = 3m + 1, m = 3q^2 + 4q + 1.$

OR

We represent 6,72 and 120 in their prime factors.

$$6 = 2 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$= 2^3 \times 3 \times 5$$

Now, HCF $= 2 \times 3 = 6$

And LCM $= 2^3 \times 3^2 \times 5$

$= 360.$

14. ***See Worksheet-15 Sol.6***

15. Let the three terms of an A.P. be $a - d$, a and $a + d$.

Sum $= a - d + a + a + d = 24$

$\Rightarrow \quad 3a = 24$

$\Rightarrow \quad a = 8$

Sum of squares $= (a-d)^2 + a^2 + (a + d)^2 = 194$

$\Rightarrow \quad a^2 - 2ad + d^2 + a^2 + a^2 + 2ad + d^2 = 194$

$\Rightarrow \quad 3a^2 + 2d^2 = 194$

$\Rightarrow \quad 192 + 2d^2 = 194 \quad (\because a = 8)$

$\Rightarrow \quad 2d^2 = 2$

$\Rightarrow \quad d^2 = 1 \Rightarrow d = \pm 1$

If $d = 1$, numbers are 7, 8, 9

If $d = -1$, numbers are 9, 8, 7

Hence, the required numbers are 7, 8, 9 or 9, 8, 7.

OR

$S_n = 3n^2 + 2n$

Replacing n by $n - 1$, we get

$S_{n-1} = 3(n - 1)^2 + 2(n - 1)$

$= (n - 1)(3n - 3 + 2)$

$= (n - 1)(3n - 1)$

$= 3n^2 - n - 3n + 1$

$= 3n^2 - 4n + 1$

We know that nth terms is given by

$a_n = S_n - S_{n-1}$

$\therefore \quad a_n = 3n^2 + 2n - 3n^2 + 4n - 1$

$= 6n - 1.$

16. Given system of linear equations can be written as:

$(a - b)x + (a + b)y - (a^2 - 2ab - b^2) = 0$

$(a + b)x + (a + b)y - (a^2 + b^2) = 0$

By cross-multiplication,

$$\frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}$$

$$= \frac{-y}{-(a-b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}$$

$$= \frac{1}{(a-b)(a+b)-(a+b)(a+b)}$$

$$\Rightarrow \frac{x}{-2b(a+b)^2} = \frac{-y}{-4ab^2} = \frac{1}{-2b(a+b)}$$

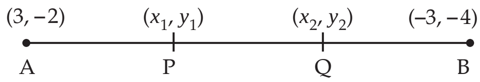
Hence, the solution of given system of equations is

$x = a + b,$

$y = -\frac{2ab}{a+b}.$

17. Let the points of trisection be $P(x_1, y_1)$ and $Q(x_2, y_2)$ such that P is the mid-point of A(3, – 2), $Q(x_2, y_2)$ and Q is the mid-point of $P(x_1, y_1)$, B(–3, – 4).

i.e., AP = PQ = QB



$\Rightarrow$ AP : PB = 1 : 2

Using section formula,

$$\therefore \quad x_1 = \frac{-3+2\times3}{1+2}, y_1 = \frac{-4+2\times(-2)}{1+2}$$

$$\Rightarrow \quad x_1 = 1, y_1 = -\frac{8}{3}$$

Again AQ : QB = 2 : 1

$$\therefore \quad x_2 = \frac{2\times(-3)+3}{2+1}, y_2 = \frac{2\times(-4)-2}{2+1}$$

$$\Rightarrow \quad x_2 = -1, y_2 = -\frac{10}{3}$$

Hence, the required points are $P\left(1, -\frac{8}{3}\right)$ and $Q\left(-1, -\frac{10}{3}\right)$

OR

Let $A(2, 3) \equiv A(x_1, y_1)$

$B(-1, 0) \equiv B(x_2, y_2)$

and $C(2, -4) \equiv C(x_3, y_3)$

Now, area of ΔABC

$$= \left|\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}\right|$$

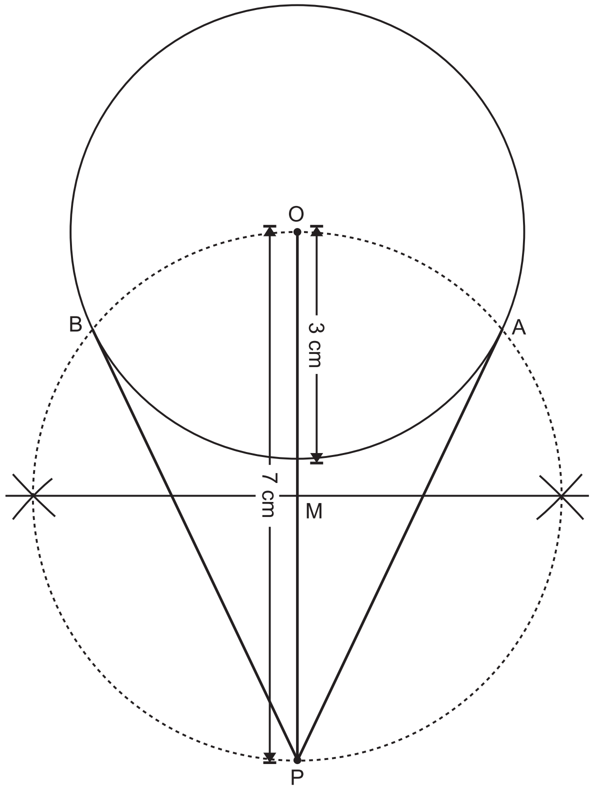
$$= \left|\frac{1}{2}\{2(0-(-4)) + (-1)(-4-3) + 2(3-0)\}\right|$$

$$= \left|\frac{1}{2}(2\times4+7+6)\right|$$

$$= \left|\frac{1}{2}(8+7+6)\right| = \left|\frac{1}{2}\times21\right| = \frac{21}{2} \text{ sq. units.}$$

18. To draw a pair of tangents from P to the circle with centre O, we follow the steps as given:

(*a*) Join OP and find its mid-point M.



(*b*) Taking M as centre and radius = MP = MO, draw a circle to intersect the given circle at A and B.

(*c*) Join PA and PB.

PA and PB are the required tangents.

On measuring, PA = 6.35 cm and PB = 6.35 cm. Clearly, PA and PB are of same length.

19. Statement: In a triangle, if square of the largest side is equal to the sum of the squares of the other two sides, then the angle opposite to the largest side is a right angle.

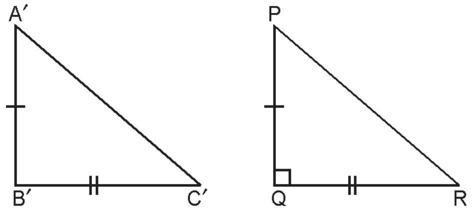
Proof: We are given a triangle A′ B′ C′ with

$$A'C'^2 = A'B'^2 + B'C'^2 \quad ...(i)$$

We have to prove that $\angle B' = 90°$

Let us construct a ΔPQR with $\angle Q = 90°$ such that

$$PQ = A'B' \text{ and } QR = B'C' \quad ...(ii)$$



In ΔPQR,

$PR^2 = PQ^2 + QR^2$ (Pythagoras Theorem)

$= A'B'^2 + B'C'^2$ ...(*iii*) [From (ii)]

But $A'C'^2 = A'B'^2 + B'C'^2$ ...(*iv*) [From (*i*)]

From equations (*iii*) and (*iv*), we have

$PR^2 = A'C'^2$

$\Rightarrow$ PR = A′C′ ...(*v*)

Now, in ΔA′B′C′ and ΔPQR,

A′B′ = PQ [From (*ii*)]

B′C′ = QR [From (*ii*)]

A′C′ = PR [From (*v*)]

Therefore, ΔA′B′C′ ≅ ΔPQR (SSS congruence rule)

$\Rightarrow$ ∠B′ = ∠Q (CPCT)

But ∠Q = 90°

∴ ∠B′ = 90°. **Hence proved.**

20. ∵ Modal class is 30-40,

∴ $l = 30, f_1 = 16,$

$f_0 = x, f_2 = 12, h = 10$

and Mode = 3

Using the formula: Mode

$$= l + h\left\{\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right\} \text{ we get,}$$

$$32 = 30 + 10x\left[\frac{16 - x}{32 - x - 12}\right]$$

$$\Rightarrow \frac{160 - 10x}{20 - x} = 2 \Rightarrow 160 - 10x = 40 - 2x$$

$\Rightarrow$ $120 = 8x \Rightarrow x = 15$

∴ Missing frequency is 15.

21. $$\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

Dividing numerator and denominator by sin A, we get

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)[1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1}$$

= cosec A + cot A = RHS.

OR

$$\text{LHS} = \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}}$$

$$= \sqrt{\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}} = \sqrt{\frac{(1 - \sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1 - \sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

= sec θ – tan θ.

22. $\sin(x + y) = 1$ and $\cos(x - y) = \frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin(x + y) = \sin 90°$ and $\cos(x - y) = \cos 30°$

$\Rightarrow$ $x + y = 90°$ and $x - y = 30°$

Adding and subtracting, we get respectively

$2x = 120°$ and $2y = 60°$

i.e., $x = 60°$ and $y = 30°$.

Section-D

23. To draw a line, we need atleast two solutions of its corresponding equation.

$x + 3y = 6$; at $x = 0, y = 2$ and $x = 3, y = 1$.

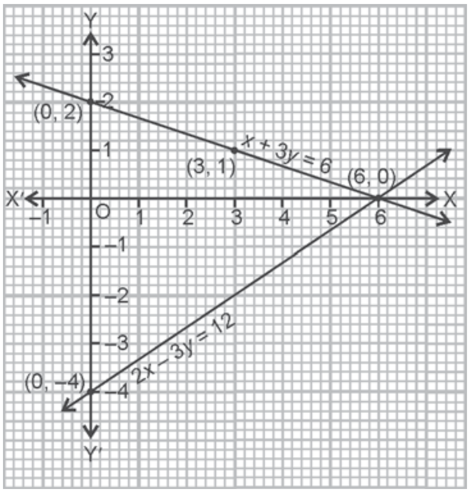
So, two solutions of $x + 3y = 6$ are:

x	0	3
y	2	1

$2x - 3y = 12$; at $x = 0, y = -4$ and at $x = 6, y = 0$

So, two solutions of $2x - 3y = 12$ are:

x	0	6
y	–4	0



Now, we draw the graph of given system of equations by using their corresponding solutions obtained in the above tables.

From the graph, the two lines intersect the y-axis at (0, 2) and (0, –4).

24. (*i*) Let usual speed = x km/hr.

As distance = Time × Speed

$$\therefore \quad \text{Usual time} = \frac{1500}{x}$$

$$\text{New speed} = (x + 250)$$

$$\therefore \quad \text{New time} = \frac{1500}{(x+250)}$$

According to question,

Usual time of flight – New time of flight = 30 minutes = $\frac{1}{2}$ hr.

$$\Rightarrow \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$$\Rightarrow \quad 1500\left[\frac{x+250-x}{x(x+250)}\right] = \frac{1}{2}$$

$$\Rightarrow \quad 3000 \times 250 = x(x+250)$$

$$\Rightarrow \quad x^2 + 250x - 750000 = 0$$

$$\Rightarrow \quad x^2 + 1000x - 750x - 750000 = 0$$

$$\Rightarrow \quad x(x+1000) - 750(x+1000) = 0$$

$$\Rightarrow \quad (x-750)(x+1000) = 0$$

$$\Rightarrow \quad x = 750 \text{ or } x = -1000$$

(Rejected)

$\therefore$ Usual speed = 750 km/h

(*ii*) Formation and solving a quadratic equation by splitting the middle term.

(*iii*) Punctuality of pilot is reflected in this problem.

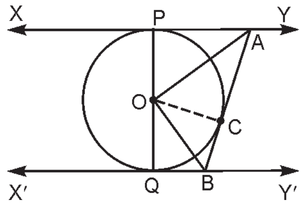
25. Hint:

As PA = AC

$\therefore$ $\Delta PAO \cong \Delta CAO$ (SSS)

$\Rightarrow$ $\angle PAO = \angle CAO$ (CPCT)

$\Rightarrow$ $\angle PAC = 2\angle CAO$



Similarly,

$$\angle CBQ = 2\angle CBO$$

As $\angle PAC + \angle CBQ = 180°$

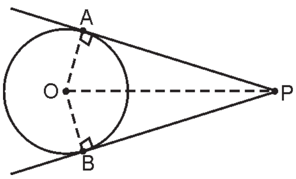
$$\Rightarrow \quad \frac{1}{2}\angle PAC + \frac{1}{2}\angle CBQ = \frac{1}{2} \times 180°$$

$$\Rightarrow \quad \angle CAO + \angle CBO = 90°$$

$$\therefore \quad \angle AOB = 90°.$$

OR

Given: Let PA and PB be two tangents drawn from an external point P to a circle C(O, r).



To prove: PA = PB.

Construction: Join OA, OB and OP.

Proof: $\angle OAP = 90°$ ...(*i*)

[Tangent is perpendicular to radius at the point of contact]

Similarly, $\angle OBP = 90°$ ...(*ii*)

From (*i*) and (*ii*), we get

$\angle OAP = \angle OBP = 90°$ ...(*iii*)

Now, in right Δs OAP and OBP,

OP = OP [Common]

OA = OB [Radii]

$\angle OAP = \angle OBP = 90°$ [From (*iii*)]

$\therefore \quad \Delta OAP \cong \Delta OBP$ [SAS]

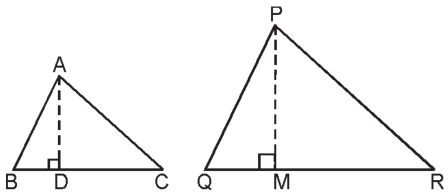
$\Rightarrow \quad PA = PB.$ [CPCT]

Note: Theorem can also be proved by using Pythagoras theorem as

$$AP^2 = OP^2 - OA^2$$
$$= OP^2 - OB^2 = PB^2$$

$\Rightarrow \quad AP = PB.$ [$\because$ OA = OB]

26. Let the two given triangles be ABC and PQR such that $\Delta ABC \sim \Delta PQR$



$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} \quad ...(i)$$

Let us draw perpendiculars AD and PM from A and P to BC and QR respectively.

$$\therefore \quad \angle ADB = \angle PMQ = 90° \quad ...(ii)$$

Now, in ΔABD and ΔPQM,

$$\angle B = \angle Q$$

($\because \Delta ABC \sim \Delta PQR$)

$$\angle ADB = \angle PMQ \quad \text{[From (ii)]}$$

So, by AA rule of similarity, we have

$$\Delta ABD \sim \Delta PQM$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM} \quad ...(iii)$$

From equations (*i*) and (*iii*), we get

$$\frac{BC}{QR} = \frac{AD}{PM} \quad ...(iv)$$

$$\text{Now,} \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PM}$$

$$= \frac{\frac{1}{2} \times BC \times BC}{\frac{1}{2} \times QR \times QR}$$

[Using (iv)]

$$= \left(\frac{BC}{QR}\right)^2 \quad ...(v)$$

Similarly, we can prove that

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 \quad ...(vi)$$

$$\text{and} \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AC}{PR}\right)^2 \quad ...(vii)$$

From equations (*v*), (*vi*) and (*vii*), we obtain

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$
$$= \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2.$$

Hence, the theorem

Further, in the question

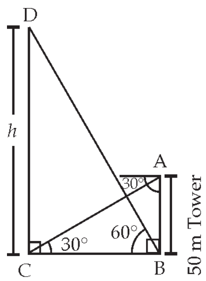
$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \quad \frac{64}{121} = \frac{BC^2}{15.4 \times 15.4}$$

$$\Rightarrow \quad BC = \sqrt{\frac{64 \times 15.4 \times 15.4}{121}}$$

$$= \frac{8}{11} \times 15.4 = 11.2 \text{ cm.}$$

27. Let height of hill = h m



In right ΔABC, $\tan 30° = \frac{AB}{BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{BC}$$

$$\Rightarrow \quad BC = 50\sqrt{3} \text{ m}$$

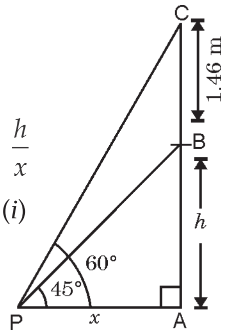
In right ΔDCB;

$$\tan 60° = \frac{DC}{BC} \Rightarrow \sqrt{3} = \frac{h}{50\sqrt{3}}$$

$$\Rightarrow \quad h = 50 \times 3 = 150 \text{ m.}$$

OR

In the adjoining figure, AB is the pedestal, BC is the statue and P is the point of observation.



Let AB = h and AP = x

In right ΔPAB,

$$\tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x}$$

$$\Rightarrow \quad x = h \quad \ldots(i)$$

In right ΔPAC,

$$\tan 60° = \frac{h + 1.46}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h + 1.46}{h} \quad \text{[Using } (i)\text{]}$$

$$\Rightarrow \quad \sqrt{3}h = h + 1.46$$

$$\Rightarrow \quad (\sqrt{3} - 1)h = 1.46$$

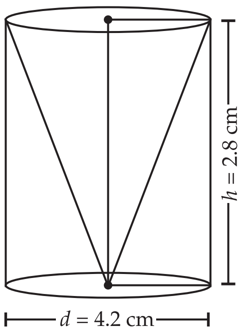
$$\Rightarrow \quad h = \frac{1.46}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 0.73 \times (1.73 + 1)$$

$$= 1.9929$$

Hence, the height of the pedestal is 1.99 m.

28. $r = \frac{4.2}{2} = 2.1$ cm and $h = 2.8$ cm



$$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + (2.8)^2}$$

$$= \sqrt{4.41 + 7.84} = \sqrt{12.25}$$

$$= 3.5 \text{ cm}$$

∴ T.S.A. of remaining solid

= C.S.A. of cylinder
+ Area of circular base
+ C.S.A. of cone

$$= 2\pi rh + \pi r^2 + \pi rl$$

$$= \pi r(2h + r + l)$$

$$= \frac{22}{7} \times 2.1\,(2 \times 2.8 + 2.1 + 3.5)$$

$$= \frac{22}{7} \times \frac{21}{10} \times (5.6 + 2.1 + 3.5)$$

$$= \frac{66}{10} \times 11.2$$

$$= 73.92 \text{ cm}^2.$$

29. Volume of cylindrical tank = $\pi(5)^2 \times 2$

$= 50\pi$ cm^3

∴ Volume of water that flows through pipe in x hours

= Volume of cylinder of radius 10 cm and length (= 4x km) = 4000x m.

$$= \pi \times \left(\frac{1}{10}\text{m}\right)^2 \times 4000x \text{ m} = 40\pi x \text{ m}^3$$

$$\therefore \quad 40\pi x = 50\pi$$

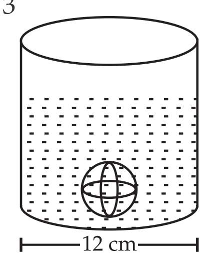
$$\Rightarrow \quad x = \frac{5}{4} \text{ hrs.} = 1 \text{ hr } 15 \text{ min.}$$

OR

Radius of sphere = $\frac{6}{2}$ = 3 cm

Volume of sphere = $(V_1) = \frac{4}{3}\pi(3)^3$

$= 4\pi \times 9 = 36\pi$ cm^3



Let r = radius of circular base of cylinder

$$\therefore \quad r = \frac{12}{2} = 6 \text{ cm.}$$

Let level of water raised by h m

∴ Volume of water raised in cylinder

= Volume of sphere

$$\Rightarrow \quad \pi r^2 h = 36\pi$$

$$\Rightarrow \quad 6 \times 6 \times h = 36$$

$$\Rightarrow \quad h = 1 \text{ cm.}$$

⇒ Water surface will raised by 1 cm.

30. ***See Worksheet 133, Sol. 8***

Practice Paper-4

Section-A

1. HCF = 9

LCM = 360

Let first number $a = 45$

2nd number $= b$

As $a \times b = \text{HCF} \times \text{LCM}$

$\Rightarrow 45 \times b = 9 \times 360$

$\Rightarrow b = \dfrac{9 \times 360}{45}$

$= \dfrac{360}{5} = 72.$

$\therefore b = 72.$

2. $p(x) = ax^2 - 3(a-1) - 1$

$x = 1$ is a zero of $p(x)$

$\Rightarrow p(1) = 0$

$\Rightarrow a(1)^2 - 3(a-1) - 1 = 0$

$\Rightarrow a - 3a + 3 - 1 = 0$

$\Rightarrow -2a + 2 = 0$

$\Rightarrow -2a = -2 \Rightarrow a = 1.$

3. Let A$(x, 7)$ and B$(-1, -5)$ be two points

As AB = 13

$\Rightarrow \text{AB}^2 = 169$

$\Rightarrow (x+1)^2 + (7+5)^2 = 169$

$\Rightarrow (x+1)^2 = 169 - 144 = 25$

$\Rightarrow x + 1 = \pm 5$

$\Rightarrow x + 1 = 5$ or $x + 1 = -5$

$\Rightarrow x = 4$ or $x = -6.$

4. From figure;

As $\text{OT} \perp \text{TP} \Rightarrow \angle\text{OTP} = 90°$

$\therefore$ In ΔOTP:

$\angle\text{TOP} = 180° - (90 + 30°) = 60°$

$\therefore \angle\text{POS} = \angle\text{TOP}$

$\therefore \angle\text{TOS} = \angle\text{TOP} + \angle\text{POS}$

$= 60 + 60 = 120°.$

5. Median $= \dfrac{(x+2)+(x+3)}{2}$

$\Rightarrow 27.5 = \dfrac{2x+5}{2} \Rightarrow 55 = 2x + 5$

$\Rightarrow 2x = 50 \Rightarrow x = 25.$

6. Probability of losing

= 1 – probability of wining

$= 1 - 0.7 = 0.3.$

Section-B

7. Let if possible that $(x + \sqrt{y})$ is rational as x is rational (Given)

$\therefore (x + \sqrt{y}) - x$ should be rational

$\Rightarrow \sqrt{y}$ should be rational

Not possible as it is given that $\sqrt{y}$ is irrational.

Hence $x + \sqrt{y}$ can't be rational

$\Rightarrow x + \sqrt{y}$ is irrational. **Hence proved.**

8. We know $a_n = a + (n-1)d$

$\therefore$ According to question,

$a_2 = 4$ and $a_7 = -11$

$\Rightarrow 4 = a + (2-1)d$

$\Rightarrow 4 = a + d$ ...(*i*)

and $-11 = a + (7-1)d$

$\Rightarrow -11 = a + 6d$ ...(*ii*)

Solving (*i*) and (*ii*), we get

$a = 7; d = -3$

$\therefore a_{16} = a + 15d$

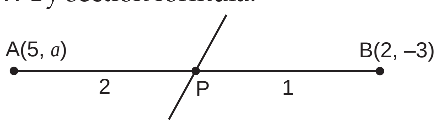
$= 7 + 15(-3) = 7 - 45$

$a_{16} = -38.$

9. As P lies on x-axis, its ordinate is zero.

Let $(x, 0)$ be coordinate of P

$\therefore$ By section formula:



$x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}$ and $y = \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\Rightarrow \quad x = \frac{2(2)+1(5)}{2+1} = 3$ and $y = \frac{2(-3)+1(a)}{2+1}$

$\Rightarrow \quad 0 = -6 + a \Rightarrow a = 6$

$\therefore$ (*i*) $a = 6$

(*ii*) Coordinate of P = (3, 0).

10. $\frac{4}{3} \times \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + \frac{3}{4} \times (\sqrt{3})^2$

$-2 \times (1)^2$

$= \frac{4}{3} \times \frac{1}{3} + \frac{3}{4} - \frac{3}{4} + \frac{3}{4} \times 3 - 2$

$= \frac{4}{9} + \frac{3}{4} - \frac{3}{4} + \frac{9}{4} - 2$

$= \frac{4}{9} + \frac{9}{4} - 2 = \frac{16+81-72}{36} = \frac{25}{36}$.

11. Shaded part = Aarea of square – 4 × Area of circle

$= (14)^2 - 4(\pi r^2)$

$= 196 - 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 196 - 154 = 42 \text{ cm}^2$.

12. (*i*) $\frac{1}{12}$ (*ii*) $\frac{1}{12}$

Section-C

13. Let a be any positive odd integer.

Let 6 be divisor q be quotient and r = remainder

$\therefore$ By Euclid's lemma:

$a = 6q + r, 0 \le r < 6, r \in z$

$\Rightarrow \quad a = 6q + 0$

$a = 6q + 1$

$a = 6q + 2$

$a = 6q + 3$

$a = 6q + 4$

$a = 6q + 5$

as a is odd

$\Rightarrow$ We can take $a = 6q + 1$ or $6q + 3$, or $6q + 5$

$\therefore$ Any positive odd integer is of the form: $6q + 1, 6q + 3, 6q + 5$.

14. Since $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of $f(x)$

$\therefore$ $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are both factors of $f(x)$

$\Rightarrow (x^2 - 5)$ is a factor of $f(x)$

$\therefore$ On dividing $f(x)$ by $x^2 - 5$, we get

$$
\begin{array}{r|l}
x^2 - 5 & 2x^4 - 3x^3 - 9x^2 + 15x - 5 \;\big(\; 2x^2 - 3x + 1 \\
 & \underline{2x^4 \qquad\quad - 10x^2} \\
 & \quad - \qquad\quad + \\
 & -3x^3 + x^2 + 15x - 5 \\
 & \underline{-3x^3 \qquad + 15x} \\
 & \quad + \qquad\quad - \\
 & x^2 - 5 \\
 & \underline{x^2 - 5} \\
 & - \quad + \\
 & 0
\end{array}
$$

$\therefore \quad f(x) = (2x^2 - 3x + 1)(x^2 - 5)$

$= (x - 1)(2x - 1)(x^2 - 5)$

$\therefore$ Zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow \quad x - 1 = 0$ or $2x - 1 = 0$ or $x^2 - 5 = 0$

$\Rightarrow \quad x = 1$ or $x = \frac{1}{2}$ or $x = \pm\sqrt{5}$.

OR

Sum of zeroes $= \frac{1}{2} + 2\sqrt{3} + \frac{1}{2} - 2\sqrt{3}$

$= 1$

Product of zeroes $= \left(\frac{1}{2} + 2\sqrt{3}\right)\left(\frac{1}{2} - 2\sqrt{3}\right)$

$= \left(\frac{1}{2}\right)^2 - (2\sqrt{3})^2$

$= \frac{1}{4} - 12 = -\frac{47}{4}$.

Required polynomial: $p(x) = x^2 -$ (Sum of zeroes) x + Product of zeroes

i.e., $p(x) = x^2 - (1)x + \left(\frac{-47}{4}\right)$ *i.e.*, $p(x) = x^2 - x - \frac{47}{4}$.

15. $S_n = \frac{n}{2}[2a + (n-1).d]$

$S_9 = \frac{9}{2}[2 \times a + (9-1).d]$

$\Rightarrow \quad 351 = \frac{9}{2}(2a + 8d)$

$\Rightarrow \quad 78 = 2a + 8d \qquad \ldots(i)$

Also $\quad S_{20} = \frac{20}{2}[2a + (20 - 1)d]$

$\Rightarrow \quad 1770 = 10(2a + 19d)$

$\Rightarrow \quad 177 = 2a + 19d \quad \ldots(ii)$

Subtracting (*i*) from (*ii*): we get

$99 = 11d \Rightarrow d = 9$

$\therefore$ From (*i*) $\Rightarrow 78 = 2a + 72$

$\Rightarrow \quad 39 = a + 36$

$\Rightarrow \quad a = 3$

OR

$a = 7, d = 11 - 7 = 4.$

Let $\quad a_n = 111$

$\Rightarrow \quad a + (n - 1) \times d = 111$

$\Rightarrow \quad 7 + (n - 1) \times 4 = 111$

$\Rightarrow \quad 7 + 4n - 4 = 111 \Rightarrow 4n = 108$

$\Rightarrow \quad n = 27$

$\therefore \quad S_n = S_{27} = \frac{27}{2}(7 + 111)$

$= \frac{27}{2} \times 118 = 27 \times 59$

$= 1593.$

16. From given equations

$$a_1 = \frac{1}{a};\ b_1 = \frac{1}{b};\ c_1 = 2$$

$$a_2 = a;\quad b_2 = -b;\ c_2 = a^2 - b^2$$

$\therefore$ Using cross-multiplication method:

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{-1}{a_1 b_2 - a_2 b_1}$$

$$\begin{matrix} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{matrix}$$

$$\Rightarrow \quad \begin{matrix} \frac{1}{b} & 2 & \frac{1}{a} & \frac{1}{b} \\ -b & a^2 - b^2 & a & -b \end{matrix}$$

$$\Rightarrow \quad \frac{x}{\left(\frac{a^2 - b^2}{b} + 2b\right)} = \frac{y}{\left(2a - \frac{a^2 - b^2}{a}\right)} = \frac{-1}{-\frac{b}{a} - \frac{a}{b}}$$

$$\Rightarrow \quad \frac{x}{\left(\frac{a^2 + b^2}{b}\right)} = \frac{y}{\left(\frac{b^2 - a^2}{a}\right)} = \frac{-1}{\frac{-(b^2 + a^2)}{ab}}$$

$$\Rightarrow \quad x = \frac{\frac{(a^2 + b^2)}{b}}{+\frac{(b^2 + a^2)}{ab}} = +\frac{ab}{b} = +a$$

and $\quad y = \frac{-\frac{(b^2 + a^2)}{a}}{\frac{(b^2 + a^2)}{ab}} = \frac{ab}{a} = b$

$\therefore \quad x = a;\ y = b.$

OR

Given: $\quad x + y = 7 \quad \ldots(i)$

$12x + 5y = 7 \quad \ldots(ii)$

Taking equation (*i*), we have

$x = 7 - y$

Putting $x = 7 - y$ in equation (*ii*), we get

$12(7 - y) + 5y = 7$

$\Rightarrow \quad 84 - 12y + 5y = 7$

$\Rightarrow \quad 84 - 7y = 7$

$\Rightarrow \quad -7y = 7 - 84$

$\Rightarrow \quad -7y = -77$

$\Rightarrow \quad y = \frac{-77}{-7} \Rightarrow y = 11$

Putting $y = 11$ in equation (*i*), we have

$x + 11 = 7$

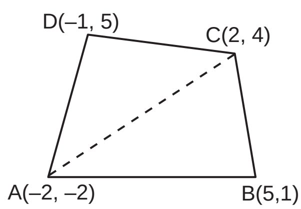
$\Rightarrow \quad x = 7 - 11$

$\Rightarrow \quad x = -4$

Therefore, $x = -4$, $y = 11$ is the required solution.

17. Join AC

ar of quad (ABCD) = *ar* (ΔABC) + *ar*(ΔACD)



We know

$$ar \text{ of } \Delta = \frac{1}{2}\,|\,x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\,|$$

$$\therefore\ ar \text{ of } (\Delta ABC) = \frac{1}{2}\,|-2(1-4) + 5(4+2) + 2(-2-1)\,|$$

$$= \frac{1}{2}(30) = 15 \text{ sq. unit}$$

and ar of (ΔACD)

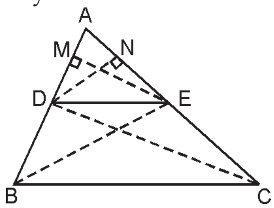
$$= \frac{1}{2}\,|-2(4-5) + 2(5+2) + (-1)(-2-4)\,|$$

$$= \frac{1}{2}(22) = 11 \text{ sq. unit}$$

$\therefore$ Area of quad ABCD = 15 + 11 = 26 sq. unit

18. Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio.

Proof: ABC is a given triangle in which DE || BC. DE intersects AB and AC at D and E respectively.



We have to prove

$$\frac{AB}{BD} = \frac{AE}{CE}$$

Let us draw EM $\perp$ AB and DN $\perp$ AC. Join BE and CD.

Now, $ar(\Delta ADE) = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times AD \times EM \quad ...(i)$$

Also, $ar(\Delta ADE) = \frac{1}{2} \times AE \times DN \quad ...(ii)$

$$ar(\Delta BDE) = \frac{1}{2} \times BD \times EM \quad ...(iii)$$

$$ar(\Delta CDE) = \frac{1}{2} \times CE \times DN \quad ...(iv)$$

Dividing equation (*i*) by equation (*iii*) and equation (*ii*) by equation (*iv*), we have

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{BD} \quad ...(v)$$

and $\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{CE} \quad ...(vi)$

But $ar(\Delta BDE) = ar(\Delta CDE) \quad ...(vii)$

(Triangles are on the same base DE and between the same parallels BC and DE)

Comparing equations (*v*), (*vi*) and (*vii*), we have

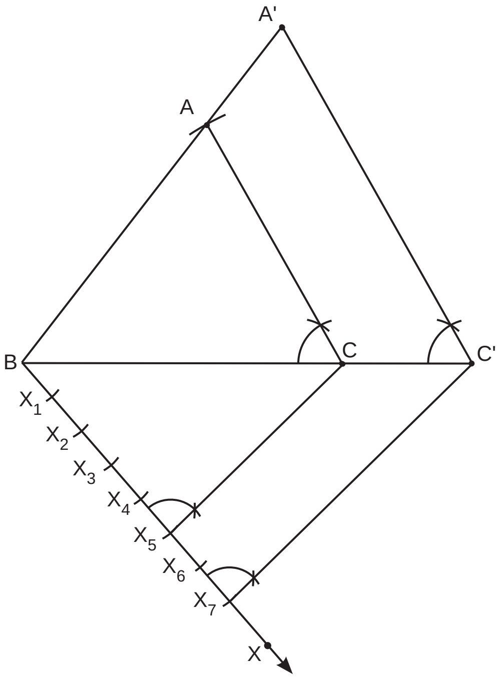
$$\frac{AD}{BD} = \frac{AE}{CE}.$$

19. A′BC′ is required Δ

$$\Delta A'BC' \sim \Delta ABC$$

$$\frac{A'B}{AB} = \frac{BC'}{BC} = \frac{A'C'}{AC}$$

$$= \frac{7}{5}$$



20. Let $h = 10$; assumed mean $= a = 35$

Class interval	Mid-points x_i	No. of persons f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-10	5	100 – 90 = 10	–3	–30
10-20	15	90 – 75 = 15	–2	–30
20-30	25	75 – 50 = 25	–1	–25
30-40	35	50 – 25 =25	0	0
40-50	45	25 – 15 = 10	1	10
50-60	55	15 – 5 = 10	2	20
60-70	65	5 – 0 = 5	3	15
Total		$\Sigma f_i = 100$		$\Sigma f_i u_i = -40$

$$\therefore \quad \text{Mean } \bar{x} = a + h.\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 35 + 10 \times \frac{-40}{100}$$

$$= 35 - 4 = 31$$

$\therefore$ Mean age = 31.

21. $\text{LHS} = \frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta}$

$$= \frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{\cos\theta(1+\cos\theta)}$$

$$= \frac{\sin\theta\cos\theta(1+\cos\theta)+\sin\theta(1-\cos\theta)}{\cos\theta(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{\sin\theta\cos\theta+\sin\theta\cos^2\theta+\sin\theta-\sin\theta\cos\theta}{\cos\theta(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{\sin\theta(\cos^2\theta+1)}{\cos\theta(1-\cos^2\theta)} = \frac{\sin\theta(1+\cos^2\theta)}{\cos\theta.\sin^2\theta}$$

$$= \frac{(1+\cos^2\theta)}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$= \text{cosec}\,\theta \sec\theta + \cot\theta$

= RHS.

OR

$\text{cosec A} = \sqrt{10}$

$$\sin A = \frac{1}{\text{cosec A}} = \frac{1}{\sqrt{10}}$$

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\frac{1}{10}}$$

$$= \frac{3}{\sqrt{10}}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{1}{3}$$

$$\cot A = \frac{1}{\tan A} = 3$$

$$\sec A = \frac{1}{\cos A} = \frac{\sqrt{10}}{3}.$$

22. $\cos(40° + \theta) = \sin\{90° - (40° + \theta)\}$

$= \sin(50° - \theta)$

$\cos 40° = \cos(90° - 50°)$

$= \sin 50°.$

and $\sin 40° = \sin(90° - 50°) = \cos 50°$

Now, given expression

$$= \sin(50° - \theta) - \sin(50° - \theta) + \frac{\sin^2 50° + \cos^2 50°}{\cos^2 50 + \sin^2 50°}$$

$$= 0 + \frac{1}{1} = 1.$$

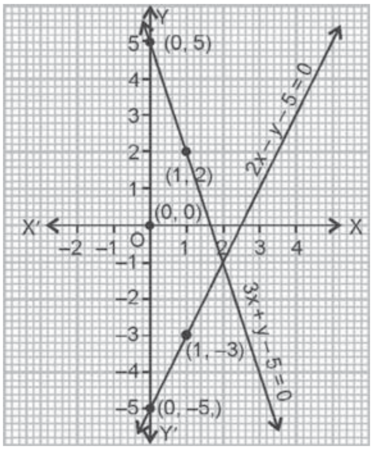
Section-D

23. Table for values of x and y as regarding equation $3x+y-5=0$ is

x	0	1
y	5	2

Similarly table for equation $2x - y - 5 = 0$ is

x	0	1
y	–5	–3



Let us draw the graph of lines using the tables obtained above.

The lines intersect y-axis at (0, 5) and (0, –5).

24. Let original number of children = x

According to question $\dfrac{6500}{x} - \dfrac{6500}{x+15} = 30$

$\Rightarrow \quad x^2 + 15x - 3250 = 0$

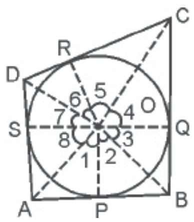
$\Rightarrow \quad (x + 65)(x - 50) = 0$

$\Rightarrow x = 50$ or $x = -65$ not possible

$\therefore$ Number of children are = 50

Moral value: Kindness and generosity.

25. Let the given quadrilateral be ABCD subscribing a circle with centre O. Let the sides AB, BC, CD and DA touch the circle at P, Q, R and S respectively (see figure).



Join OA, OB, OC, OD, OP, OQ, OR and OS.

We need to prove

$\angle AOB + \angle COD = \angle BOC + \angle DOA = 180°$.

Proof: In ΔAOP and ΔAOS,

OP = OS (Radii of same circle)

AP = AS (Tangents from external points)

AO = AO (Common)

$\therefore \quad \Delta AOP \cong \Delta AOS$ (SSS axiom of congruence)

$\therefore \quad \angle 1 = \angle 8$ ...(*i*) (CPCT)

Similarly, we can prove that

$\angle 2 = \angle 3$, $\angle 4 = \angle 5$ and $\angle 6 = \angle 7$ ... (*ii*)

As, $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7$ and $\angle 8$ are subtended at a point

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 2 + \angle 2 + \angle 5 + \angle 5 + \angle 6 + \angle 6 = 360°$

Also, $\angle 8 + \angle 8 + \angle 3 + \angle 3 + \angle 4 + \angle 4 + \angle 7 + \angle 7 = 360°$

[Using results from equations (*i*) and (*ii*)]

$\Rightarrow \quad 2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360°$

Also, $\quad 2(\angle 3 + \angle 4) + 2(\angle 7 + \angle 8) = 360°$

$\Rightarrow \quad 2\angle AOB + 2\angle COD = 360°$

Also, $\quad 2\angle BOC + 2\angle DOA = 360°$

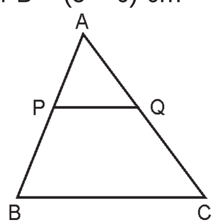
$\Rightarrow \angle AOB + \angle COD = \angle BOC + \angle DOA = 180°$

Hence proved.

OR

We have,

AB = AP + PB = (3 + 6) cm = 9 cm



and $\quad$ AC = AQ + QC = (5 + 10) cm = 15 cm.

$\therefore \quad \dfrac{AP}{AB} = \dfrac{3}{9} = \dfrac{1}{3}$ and $\dfrac{AQ}{AC} = \dfrac{5}{15} = \dfrac{1}{3}$

$$\Rightarrow \quad \frac{AP}{AB} = \frac{AQ}{AC}$$

Thus, in triangles APQ and ABC, we have

$$\frac{AP}{AB} = \frac{AQ}{AC} \text{ and } \angle A = \angle A$$

[Common]

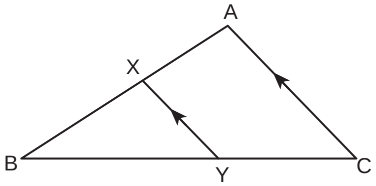
Therefore, by SAS-criterion of similarity, we have

$\Delta APQ \sim \Delta ABC$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC} \Rightarrow \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{PQ}{BC} = \frac{5}{15} \Rightarrow \frac{PQ}{BC} = \frac{1}{3} \Rightarrow BC = 3PQ.$$

26.



Given (*i*) XX || AC

(*ii*) $ar(\Delta BXY)$ = area of trapezium (XYCA)

To find $\frac{AX}{AB}$

As $ar(\Delta XBY) = ar$ of trapezium (AXYC)

$$\Rightarrow \quad ar(\Delta ABC) = ar(\Delta XBY) + ar(AXYC)$$

$$= ar(\Delta XBY) + ar\, XBY$$

$$ar(\Delta ABC) = 2\, ar(\Delta XBY)$$

$$\Rightarrow \quad \frac{ar(\Delta XBY)}{ar(\Delta ABC)} = \frac{1}{2} \qquad \ldots(i)$$

Also clearly $\Delta XBY \sim \Delta ABC$

$$\Rightarrow \quad \frac{ar(\Delta XBY)}{ar(\Delta ABC)} = \frac{XB^2}{AB^2} \Rightarrow \frac{XB^2}{AB^2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{XB}{AB} = \frac{1}{\sqrt{2}} \Rightarrow \frac{AB - AX}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{AB}{AB} - \frac{AX}{AB}$$

$$\Rightarrow \quad \frac{AX}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2-\sqrt{2}}{2}$$

$$\Rightarrow \quad \frac{AX}{AB} = \frac{2-\sqrt{2}}{2}.$$

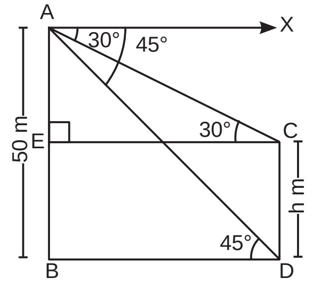
27. Let AB = 50 m

CD = h (height of pole)

$\therefore$ In right ΔABD;

$$\cot 45° = \frac{BD}{AB}$$

$$\Rightarrow \quad 1 = \frac{BD}{50} \Rightarrow BD = 50 \text{ m}$$



Also AE = AB – EB = $(50 - h)$ m

In right $\angle \Delta AEC$; $\tan 30° = \frac{AE}{EC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50-h}{EC}$$

$$\Rightarrow \quad EC = (50-h)\sqrt{3} = BD$$

$$\therefore \quad 50 = (50-h)\sqrt{3}$$

$$\Rightarrow \quad h = \frac{50\sqrt{3}-50}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{50(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

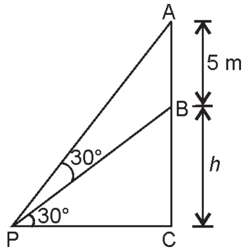
$$\Rightarrow \quad h = \frac{50(3-\sqrt{3})}{3} = \frac{50 \times 1.27}{3}$$

$$= 21.166 \text{ m}$$

OR

Let the tower be BC the flagstaff be AB and the point on the plane be P.

Let BC = h



In right-angled ΔBCP,

$$\tan 30° = \frac{h}{PC}$$

$\Rightarrow \quad PC = h \cot 30° \quad ...(i)$

In right-angled ΔACP,

$$\tan 60° = \frac{5+h}{PC}$$

$\Rightarrow \quad PC = (5+h) \cot 60° \quad ...(ii)$

Comparing equations (*i*) and (*ii*), we have

$h \cot 30° = (5+h) \cot 60°$

$\Rightarrow \quad h\sqrt{3} = (5+h)\frac{1}{\sqrt{3}}$

$\Rightarrow \quad 3h = 5 + h$

$\Rightarrow \quad h = 2.5$

Hence, the height of the tower is 2.5 m.

28. Le r = radius of base and h = height of cylinder

$\therefore r = 4.2$ cm, $h = 12$ cm

TSA = CSA of cylinder + CSA of two hemisphere

$= 2\pi rh + 2 \times (2\pi r^2)$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 4.2\,(12 + 2 \times 4.2)$

$= 538.56 \text{ cm}^2$

Volume of article = Volume of cylinder – Volume of two hemisphere

$= \pi r^2 h - 2 \times \left(\frac{2}{3}\pi r^3\right)$

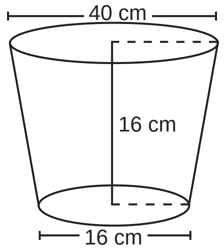
$= \pi r^2\left(h - \frac{4}{3}r\right)$

$= \frac{22}{7} \times (4.2)^2\left(12 - \frac{4}{3} \times 4.2\right)$

$= 354.816 \text{ cm}^3$.

29. Let r = radius of lower end

R = radius of upper end



$\therefore \quad r = \frac{16}{2} = 8$ cm

and $R = \frac{40}{2} = 20$ cm.

Let h = vertical height of frustum

and l = slant height of frustum

$\therefore \quad h = 16$ cm and

$l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{16^2 + (20 - 8)^2}$

$= \sqrt{256 + 144} = \sqrt{400}$

$= 20$ cm

$\therefore$ Volume of bucket

$= \frac{1}{3}\pi h(R^2 + r^2 + rR)$

$= \frac{1}{3} \times 3.14 \times 16(20^2 + 8^2 + 160)$

$= 10449.92 \text{ cm}^3$

and surface area of bucket

$= \pi l(r + R) + \pi r^2$

$= \pi[l(r + R) + r^2]$

$= 3.14[20(8 + 20) + 8^2]$

$= 1959.36 \text{ cm}^2$

$\therefore$ Cost of metal sheet used = ` $\frac{1959.36 \times 20}{100}$

= ` 391.87

OR

Let r = radius of smaller sphere = 3 cm

and R = radius of new sphere formed

$\therefore \quad V_1$ = Volume of smaller sphere

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

V_2 = Volume of bigger sphere

as density of metal = $\frac{\text{mass}}{\text{volume}}$

$\therefore \quad = \frac{1}{36\pi}$

$\Rightarrow$ Volume of bigger sphere × density = Mass

$$= \frac{\text{Mass}}{\text{density}} = \frac{7}{\left(\frac{1}{36}\pi\right)}$$

$\therefore \quad V_2 = 252\pi \text{ cm}^3$

Now, let V = Volume of new sphere

$\therefore \quad V = V_1 + V_2$

$= 36\pi + 252\pi$

$$\frac{4}{3}\pi R^3 = 288\pi$$

$$\Rightarrow \quad R^3 = \frac{288 \times 3}{4} = 72 \times 3 = 216$$

$\Rightarrow \quad R = 6 \text{ cm}$

$\therefore$ Diameter = 12 cm.

30. Let us rewrite the table with class intervals.

Class interval	*f*	*cf*
36-38	0	0
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35
	N = 35	

We mark the upper class limits on *x*-axis and cumulative frequencies on *y*-axis with a suitable scale.

We plot the points (38, 0); (40, 3); (42, 5); (44, 9); (46, 14); (48, 28); (50, 32) and (52, 35).

These points are joined by a free hand smooth curve to obtain a less than type ogive as shown in the given figure.

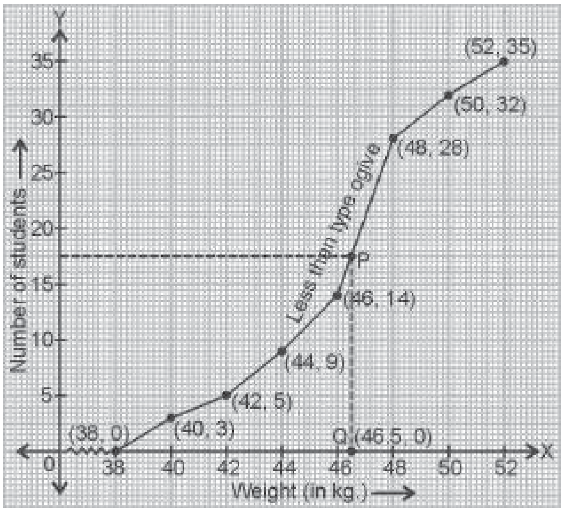


Fig.: *Less than* type ogive

To obtain median from the graph:

We first locate the point corresponding to $\frac{N}{2} = \frac{35}{2}$ = 17.5 students on the *y*-axis.

From this point, draw a line parallel to the *x*-axis to cut the curve at P. From the point P, draw a perpendicular PQ on the *x*-axis to meet it at Q. The *x*-coordinate of Q is 46.5. Hence, the median is 46.5 kg.

Let us verify this median using the formula.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$$

$$= 46 + \frac{7}{14}$$

$$= 46 + 0.5$$

$$= 46.5 \text{ kg.}$$

Thus, the median is the same in both methods.

Practice Paper-5

Section-A

1. LCM $(p, q) = x^3 y^2 z^3$.

2. Let one zero be α, then the other one will be $\frac{1}{\alpha}$.

So, $$\alpha \cdot \frac{1}{\alpha} = \frac{4a}{a^2+4}$$

$\Rightarrow$ $a^2 - 4a + 4 = 0$

$\Rightarrow$ $(a-2)^2 = 0$

$\Rightarrow$ $a = 2$.

3. Given vertices are:

A(– 3, 0), B(5, – 2) and C(– 8, 5).

We know that centroid G is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$\therefore$ Centroid $= \left(\frac{-3+5-8}{3}, \frac{0-2+5}{3}\right)$

$= (-2, 1)$.

4. *Try yourself*

5. Let us rewrite the given distribution in the other manner

Marks	No. of students
0-10	3
10-20	9
20-30	15
30-40	30
40-50	18
50-60	5

Clearly, the modal class is 30-40.

6. Total balls = 5 + 8 + 4 + 7 = 24

Let G = getting a green ball

Total green ball = 4.

$\therefore$ $P(G) = \frac{4}{24} = \frac{1}{6}$

$\therefore$ $P(\text{not } G) = 1 - \frac{1}{6} = \frac{5}{6}$.

Section-B

7. $7 \times 5 \times 3 \times 2 + 3 = 3\,(7 \times 5 \times 2 + 1)$

$= 3\,(70 + 1) = 3 \times 71$.

This is the product of primes.

By Fundamental theorem of arithmetic, every composite number can be expressed as the product of primes.

$\therefore$ $7 \times 5 \times 3 \times 2 + 3$ is a composite number.

8. nth term is: $a_n = 3 + 2n$

This is an A.P.

First term $= a_1 = 3 + 2(1) = 5$

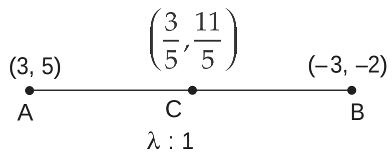
24th term $= a_{24} = 3 + 2(24) = 51$

Now, sum of first 24 terms:

$$S_{24} = \frac{24}{2}(a_1 + a_{24}) = 12 \times (5 + 51)$$

$= 12 \times 56 = 672$.

9. Let the ratio be $\lambda : 1$.



Let use section formula.

$$\frac{3}{5} = \frac{1 \times 3 + \lambda(-3)}{\lambda + 1}$$

$\Rightarrow$ $3\lambda + 3 = 15 - 15\lambda$

$\Rightarrow$ $18\lambda = 12 \Rightarrow \lambda = \frac{2}{3}$

$\therefore$ $\lambda : 1 = 2 : 3$

Thus, the required ratio is 2 : 3.

10. $\cot\theta = \frac{15}{8}$ [Given]

Now, $$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$$

$$= \frac{2(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)\,2(1-\cos\theta)}$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta = \left(\frac{15}{8}\right)^2 = \frac{225}{64}.$$

11. ACB is the minor segment of a circle with centre O and radius OA = OB = r = 14 m

Join AB.

In ΔOAB,

OA = OB (Radii of same circle)

$\Rightarrow$ $\angle 2 = \angle 1$ ...(i)

(Angles opposite to equal sides)

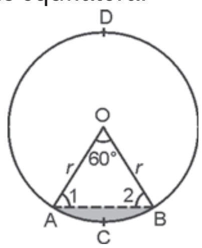
$\therefore$ $\angle 1 + \angle 2 + 60° = 180°$

(Angle sum property for a triangle)

$\Rightarrow$ $\angle 1 + \angle 1 + 60° = 180°$ [Using (i)]

$\Rightarrow$ $\angle 1 = \angle 2 = 60°$ [Using (i)]

$\Rightarrow$ ΔAOB is equilateral



$\therefore$ $ar(\Delta AOB)\ \frac{\sqrt{3}}{4}r^2 = \frac{\sqrt{3}}{4} \times 14^2$

$= 49\sqrt{3}$ m^2

ar(sector AOBC) $= \pi r^2 \times \frac{60°}{360°}$

$= \frac{22}{7} \times 14 \times 14 \times \frac{1}{6}$

$= \frac{308}{3}$ m^2

Now, ar(segment ACB)

$= ar$(sector AOBC) – $ar(\Delta AOB)$

$= \left(\frac{308}{3} - 49\sqrt{3}\right)\text{m}^2$

12. Sample space is:

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

$\therefore$ $n(s) = 8$

(i) Let E = getting at least 2 heads

= {THH, HTH, HHT, HHH}

$\therefore$ $n(E) = 4$

$\therefore$ $P(E) = \frac{4}{8} = \frac{1}{2}$.

(ii) Let F = getting at most 2 heads

$\therefore$ F = {TTT, TTH, THT, HTT, HHT, HTH, THH}

$\therefore$ $n(F) = 7$ $\therefore$ $P(F) = \frac{7}{8}$.

Section-C

13. Let if possible that $\sqrt{5} - 3\sqrt{2}$ is rational number

and let $\sqrt{5} - 3\sqrt{2} = x$; where x is rational

$\Rightarrow$ $\sqrt{5} = x + 3\sqrt{2}$

Squaring both sides, we get

$\Rightarrow$ $5 = x^2 + 18 + 6x\sqrt{2}$

$\Rightarrow$ $-6x\sqrt{2} = x^2 + 18 - 5$

$\Rightarrow$ $\sqrt{2} = \frac{x^2 + 13}{-6x}$ ...(i)

Clearly RHS of (i) above is rational number but $\sqrt{2}$ is irrational so our assumption leads to a contradiction.

Hence $\sqrt{5} - 3\sqrt{2}$ is irrational.

14. According to the division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow$ $x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

(As given in question)

$\Rightarrow$ $g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$

To find $g(x)$, we proceed as given below.

$$\begin{array}{r|l} & x^2 - x + 1 \\ x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & \underline{- \quad +} \\ & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & \underline{+ \quad -} \\ & x - 2 \\ & x - 2 \\ & \underline{- \quad +} \\ & 0 \end{array}$$

Thus, $g(x) = x^2 - x + 1$.

15. Let the first term and common difference of first A.P. be A and D respectively and that of the second A.P. be a and d respectively.

$$\frac{\frac{n}{2}[2A+(n-1)D]}{\frac{n}{2}[2a+(n-1)d]} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{2A+(n-1)D}{2a+(n-1)d} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{A+\left(\frac{n-1}{2}\right)D}{a+\left(\frac{n-1}{2}\right)d} = \frac{7n+1}{4n+27}$$

To get 5th term of an A.P. the coefficient of common difference should be 4

$\therefore$ we should put $\frac{n-1}{2} = 4$, *i.e.*, $n = 9$.

Therefore, $$\frac{A+4D}{a+4d} = \frac{7\times 9+1}{4\times 9+27}$$

$$\Rightarrow \quad \frac{A_5}{a_5} = \frac{64}{63}$$

Hence, the required ratio is 64 : 63.

OR

Let the first term be a and the common difference be d.

A.P. = a, $a + d$, $a + 2d$,........

According to question,

$T_3 = 16$ and $T_7 = 12 + T_5$

$\Rightarrow$ $a + 2d = 16$ and $a + 6d = 12 + a + 4d$

$\Rightarrow$ $a + 2d = 16$ and $d = 6$

$\Rightarrow$ $a = 4$ and $d = 6$

So, the required A.P. will be 4, 10, 16, ...

16. Rewriting the given system of linear equations, we have

$2ax + 3by - (a + 2b) = 0$

$3ax + 2by - (2a + b) = 0$

By cross-multiplication,

$$\frac{x}{-3b(2a+b)+2b(a+2b)} = \frac{-y}{-2a(2a+b)+3a(a+2b)} = \frac{1}{2a\times 2b - 3a\times 3b}$$

$$\Rightarrow \frac{x}{-6ab-3b^2+2ab+4b^2} = \frac{-y}{-4a^2-2ab+3a^2+6ab} = \frac{1}{4ab-9ab}$$

$$\Rightarrow \quad \underset{(i)}{\frac{x}{b^2-4ab}} = \underset{(ii)}{\frac{-y}{4ab-a^2}} = \underset{(iii)}{\frac{1}{-5ab}}$$

Taking (i) and (iii), we get

$$x = \frac{4ab-b^2}{5ab} = \frac{4a-b}{5a}$$

Taking (ii) and (iii), we get

$$y = \frac{4ab-a^2}{5ab} = \frac{4b-a}{5b}.$$

OR

The given system of equations can be written as

$ax + by - (a - b) = 0;$

$a_1 = a,\ b_1 = b;\ c_1 = -(a-b)$

$bx - ay - (a + b) = 0;$

$a_2 = b,\ b_2 = -a;\ c_2 = -(a+b)$

By cross-multiplication,

$$\underset{(i)}{\frac{x}{-b(a+b)-a(a-b)}} = \underset{(ii)}{\frac{-y}{-a(a+b)+b(a-b)}} = \underset{(iii)}{\frac{1}{-a^2-b^2}}$$

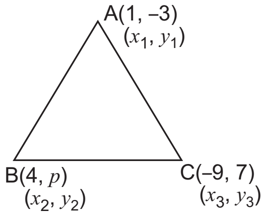
Taking (i) and (iii), we get

$$x = \frac{-ab-b^2-a^2+ab}{-a^2-b^2} = 1$$

Taking (ii) and (iii), we get

$$y = \frac{-a^2-ab+ab-b^2}{-a^2-b^2} = -1$$

Hence $x = 1$, $y = -1$ is the solution of the given system of equations.

17. 

Area of ΔABC

$= \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$

$\Rightarrow 15 = \frac{1}{2}[1(p - 7) + 4(7 + 3) + (-9)(-3 - p)]$

$30 = [p - 7 + 40 + 27 + 9p]$

$\Rightarrow \quad 30 = [10p + 60] \Rightarrow 10p = -30$

$\Rightarrow \quad p = -3.$

OR

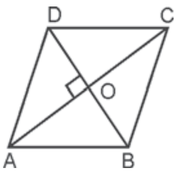
Area of triangle

$= \left|\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$

$= \left|\frac{1}{2}\left[2(0 + 4) - 1(-4 - 3) + 2(3 - 0)\right]\right|$

$= \frac{1}{2}[8 + 7 + 6] = \frac{21}{2}$ sq. units.

18. Let ABCD be a rhombus



Since, diagonals of a rhombus bisect each other at right angles,

$\therefore \quad AO = CO,$

$BO = DO,$

$\angle AOD = \angle DOC$

$= \angle COB = \angle BOA = 90°$

Now, in ΔAOD

$AD^2 = AO^2 + OD^2 \quad ...(i)$

Similarly, $DC^2 = DO^2 + OC^2 \quad ...(ii)$

$CB^2 = CO^2 + BO^2 \quad ...(iii)$

and $BA^2 = BO^2 + AO^2 \quad ...(iv)$

Adding equations (*i*), (*ii*), (*iii*) and (*iv*), we have

$AD^2 + DC^2 + CB^2 + BA^2$

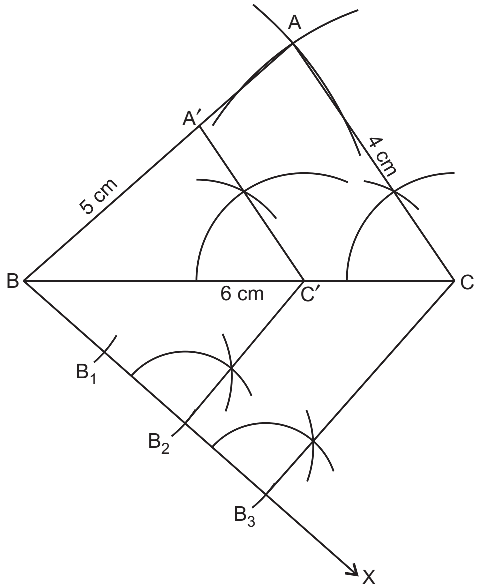
$= 2(DO^2 + CO^2 + BO^2 + AO^2)$

$= 2\left(\frac{BD^2}{4} + \frac{AC^2}{4} + \frac{BD^2}{4} + \frac{CA^2}{4}\right)$

$= BD^2 + CA^2.$ **Hence proved**

19. Steps of construction:

1. Draw a ΔABC in which AB = 5 cm, BC = 6 cm and AC = 4 cm.
2. Make any acute ∠CBX.
3. With suitable distances divide $BB_1 = B_1B_2 = B_2B_3$.
4. Join B_2C.
5. Draw B_2C' || to B_3C.
6. Draw $C'A'$|| to CA.



7. ΔA'B'C' is the required triangle whose side is $\frac{2}{3}$ of the corresponding sides of given ΔABC.

20.

C.I.	Frequency (f_i)	Mid value (x_i)	f_ix_i
0-10	2	5	10
10-20	3	15	45

20-30	5	25	125
30-40	3	35	105
40-50	p	45	$45p$
	$\Sigma f_i = 13 + p$		$\Sigma f_i x_i = 285 + 45p$

Mean $\bar{x} = 25$ [Given]

Using the formula:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\Rightarrow \quad 25 = \frac{285 + 45p}{13 + p}$$

$$\Rightarrow \quad 25(13 + p) = 285 + 45p$$

$$\Rightarrow \quad 325 + 25p = 285 + 45p$$

$$\Rightarrow \quad 325 - 285 = 45p - 25p$$

$$\Rightarrow \quad 40 = 20p \Rightarrow p = \frac{40}{20}$$

$$\Rightarrow \quad p = 2.$$

21. $7\sin^2\theta + 3(1 - \sin^2\theta) = 4$

Let $\sin\theta = x$

$\therefore \quad 7x^2 + 3 - 3x^2 = 4$

$$\Rightarrow 4x^2 = 1 \Rightarrow x^2 = \frac{1}{4}$$

$$\Rightarrow \quad x = \pm\frac{1}{2}$$

$$\therefore \quad \sin\theta = \frac{1}{2}$$

or $$\sin\theta = \frac{-1}{2}$$

$\sin\theta = -\frac{1}{2}$ is not possible as θ is acute.

$$\Rightarrow \quad \operatorname{cosec}\theta = 2$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \sec\theta + \operatorname{cosec}\theta = \frac{2}{\sqrt{3}} + 2.$$

Hence proved

OR

$$\text{LHS} = \frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

$$= \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)} = \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1}$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \text{RHS}$$

22. Given expression

$= 8\sqrt{3}\ \operatorname{cosec}^2 30°.\ \sin 60°.\ \cos 60°.\ \cos^2 45°.$

$\sin 45°.\ \tan 30°.\ \operatorname{cosec}^3 45°.$

$$= 8\sqrt{3} \times \frac{1}{\sin^2 30°}.\ \sin(90° - 30°).$$

$$\cos(90° - 30°)\ \cos^2(90° - 45°).\ \sin 45°.$$

$$\frac{\sin 30°}{\cos 30°}.\frac{1}{\sin^3 45°}$$

$$= 8\sqrt{3} \times \frac{1}{\sin^2 30°} \times \cos 30°.\ \sin 30°.\ \sin^2 45°.$$

$$\sin 45°.\ \frac{\sin 30°}{\cos 30°} \times \frac{1}{\sin^3 45°}$$

= [illegible] $\times \left(\frac{\sin 30°.\ \sin 30°}{\sin^2 30°}\right) \times \frac{\cos 30°}{\cos 30°}$

$$\times \frac{\sin^2 45° \sin 45°}{\sin^3 45°}$$

$$= 8\sqrt{3} \times 1 \times 1 \times 1 = 8\sqrt{3}.$$

Section-D

23. Tables for equations $3x + y - 11 = 0$ and $x - y - 1 = 0$ are respectively

x	3	4
y	2	–1

and

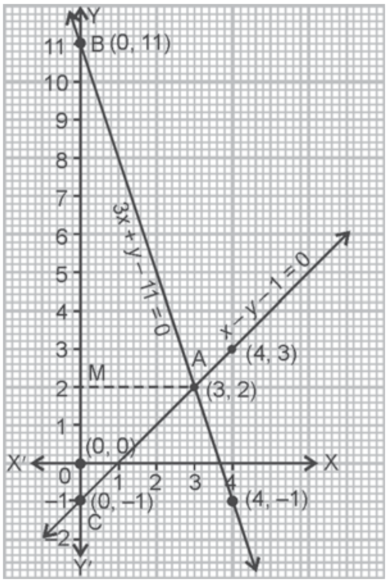
x	0	4
y	–1	3

Let us draw the graph.

From the graph, it is clear that the lines intersect each other at a point A(3, 2). So the solution is $x = 3$, $y = 2$.

The line $3x + y - 11 = 0$ intersects the y-axis at B(0, 11) and the line $x - y - 1 = 0$ intersects the

y-axis at C (0, – 1). Draw the perpendicular AM from A on the y-axis to intersect it at M.



Now, in ΔABC,

base BC = 11 + 1 = 12 units,

height AM = 3 units.

$$\therefore \quad ar(\Delta ABC) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 12 \times 3$$

$$= 18 \text{ sq. units}$$

Hence, $x = 3$, $y = 2$; area = 18 sq. units..

24. Let the time taken to fill the tank by only the larger tap be x hours.

Then the same time by the smaller tap = $(x + 10)$ hours.

The part of the tank filled by only the larger tap in 1 hour = $\frac{1}{x}$

The part of the tank filled by only the smaller tap in 1 hour = $\frac{1}{x+10}$

So the part of the tank filled by both the taps together in 1 hour = $\frac{1}{x} + \frac{1}{x+10}$

But this is given to be $\frac{1}{9\frac{3}{8}}$, *i.e.*, $\frac{8}{75}$

Therefore, $\frac{1}{x} + \frac{1}{x+10} = \frac{8}{75}$

$$\Rightarrow \quad \frac{x+10+x}{x(x+10)} = \frac{8}{75}$$

$$\Rightarrow \quad 75(2x + 10) = 8x^2 + 80x$$

$$\Rightarrow \quad 8x^2 - 70x - 750 = 0$$

$$\Rightarrow \quad 4x^2 - 35x - 375 = 0$$

$$\Rightarrow \quad 4x^2 - 60x + 25x - 375 = 0$$

$$\Rightarrow \quad 4x(x - 15) + 25(x - 15) = 0$$

$$\Rightarrow \quad (4x + 25)(x - 15) = 0$$

$$\Rightarrow \quad x = -\frac{25}{4} \text{ or } x = 15$$

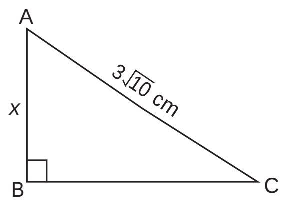
$x \neq -\frac{25}{4}$ as time cannot be negative

$\therefore$ $x = 15$ hours

$\therefore$ $x + 10 = 25$ hours.

Thus, the larger tap and the smaller tap can fill the tank in 15 hours and 25 hours respectively.

OR



Consider ΔABC; $\angle$B = 90°

$\therefore$ Hypotenuse AC = $3\sqrt{10}$ cm

Let smaller side = x = AB

and longer leg = BC.

$\therefore$ As $\text{Hyp}^2 = AB^2 + BC^2$

$$\Rightarrow \quad \left(3\sqrt{10}\right)^2 = x^2 + BC^2$$

$$\Rightarrow \quad BC^2 = 90 - x^2$$

$$BC = \sqrt{90 - x^2}$$

$\therefore$ longer leg = $\sqrt{90 - x^2}$

Now, new smaller leg = $3x$

and new longer leg = $2\sqrt{90 - x^2}$

New hypotenuse = $9\sqrt{5}$ cm

∴ Again using Pythagoras theorem, we get

$$(9\sqrt{5})^2 = (3x)^2 + \left(2\sqrt{90 - x^2}\right)^2$$

$\Rightarrow$ $405 = 9x^2 + 4(90 - x^2)$

$\Rightarrow$ $405 = 9x^2 + 360 - 4x^2$

$\Rightarrow$ $5x^2 = 45$

$\Rightarrow$ $x^2 = 9$

$\Rightarrow$ $x^2 - 9 = 0$

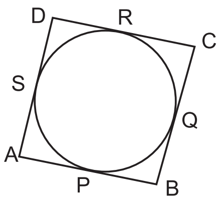
$\Rightarrow$ $(x - 3)(x + 3) = 0$

$\Rightarrow$ $x = 3$ or $x = -3$

∴ Smaller leg = 3 cm

longer leg $\sqrt{90 - 9} = \sqrt{81} = 9$ cm.

25. Let the given parallelogram be ABCD whose sides touches a circles at P, Q, R and S as shown in the adjoining figure.



Since, length of two tangents drawn from an external point to a circle are equal.

∴ AP = AS ...(*i*)

Similarly, we have

PB = BQ ...(*ii*)

DR = SD ...(*iii*)

RC = QC ...(*iv*)

Adding these four equations, we have

AP + PB + DR + RC = AS + BQ + SD + QC

$\Rightarrow$ (AP + PB) + (DR + RC) = (AS + SD) + (BQ + QC)

$\Rightarrow$ AB + DC = AD + BC

∵ AB = DC and AD = BC (ABCD is a parallelogram)

∴ AB = BC

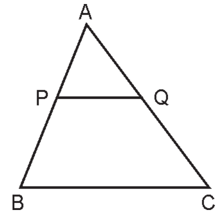
Thus, AB = BC = CD = DA

Hence, ABCD is a rhombus.

OR

We have,

AB = AP + PB = (3 + 6) cm = 9 cm



and AC = AQ + QC = (5 + 10) cm = 15 cm.

$$\therefore \frac{AP}{AB} = \frac{3}{9} = \frac{1}{3} \text{ and } \frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$$

Thus, in triangles APQ and ABC, we have

$$\frac{AP}{AB} = \frac{AQ}{AC} \text{ and } \angle A = \angle A$$

[Common]

Therefore, by SAS-criterion of similarity, we have

$\Delta APQ \sim \Delta ABC$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC} \Rightarrow \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{PQ}{BC} = \frac{5}{15} \Rightarrow \frac{PQ}{BC} = \frac{1}{3} \Rightarrow BC = 3PQ.$$

26. Statement: In a triangle, if square of the largest side is equal to the sum of the squares of the other two sides, then the angle opposite to the largest side is a right angle.

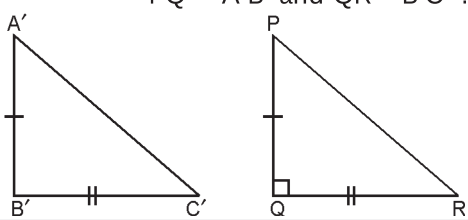
Proof: We are given a triangle A' B' C' with

$A'C'^2 = A'B'^2 + B'C'^2$ ...(*i*)

We have to prove that $\angle B' = 90°$

Let us construct a ΔPQR with $\angle Q = 90°$ such that

PQ = A′B′ and QR = B′C′ ...(*ii*)



In ΔPQR,

$$PR^2 = PQ^2 + QR^2 \quad \text{(Pythagoras Theorem)}$$

$$= A'B'^2 + B'C'^2 \quad ...(iii) \quad \text{[From (ii)]}$$

But $A'C'^2 = A'B'^2 + B'C'^2$ ...(*iv*) [From (*i*)]

From equations (*iii*) and (*iv*), we have

$$PR^2 = A'C'^2$$

$\Rightarrow$ $PR = A'C'$ ...(*v*)

Now, in $\Delta A'B'C'$ and ΔPQR,

$A'B' = PQ$ [From (*ii*)]

$B'C' = QR$ [From (*ii*)]

$A'C' = PR$ [From (*v*)]

Therefore, $\Delta A'B'C' \cong \Delta PQR$ (SSS congruence rule)

$\Rightarrow$ $\angle B' = \angle Q$ (CPCT)

But $\angle Q = 90°$

$\therefore$ $\angle B' = 90°$.

Hence proved.

2nd Part

In ΔADC, $\angle D = 90°$

$$\therefore AC^2 = AD^2 + DC^2 = 6^2 + 8^2 = 36 + 64 = 100$$

In ΔABC,

$$AB^2 + AC^2 = 24^2 + 100 = 676$$

and $BC^2 = 26^2 = 676$

Clearly, $BC^2 = AB^2 + AC^2$

Hence, by converse of Pythagoras Theorem, in ΔABC,

$$\angle BAC = 90°$$

$\Rightarrow$ ΔABC is a right triangle.

OR

Given: ΔABC in which $AB^2 + BC^2 = AC^2$

To prove: $\angle B = 90°$

Construction: Draw right ΔPQR right angled at Q such that PQ = AB and QR = BC

Proof: In right ΔPQR,

$$PR^2 = PQ^2 + QR^2 \quad \text{[By Pythagoras theorem]}$$

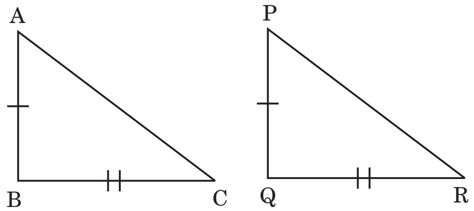
$$= AB^2 + BC^2$$

[$\because$ By construction PQ = AB, QR = BC]

$$PR^2 = AB^2 + BC^2$$

But $AB^2 + BC^2 = AC^2$ [Given]

$\therefore$ $PR^2 = AC^2 \Rightarrow PR = AC$



In ΔABC and ΔPQR,

AB = PQ [By construction]

BC = QR [By construction]

AC = PR [Proved above]

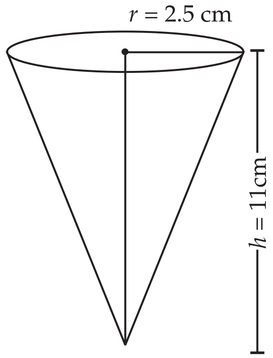
$\therefore$ $\Delta ABC \cong \Delta PQR$ [SSS criterion]

$\therefore$ $\angle B = \angle Q$ [By CPCT]

$\Rightarrow$ $\angle B = 90°$ [$\because \angle Q = 90°$]

27. Volume (V_1) of cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (2.5)^2 \times 11$$



Volume (V_2) of each spherical ball

$$= \frac{4}{3}\pi\left(\frac{0.5}{2}\right)^3$$

Let '*n*' balls be inserted in vessel.

$\therefore$ According to question,

$$n(V_2) = \frac{2}{5} \times V_1$$

$$\Rightarrow \quad n = \frac{2}{5} \times \frac{V_1}{V_2}$$

$$= \frac{2}{5} \times \frac{\frac{1}{3}\pi (2.5)^2 \times 11}{\frac{4}{3}\pi\left(\frac{0.5}{2}\right)^3}$$

$$= \frac{2}{5} \times \frac{2.5 \times 2.5 \times 11 \times 2 \times 2 \times 2}{4 \times 0.5 \times 0.5 \times 0.5}$$

$$= \frac{2 \times 25 \times 25 \times 11 \times 2 \times 10}{5 \times 5 \times 5 \times 5}$$

= 44 × 10 = 440 balls

Value: Love towards nature.

28. It must be noted that the funnel is open from both the sides top and bottom.

Let r_1 = Radius of cylindrical portion

$$= \frac{8}{2} = 4 \text{ cm}$$

and h_1 = Height of cylindrical portion

= 10 cm

Curved surface area of the cylindrical portion

$C_1 = 2\pi rh = 2\pi \times 4 \times 10$

$= 80\pi \text{ cm}^2$

r_2 = Radius of the top of frustum

$$= \frac{18}{2} = 9 \text{ cm}$$

h_2 = Height of frustum = 22 – 10 = 12 cm

l = slant height of the frustum

$$= \sqrt{h_2^2 + (r_2 - r_1)^2}$$

$$= \sqrt{12^2 + (9-4)^2} = \sqrt{169}$$

= 13 cm.

Curved surface area of the frustum

$r_2 = \pi(r_1 + r_2)l.$

$= \pi \times 13 \times 13 = 169\pi \text{ cm}^2.$

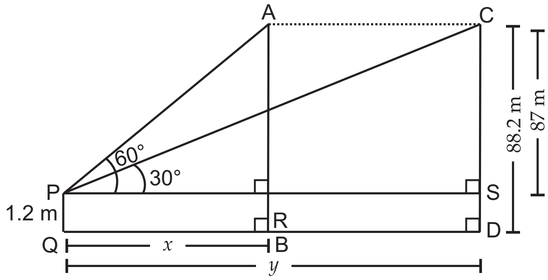
Now, area of the sheet required = $c_1 + c_2$

$= 80\pi + 169\pi = 249\pi$

$$= 249 \times \frac{22}{7} = \frac{5478}{7}$$

$$= 782\frac{4}{7} \text{ cm}^2.$$

29.



Let PQ = height of girl = 1.2 m

and AB = CD = 88.2 m

= height of balloon from ground

∴ AR = CS = 88.2 – 1.2 = 87 m

Let PR = x and PS = y

∴ In right-angled ΔARP,

$$\tan 60° = \frac{AR}{PR}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AR}{x}$$

$$\Rightarrow \quad x = \frac{87}{\sqrt{3}} \text{ m.}$$

Also in right-angled ΔCSP,

$$\tan 30° = \frac{CS}{PS} = \frac{87}{y}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{y}$$

$$\Rightarrow \quad y = 87\sqrt{3} \text{ m}$$

∴ Distance travelled by balloon = $y - x$

$$= 87\sqrt{3} - \frac{87}{\sqrt{3}}$$

$$= 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= 87 \times \left(\frac{3-1}{\sqrt{3}}\right)$$

$$= 87 \times \frac{2}{\sqrt{3}}$$

$$= 58\sqrt{3} \text{ m.}$$

30. *Try yourself*

❑❑